

# Electroproduction of hybrid 1+ mesons

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## Exotic hybrid mesons

## Spectroscopy

## Quark model and meson spectroscopy

- spectroscopy:  $\vec{J} = \vec{L} + \vec{S}$ ; neglecting any spin-orbital interaction  
 $\Rightarrow S, L =$  additional quantum numbers to classify hadron states

$$\vec{J}^2 = J(J+1), \quad \vec{S}^2 = S(S+1), \quad \vec{L}^2 = L(L+1),$$

with  $J = |L - S|, \dots, L + S$

- In the usual quark-model: meson =  $q\bar{q}$  bound state with

$$C = (-)^{L+S} \quad \text{and} \quad P = (-)^{L+1}.$$

- Thus:

$$S = 0, \quad L = J, \quad J = 0, 1, 2, \dots : \quad J^{PC} = 0^{-+}, 1^{+-}, 2^{-+}, 3^{+-}, \dots$$

$$S = 1, \quad L = 0, \quad J = 1 : \quad J^{PC} = 1^{--} \quad \rho \text{ - meson}$$

$$L = 1, \quad J = 0, 1, 2 : \quad J^{PC} = 0^{++}, 1^{++}, 2^{++}$$

$$L = 2, \quad J = 1, 2, 3 : \quad J^{PC} = 1^{--}, 2^{--}, 3^{--}$$

...

- $\Rightarrow$  the exotic mesons with  $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, \dots$  are forbidden

# Exotic hybrid mesons

## Experimental status

### Experimental candidates for light hybrid mesons (1)

three candidates:

- $\pi_1(1400)$ 
  - **GAMS '88 (SPS, CERN)**: in  $\pi^- p \rightarrow \eta \pi^0 n$  (through  $\eta \pi^0 \rightarrow 4\gamma$  mode)  
 $M = 1406 \pm 20 \text{ MeV}$     $\Gamma = 180 \pm 30 \text{ MeV}$
  - **E852 '97 (BNL)**:  $\pi^- p \rightarrow \eta \pi^- p$   
 $M = 1370 \pm 16 \text{ MeV}$     $\Gamma = 385 \pm 40 \text{ MeV}$
  - **VES '01 (Protvino)** in  $\pi^- Be \rightarrow \eta \pi^- Be$ ,  $\pi^- Be \rightarrow \eta' \pi^- Be$ ,  
 $\pi^- Be \rightarrow b_1 \pi^- Be$   
 $M = 1316 \pm 12 \text{ MeV}$     $\Gamma = 287 \pm 25 \text{ MeV}$   
 but resonance hypothesis ambiguous
  - **Crystal Barrel (LEAR, CERN) '98 '99** in  $\bar{p} n \rightarrow \pi^- \pi^0 \eta$  and  $\bar{p} p \rightarrow 2\pi^0 \eta$   
 (through  $\pi\eta$  resonance)  
 $M = 1400 \pm 20 \text{ MeV}$     $\Gamma = 310 \pm 50 \text{ MeV}$   
 and  $M = 1360 \pm 25 \text{ MeV}$     $\Gamma = 220 \pm 90 \text{ MeV}$

## Exotic hybrid mesons

## Experimental status

## Experimental candidates for light hybrid mesons (2)

- $\pi_1(1600)$ 
  - **E852 (BNL)**: in peripheral  $\pi^- p \rightarrow \pi^+ \pi^- \pi^- p$  (through  $\rho\pi^-$  mode) '98 '02,  $M = 1593 \pm 8$  MeV  $\Gamma = 168 \pm 20$  MeV  $\pi^- p \rightarrow \pi^+ \pi^- \pi^- \pi^0 \pi^0 p$  (in  $b_1(1235)\pi^- \rightarrow (\omega\pi^0)\pi^- \rightarrow (\pi^+ \pi^- \pi^0)\pi^0 \pi^-$  '05 and  $f_1(1285)\pi^-$  '04 modes), in peripheral  $\pi^- p$  through  $\eta'\pi^-$  '01  
 $M = 1597 \pm 10$  MeV  $\Gamma = 340 \pm 40$  MeV  
 but **E852 (BNL)** '06: no exotic signal in  $\pi^- p \rightarrow (3\pi)^- p$  for a larger sample of data!
  - **VES '00 (Protvino)**: in peripheral  $\pi^- p$  through  $\eta'\pi^-$  '93, '00,  $\rho(\pi^+ \pi^-)\pi^-$  '00,  $b_1(1235)\pi^- \rightarrow (\omega\pi^0)\pi^-$  '00
  - **Crystal Barrel (LEAR, CERN)** '03  $\bar{p}p \rightarrow b_1(1235)\pi\pi$
  - **COMPASS '10 (SPS, CERN)**: diffractive dissociation of  $\pi^-$  on  $Pb$  target through Primakov effect  $\pi^- \gamma \rightarrow \pi^- \pi^- \pi^+$  (through  $\rho\pi^-$  mode)  
 $M = 1660 \pm 10$  MeV  $\Gamma = 269 \pm 21$  MeV
- $\pi_1(2000)$ : seen only at **E852 (BNL)** '04 '05 (through  $f_1(1285)\pi^-$  and  $b_1(1235)\pi^-$ )

# Exotic hybrid mesons

## Motivation for hard production

### What about hard processes?

- Is there a hope to see such states in **hard processes**, with high counting rates, and to exhibit their light-cone wave-function?
- **hybrid mesons** =  $q\bar{q}g$  states    T. Barnes '77; R. L. Jaffe, K. Johnson, and Z. Ryzak, G. S. Bali
- popular belief:  $q\bar{q}g \Rightarrow$  higher Fock-state component  $\Rightarrow$  twist-3  
 $\Rightarrow$  hard electroproduction suppressed as  $1/Q$
- **This is not true!!** Electroproduction of hybrid is similar to electroproduction of usual  $\rho$ -meson: it is twist 2

# A short journey in collinear factorization

Extensions from DIS

## A short journey in collinear factorization

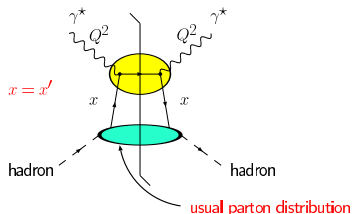
- DIS: inclusive process  $\rightarrow$  forward amplitude ( $t = 0$ )

(DIS: Deep Inelastic Scattering)

ex:  $e^\pm p \rightarrow e^\pm X$  at HERA

Structure Function

$$= \text{Coefficient Function (hard)} \otimes \text{Parton Distribution Function (soft)}$$

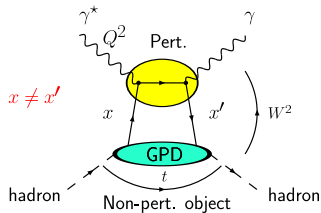


- DVCS: exclusive process  $\rightarrow$  non forward amplitude ( $-t \ll s = W^2$ )

(DVCS: Deep Virtual Compton Scattering)

Amplitude

$$= \text{Coefficient Function (hard)} \otimes \text{Generalized Parton Distribution (soft)}$$

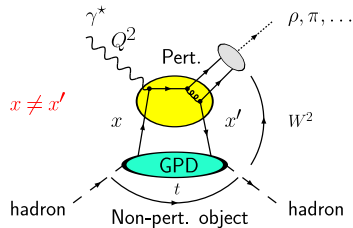


# A short journey in collinear factorization

Extensions from DIS

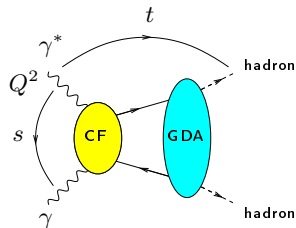
- **Meson production:**  $\gamma$  replaced by  $\rho, \pi, \dots$

$$\text{Amplitude} = \text{GPD (soft)} \otimes \text{CF (hard)} \otimes \text{Distribution Amplitude (soft)}$$



- **Crossed process:**  $s \ll -t$

$$\text{Amplitude} = \text{Coefficient Function (hard)} \otimes \text{Generalized Distribution Amplitude (soft)}$$



Diehl, Gousset, Pire, Teryaev '98

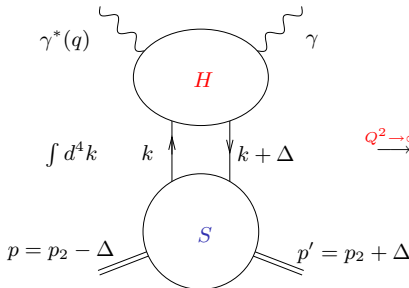


$\rho$ -electroproduction  
DVCS and GPD

Two steps for factorization

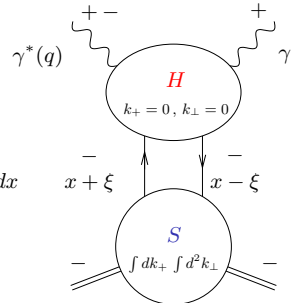
- momentum factorization: use Sudakov decomposition

$$k = \underbrace{\alpha p_1}_+ + \underbrace{\beta p_2}_- + \underbrace{k_\perp}_\perp \quad (p_1^2 = p_2^2 = 0, 2p_1 \cdot p_2 = s)$$



$Q^2 \rightarrow \infty$

$$\int dk^- = p_2^- \int dx$$



$$\int d^4k S(k, k + \Delta) H(q, k, k + \Delta) = \int dk^- \int dk^+ d^2k_\perp S(k, k + \Delta) H(q, k^-, k^- + \Delta^-)$$

- supplement with Fierz identity in spinor + color space

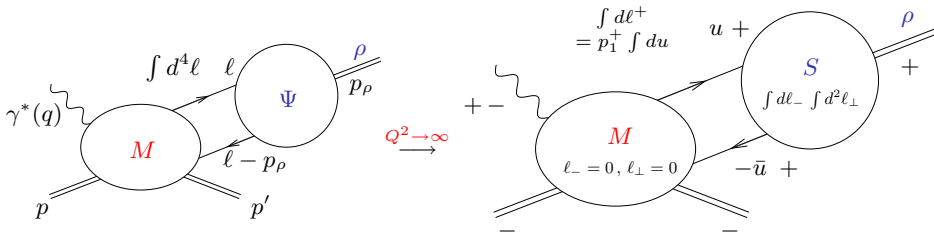
$$\Rightarrow \mathcal{M} = \text{GPD} \otimes \text{Hard part}$$

# $\rho$ -electroproduction

$\rho$ -meson production: from the wave function to the DA

What is a  $\rho$ -meson in QCD?

It is described by its **wave function**  $\Psi$  which reduces in **hard processes** to its **Distribution Amplitude**



$$\int d^4 l M(q, l, l - p_\rho) \Psi(l, l - p_\rho) = \int dl^+ M(q, l^+, l^+ - p_\rho^+) \int_{|k_\perp^2| < \mu_F^2} dl^- \int d^2 l_\perp \Psi(l, l - p_\rho)$$

Hard part

DA  $\Phi(u, \mu_F^2)$

(see Chernyak, Zhitnitsky '77; Brodsky, Lepage '79; Efremov, Radyushkin '80; ... in the case of form-factors studies)

$\rho$ -electroproduction $\rho$ -meson production: factorization with a **GPD** and a **DA**

The diagram illustrates the factorization of  $\rho$ -meson electroproduction into three components: GPD, Hard part, and DA.

**Left Diagram (Initial State):** A photon  $\gamma^*(q)$  interacts with a hard part  $H$  (red circle). The hard part  $H$  is connected to a soft part  $S$  (blue circle) via two lines with momenta  $k$  and  $k + \Delta$ . The soft part  $S$  has incoming momenta  $p = p_2 - \Delta$  and outgoing momenta  $p' = p_2 + \Delta$ . The hard part  $H$  has incoming momenta  $\ell$  and  $\ell - p_\rho$ , and is connected to a  $\rho$ -meson  $\Psi$  (blue circle) with momentum  $p_\rho$ . The hard part  $H$  is integrated over  $\int d^4 \ell$ .

**Right Diagram (Final State):** The hard part  $H$  is now a red circle with internal momenta  $\ell_- = 0, \ell_\perp = 0$  and  $k_+ = 0, k_\perp = 0$ . It is connected to a soft part  $S$  (blue circle) via two lines with momenta  $x + \xi$  and  $x - \xi$ . The soft part  $S$  has incoming momenta  $-$  and  $-$ , and outgoing momenta  $-$  and  $-$ . The soft part  $S$  is integrated over  $\int dk_+ \int d^2 k_\perp$ . The hard part  $H$  is integrated over  $\int du$  and is connected to a  $\rho$ -meson  $\Psi$  (blue circle) with momentum  $u +$ . The hard part  $H$  is also integrated over  $\int d\ell_- \int d^2 \ell_\perp$ .

**Factorization Equation:**

$$\int d^4 k d^4 \ell \quad S(k, k + \Delta) \quad H(q, k, k + \Delta) \quad \Psi(\ell, \ell - p_\rho)$$

$$\xrightarrow{Q^2 \rightarrow \infty} \int dx \quad S(x + \xi, x - \xi) \quad H(x/\xi, u, \mu_{F_1}^2, \mu_{F_2}^2) \quad \Psi(u, \mu_{F_1}^2)$$

$$= \int dk^- d\ell^+ \int dk^+ \int_{|k_\perp^2| < \mu_{F_2}^2} d^2 k_\perp S(k, k + \Delta) H(q; k^-, k^- + \Delta^-; \ell^+, \ell^+ - p_\rho^+) \int d\ell^- \int_{|\ell_\perp^2| < \mu_{F_1}^2} d^2 \ell_\perp \Psi(\ell, \ell - p_\rho)$$

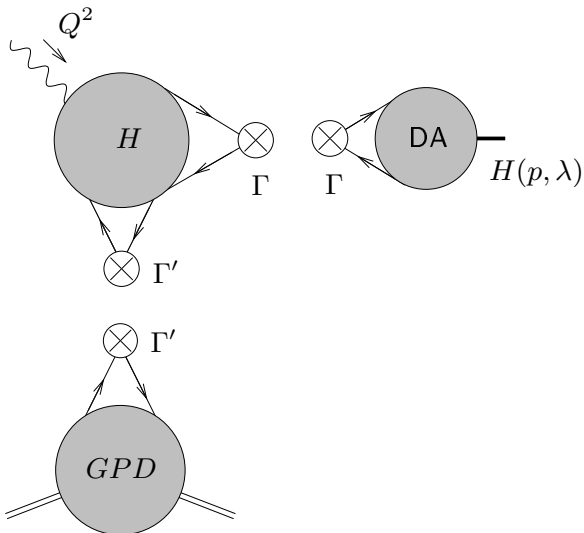
**Labels for the components:**

- GPD**  $F(x, \xi, t, \mu_{F_2}^2)$
- Hard part**  $T(x/\xi, u, \mu_{F_1}^2, \mu_{F_2}^2)$
- DA**  $\Phi(u, \mu_{F_1}^2)$

Collins, Frankfurt, Strikman '97; Radyushkin '97

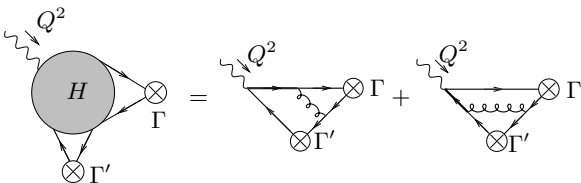
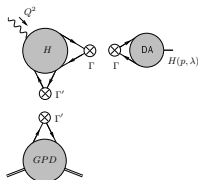
## Factorization for hybrid meson electroproduction

## Factorization framework

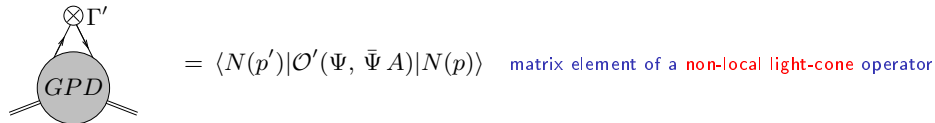
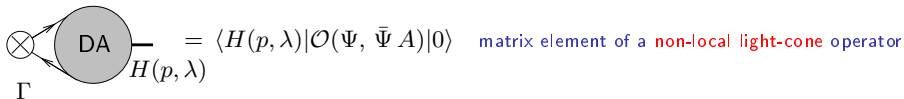


# Factorization for hybrid meson electroproduction

## The building blocks



hand-bag diagrams

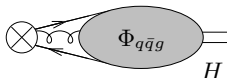


# Hybrid Distribution Amplitude

## Hybrid DA from non-local twist 2 operator

### Distribution amplitude of exotic hybrid mesons at twist 2

- One may think that to produce  $|q\bar{q}g\rangle$ , the fields  $\Psi$ ,  $\bar{\Psi}$ ,  $A$  should appear explicitly in the non-local operator  $\mathcal{O}(\Psi, \bar{\Psi}, A)$



- If one tries to produce  $H = 1^{-+}$  from a **local operator**, the dominant operator should be  $\bar{\Psi}\gamma^\mu G_{\mu\nu}\Psi$  of **twist** = dimension - spin = 5 - 1 = 4
- It means that there should be a  $1/Q^2$  suppression in the production amplitude of  $H$  with respect to usual  $\rho$ -production (which is twist 2)
- But one of the main progress is the understanding of hard exclusive processes in terms of **non-local light-cone operators**, like the **twist 2 operator**

$$\bar{\psi}(-z/2)\gamma_\mu[-z/2; z/2]\psi(z/2)$$

where  $[-z/2; z/2]$  is a **Wilson line** which thus hides gluonic degrees of freedom: **the needed gluon is there, at twist 2**. This does not require to introduce explicitly  $A$ !

# Hybrid Distribution Amplitude

## Hybrid DA from non-local twist 2 operator

### Distribution amplitude and quantum numbers: $C$ -parity

- Define the  $H$  DA as (for long. pol.)

$$\langle H(p, 0) | \bar{\psi}(-z/2) \gamma_\mu [-z/2; z/2] \psi(z/2) | 0 \rangle \Big|_{\substack{z^2=0 \\ z^+=0 \\ z_\perp=0}} = i f_H M_H e_\mu^{(0)} \int_0^1 dy e^{i(\bar{y}-y)p \cdot z/2} \phi_L^H(y)$$

- Inserting  $C$ -parity operator gives **antisymmetric DA for  $H^0$**

$$\phi_L^H(y) = -\phi_L^H(1-y) \quad \text{while the usual } \rho \text{ DA is symmetric}$$

- Expansion in terms of **local operators**


$$\langle H(p, \lambda) | \bar{\psi}(-z/2) \gamma_\mu [-z/2; z/2] \psi(z/2) | 0 \rangle = \sum_n \frac{1}{n!} z_{\mu_1} \dots z_{\mu_n} \langle H(p, \lambda) | \bar{\psi}(0) \gamma_\mu \overleftrightarrow{D}_{\mu_1} \dots \overleftrightarrow{D}_{\mu_n} \psi(0) | 0 \rangle,$$

$$D_\mu = \text{usual covariant derivative and } \overleftrightarrow{D}_\mu = \frac{1}{2}(\overrightarrow{D}_\mu - \overleftarrow{D}_\mu).$$

- Special case  $n = 0$ :

$$\langle H(p, 0) | \psi(0) \gamma_\mu \psi(0) | 0 \rangle = i f_H M_H e_\mu^{(0)} \int_0^1 dy \phi_L^H(y) = 0$$

$$C = (+) \quad C = (-)$$

no surprise: we expect here the  $C = (-)$   $\rho$ -meson 

# Hybrid Distribution Amplitude

## Hybrid DA from non-local twist 2 operator

### Distribution amplitude and quantum numbers: $C$ -parity and $P$ -parity

- the hybrid selects the **odd**-terms

$$\langle H(p, \lambda) | \bar{\psi}(-z/2) \gamma_\mu [-z/2; z/2] \psi(z/2) | 0 \rangle = \sum_{n \text{ odd}} \frac{1}{n!} z_{\mu_1} \dots z_{\mu_n} \langle H(p, \lambda) | \bar{\psi}(0) \gamma_\mu \overleftrightarrow{D}_{\mu_1} \dots \overleftrightarrow{D}_{\mu_n} \psi(0) | 0 \rangle,$$

- Special case  $n = 1$ :

$$\mathcal{R}_{\mu\nu} = S_{(\mu\nu)} \bar{\psi}(0) \gamma_\mu \overleftrightarrow{D}_\nu \psi(0),$$

$S_{(\mu\nu)}$  = symmetrization operator:  $S_{(\mu\nu)} T_{\mu\nu} = \frac{1}{2}(T_{\mu\nu} + T_{\nu\mu})$ .

- Relation with hybrid DA:

$$\langle H(p, \lambda) | \mathcal{R}_{\mu\nu} | 0 \rangle = \frac{1}{2} f_H M_H S_{(\mu\nu)} e_\mu^{(\lambda)} p_\nu \int_0^1 dy (1-2y) \phi^H(y), \quad (1)$$

- $C$ -parity:  $C(R_{\mu\nu}) = +$
- $P$ -parity:  $P(R_{k0}) = -$  ( $\leftarrow$  after going to rest-frame:  $p_i = 0$  and  $e_0 = 0$ )



## Hybrid Distribution Amplitude

## Normalization

## Non perturbative input for the hybrid DA

- We need to fix  $f_H$  (the analogue of  $f_\rho$ )
- This is a non-perturbative input
- Lattice does not yet give information
- The operator  $\mathcal{R}_{\mu\nu}$  is related to quark energy-momentum tensor  $\Theta_{\mu\nu}$  :

$$\mathcal{R}_{\mu\nu} = -i \Theta_{\mu\nu}$$

- Rely on QCD sum rules: resonance for  $M \approx 1.4$  GeV  
I. I. Balitsky, D. Diakonov, and A. V. Yung

$$f_H \approx 50 \text{ MeV}$$

$$f_\rho = 216 \text{ MeV}$$

- Note:  $f_H$  evolves according to the  $\gamma_{QQ}$  anomalous dimension

$$f_H(Q^2) = f_H \left( \frac{\alpha_S(Q^2)}{\alpha_S(M_H^2)} \right)^{K_1} \quad K_1 = \frac{2\gamma_{QQ}(1)}{\beta_0},$$

## Hybrid electroproduction

 $H$  versus  $\rho$ Amplitude for  $H$  versus  $\rho$  electroproduction

- At leading twist 2:

$$\mathcal{A}(\gamma_L^* p \rightarrow H_L^0 p) = \int_0^1 dz \int_{-1}^1 dx \Phi_H(z, \mu_F^2, \mu_R^2) H(x, z, Q^2, \mu_F^2, \mu_R^2) F(x, \mu_F^2, \mu_R^2)$$

$\mu_F^2$  = factorization scale;  $\mu_R^2$  = renormalization scale; we take  $\mu_F = \mu_R$ .

- $C$ -parity:

$$\begin{bmatrix} C_H = (+) & \text{odd DA labeled } M^- \\ C_\rho = (-) & \text{even DA labeled } M^+ \end{bmatrix} \times (C_\gamma = (-)) = \begin{bmatrix} C_{q-\bar{q}} = (-) & \text{even GPD under } x \leftrightarrow -x \\ C_{q+\bar{q}} = (+) & \text{odd GPD under } x \leftrightarrow -x \end{bmatrix}$$

$$\mathcal{A}_{\gamma_L^* p \rightarrow M_L^{(\pm)0} p} = \frac{e\pi\alpha_s f_H C_F}{\sqrt{2} N_c Q} \left[ e_u \mathcal{H}_{uu}^\pm - e_d \mathcal{H}_{dd}^\pm \right] \mathcal{V}^{(M, \pm)},$$

$$\mathcal{H}_{ff}^\pm = \frac{1}{P_-} \int_{-1}^1 dx \left[ \bar{u}(p_2) \gamma^- u(p_1) H_{ff}(x, \xi) + \bar{u}(p_2) \frac{i\sigma_{-\alpha} \Delta^\alpha}{2M} u(p_1) E_{ff}(x, \xi) \right] \left[ \frac{1}{x + \xi - i\epsilon} \pm \frac{1}{x - \xi + i\epsilon} \right]$$

$$\mathcal{V}^{(M, \pm)} = \int_0^1 dy \phi^M(y) \left[ \frac{1}{y} \pm \frac{1}{1-y} \right].$$

- No end-point singularity  $\frac{1}{y}$  since  $\phi^H(y)$  vanishes for  $y \rightarrow 0, 1$

## Hybrid electroproduction

 $H$  versus  $\rho$ Counting rates for  $H$  versus  $\rho$  electroproduction: order of magnitude

- Ratio:

$$\frac{d\sigma^H(Q^2, x_B, t)}{d\sigma^\rho(Q^2, x_B, t)} = \left| \frac{f_H (e_u \mathcal{H}_{uu}^- - e_d \mathcal{H}_{dd}^-) \mathcal{V}^{(H,-)}}{f_\rho (e_u \mathcal{H}_{uu}^+ - e_d \mathcal{H}_{dd}^+) \mathcal{V}^{(\rho,+)}} \right|^2$$

- Rough estimate:

- neglect  $\bar{q}$  i.e.  $x \in [0, 1]$

$\Rightarrow \text{Im}\mathcal{A}_H$  and  $\text{Im}\mathcal{A}_\rho$  are equal up to the factor  $\mathcal{V}^M$

- Neglect the effect of  $\text{Re}\mathcal{A}$

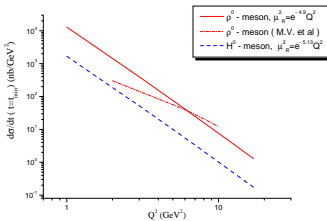
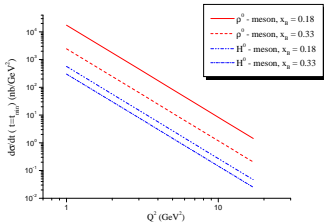
$$\frac{d\sigma^H(Q^2, x_B, t)}{d\sigma^\rho(Q^2, x_B, t)} \approx \left( \frac{5f_H}{3f_\rho} \right)^2 \approx 0.15$$

## Hybrid electroproduction

 $H$  versus  $\rho$ Counting rates for  $H$  versus  $\rho$  electroproduction: more precise study

- use standard description of GPDs based on Double Distributions
- $\mu_R^2 = Q^2$  versus BLM scale from NLO (at the level of cross-section)
 

$\xi = 0.2$	$\mu_R^2 = e^{-4.9} Q^2$	$\rho$	$\xi = 0.1$	$\mu_R^2 = e^{-4.68} Q^2$	$\rho$
(or $x_B \approx 0.33$ )	$\mu_R^2 = e^{-5.13} Q^2$	$H$	(or $x_B \approx 0.18$ )	$\mu_R^2 = e^{-5.0} Q^2$	$H$



$$\mu_R^2 = \mu_F^2 = Q^2$$

$$\mu_R^2 = \mu_F^2 = \mu_{BLM}^2 \quad x_B \approx 0.33$$

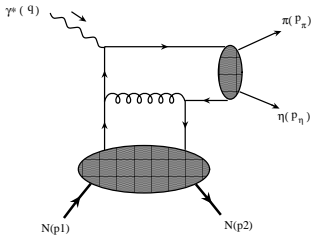
Ratio  $d\sigma^H / d\sigma^\rho$ : rather scale-fixing independent

$x_B$	0.33				0.18			
$Q^2$ (GeV <sup>2</sup> )	3.0	7.0	11.0	17.0	3.0	7.0	11.0	17.0
$\mu_R^2 = Q^2$	0.123	0.123	0.123	0.123	0.0325	0.0326	0.0326	0.0326
$\mu_R^2 = \mu_{BLM}^2$	0.131	0.133	0.133	0.134	0.0356	0.0362	0.0365	0.0367

Hybrid in electroproduction of  $\pi\eta$  pair

## Hard hybrid production study through its decay mode

- We consider here for illustration the  $\pi_1(1400)$
- Dominant decay mode:  $\pi\eta$
- $\pi\eta$  can be  $J^{PC} = 0^{++} (f_0, a_0), 1^{-+} (\pi_1), 2^{++} (a_2)$  for  $L = 0, 1, 2$
- Mass region around 1400 MeV dominated by the strong  $a_2(1329)(2^{++})$  resonance  
 ⇒ look for **interference of the amplitudes for  $H$  and  $a_2$  production**  
 ⇒ identify the hybrid production events if there is no recoil detector
- This study can be performed in the hard factorization framework:  
**Distribution Amplitude  $\phi^H \rightarrow$  Generalized Distribution Amplitude  $\phi^{\pi\eta}$**



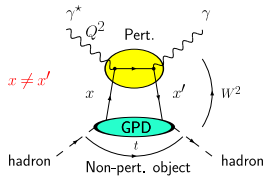
# Hybrid in electroproduction of $\pi\eta$ pair

## $\pi^0\eta$ GDA

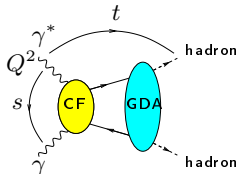
### GDA

- GDA is obtained from GPD by  $s \leftrightarrow t$  crossing

**GPD**:  $s \gg -t$  limit, with  $Q^2 \gg \Lambda_{QCD}^2$



→ **GDA**:  $-t \gg s$  limit, with  $Q^2 \gg \Lambda_{QCD}^2$



- kinematics:  $(x, \xi)$  (GPD)  $\longrightarrow$   $(y, \zeta)$  (GDA)

$$y = \frac{\text{long. momentum of the quark}}{\text{total outgoing hadronic momentum}}$$

$$\zeta = \frac{p_+}{p_+ + p'_+} = \frac{\text{long. momentum of one of the hadron}}{\text{total outgoing hadronic momentum}}$$

- For  $\pi^0\eta$ :  $\langle \pi^0(p_\pi)\eta(p_\eta) | \bar{\psi}_{f_2}(-z/2)\gamma^\mu[-z/2; z/2]\tau_{f_2 f_1}^3 \psi_{f_1}(-z) | 0 \rangle$

$$= p_{\pi\eta}^\mu \int_0^1 dy e^{i(\bar{y}-y)p_{\pi\eta} \cdot z/2} \Phi^{(\pi\eta)}(y, \zeta, m_{\pi\eta}^2) + \dots \quad (\text{only twist 2 part displayed here})$$

Hybrid in electroproduction of  $\pi\eta$  pair $\pi^0\eta$  GDA and polar angle distribution

## From GDA to polar angle distribution

- Two particles of equal mass:  $\zeta \longrightarrow$  two different particles  $\tilde{\zeta}$ :

$$\tilde{\zeta} = \frac{p_\pi^+}{(p_\pi + p_\eta)^+} - \frac{m_\pi^2 - m_\eta^2}{2m_{\pi\eta}^2}, \quad 1 - \tilde{\zeta} = \frac{p_\eta^+}{(p_\pi + p_\eta)^+} + \frac{m_\pi^2 - m_\eta^2}{2m_{\pi\eta}^2}$$

- Relation with the polar angle of the  $\pi$  meson in the  $\pi\eta$  c.m.s.:

$$2\tilde{\zeta} - 1 = \beta \cos \theta_{cm}, \quad \beta = \frac{2|\mathbf{p}_{cms}|}{m_{\pi\eta}}$$

- Expansion of the  $C = (+)$  GDA (now with  $\zeta \longrightarrow \tilde{\zeta}$ ):

$$\Phi^{q(+)}(y, \tilde{\zeta}; \mu^2) = 10z(1-z) \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl}^q(m_{\pi\eta}, \mu^2) C_n^{3/2}(2y-1) P_l(2\tilde{\zeta}-1)$$

- $l < n + 1$  from polynomiality (relation of  $y$ -moments with matrix elements of local operators, constrained by Lorentz invariance)
- as for DA, Legendre polynomials  $C_n^{3/2}$  appear due to conformal invariance: basis for QCD evolution with respect to the scale  $\mu^2$ , like  $x^n$  for PDFs

Hybrid in electroproduction of  $\pi\eta$  pair $\pi^0\eta$  GDA and polar angle distribution

## From GDA to polar angle distribution: model

- We consider the asymptotical limit  $\mu^2 \rightarrow \infty$  :

$$\Phi^{(\pi\eta), a}(y, \tilde{\zeta}, m_{\pi\eta}^2) = 10y(1-y)C_1^{(3/2)}(2y-1) \sum_{l=0}^2 B_{1l}(m_{\pi\eta}^2)P_l(\cos\theta)$$

Keeping only  $L = 1$  ( $\pi_1$ ) and  $L = 2$  ( $a_2$ ) terms:

$$\Phi^{(\pi\eta)}(y, \zeta, m_{\pi\eta}^2) = 30y(1-y)(2y-1) \left[ B_{11}(m_{\pi\eta}^2)P_1(\cos\theta) + B_{12}(m_{\pi\eta}^2)P_2(\cos\theta) \right]$$

- $B_{11}(m_{\pi\eta}^2)$  and  $B_{12}(m_{\pi\eta}^2)$  are related to corresponding Breit-Wigner amplitudes for  $m_{\pi\eta}^2 \approx M_{a_2}^2, M_H^2$ :

$$B_{11}(m_{\pi\eta}^2) \Big|_{m_{\pi\eta}^2 \approx M_H^2} = \frac{5}{3} \frac{g_{H\pi\eta} f_H M_H \beta}{M_H^2 - m_{\pi\eta}^2 - i\Gamma_H M_H}$$

and

$$B_{12}(m_{\pi\eta}^2) \Big|_{m_{\pi\eta}^2 \approx M_{a_2}^2} = \frac{10}{9} \frac{ig_{a_2\pi\eta} f_{a_2} M_{a_2}^2 \beta^2}{M_{a_2}^2 - m_{\pi\eta}^2 - i\Gamma_{a_2} M_{a_2}}$$



Hybrid in electroproduction of  $\pi\eta$  pair $\pi^0\eta$  cross-sectionDifferential cross-section for  $\pi^0\eta$  electroproduction

- Amplitude of  $\pi^0\eta$  electroproduction

(for  $e(k_1) + N(p_1) \rightarrow e(k_2) + \pi^0(p_\pi) + \eta(p_\eta) + N(p_2)$ ):

$$|T^{\pi^0\eta}|^2 = \frac{4e^2(1-y_l)}{Q^2 y_l^2} |\mathcal{A}_{(q)}^{\pi^0\eta}|^2.$$

with (for  $\gamma^*(q) + N(p_1) \rightarrow \pi^0(p_\pi) + \eta(p_\eta) + N(p_2)$ ):

$$\begin{aligned} \mathcal{A}_{(q)}^{\pi^0\eta} &= \frac{e\pi\alpha_s C_F}{N_c Q} \left[ e_u \mathcal{H}_{uu} - e_d \mathcal{H}_{dd} \right] \\ &\times \left[ B_{11}(m_{\pi\eta}^2) P_1(\cos\theta_{cm}) + B_{12}(m_{\pi\eta}^2) P_2(\cos\theta_{cm}) \right] \end{aligned}$$

twist 2 scaling like for the usual  $\rho \rightarrow \pi^+\pi^-$  channel

- Differential cross-section for

(for  $e(k_1) + N(p_1) \rightarrow e(k_2) + \pi^0(p_\pi) + \eta(p_\eta) + N(p_2)$ ):

$$\frac{d\sigma^{\pi^0\eta}}{dQ^2 dy_l d\hat{t} dm_{\pi\eta} d(\cos\theta_{cm})} = \frac{1}{4(4\pi)^5} \frac{m_{\pi\eta}\beta}{y_l \lambda^2(\hat{s}, -Q^2, m_N^2)} |T^{\pi^0\eta}|^2$$

$$\hat{t} = (p_2 - p_1)^2, \quad y_l = \frac{p_1 \cdot q}{p_1 \cdot k_1}$$

Hybrid in electroproduction of  $\pi\eta$  pair

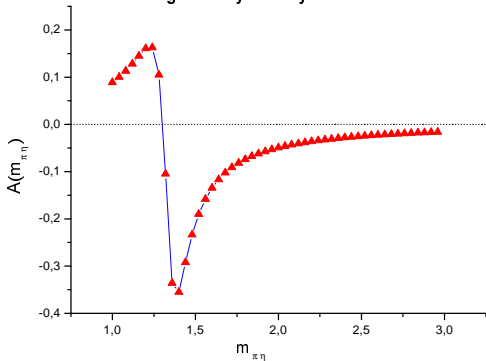
## Angular asymmetry

## Angular asymmetry to unravel the hybrid meson

- $\pi_1$  has rather small amplitude with respect to the  $a_2$  background
- Asymmetry sensitive to their interference:

$$A(Q^2, y_l, \hat{t}, m_{\pi\eta}) = \frac{\int \cos \theta_{cm} d\sigma^{\pi^0\eta}(Q^2, y_l, \hat{t}, m_{\pi\eta}, \cos \theta_{cm})}{\int d\sigma^{\pi^0\eta}(Q^2, y_l, \hat{t}, m_{\pi\eta}, \cos \theta_{cm})}$$

Angular Asymmetry



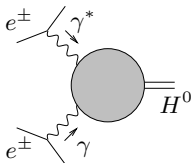
$$= \frac{\frac{8}{15} \operatorname{Re} \left[ B_{11}(m_{\pi\eta}^2) B_{12}^*(m_{\pi\eta}^2) \right]}{\frac{2}{3} \left| B_{11}(m_{\pi\eta}^2) \right|^2 + \frac{2}{5} \left| B_{12}(m_{\pi\eta}^2) \right|^2}$$

Hybrid meson production in  $\gamma^*\gamma$ 

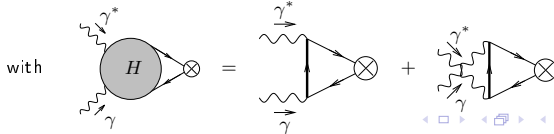
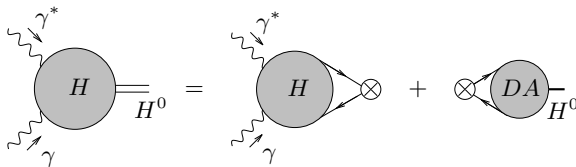
Factorized picture

Hybrid meson production in  $e^+e^-$  colliders

- Hybrid can be copiously produced in  $\gamma^*\gamma$ , i.e. at  $e^+e^-$  colliders **with one tagged out-going electron**



- This can be described in a hard factorization framework:



Hybrid meson production in  $\gamma^*\gamma$ 

## Cross-section

Counting rates for  $H^0$  versus  $\pi^0$ 

- Factorization gives:

$$\mathcal{A}^{\gamma\gamma^* \rightarrow H^0}(\gamma\gamma^* \rightarrow H_L) = (\epsilon_\gamma \cdot \epsilon_\gamma^*) \frac{(e_u^2 - e_d^2) f_H}{2\sqrt{2}} \int_0^1 dz \Phi^H(z) \left( \frac{1}{z} - \frac{1}{\bar{z}} \right)$$

- Ratio  $H^0$  versus  $\pi^0$ :

$$\frac{d\sigma^H}{d\sigma^{\pi^0}} = \left| \frac{f_H \int_0^1 dz \Phi^H(z) \left( \frac{1}{z} - \frac{1}{\bar{z}} \right)}{f_\pi \int_0^1 dz \Phi^\pi(z) \left( \frac{1}{z} + \frac{1}{\bar{z}} \right)} \right|^2$$

- This gives, with asymptotic DAs:

$$\frac{d\sigma^H}{d\sigma^{\pi^0}} \approx 38\%$$

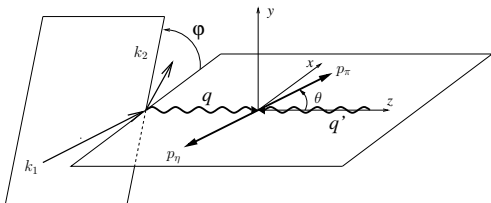
still larger than 20% at  $Q^2 \approx 1 \text{ GeV}^2$  (including kinematical twist-3 effects à la [Wandzura-Wilczek](#) for the  $H^0$  DA) and similarly

$$\frac{d\sigma^H}{d\sigma^\eta} \approx 46\%$$

# Hybrid meson production in $\gamma^*\gamma$ $\pi\eta$ channel

## Cross-section for $\gamma^*\gamma \rightarrow \pi\eta$ and angular distribution

- An estimation of the cross-section can be done using a model for the  $\pi\eta$  GDA
- It requires to model the background, and results are rather model dependent for  $\sigma^{\pi\eta}$
- A detailed study of the  $(\varphi, \theta)$  angular distribution of the  $\pi\eta$  final state could give a direct access to the strength of the twist 3 amplitude



# Hybrid meson production in $\gamma^*\gamma$

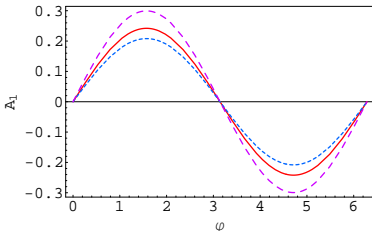
## $\pi\eta$ channel

### SSA for $\gamma^*\gamma \rightarrow \pi\eta$ with polarized lepton

- Scattering of a **longitudinally polarized lepton** on an unpolarized photon: **direct access to the interference of twist 2 with twist 3 amplitudes** through the **Single Spin Asymmetry**

$$A_1(s_{e\gamma}, Q^2, W^2; \varphi) = \frac{\int d \cos \theta_{cm} (d\sigma^{(\rightarrow)} - d\sigma^{(\leftarrow)})}{\int d \cos \theta_{cm} (d\sigma^{(\rightarrow)} + d\sigma^{(\leftarrow)})}$$

- Results for a background phase  $\alpha = 0$  and various choices of background amplitudes: **sizable asymmetry**



$W = 1.4 \text{ GeV}$ ,  $Q^2 = 5.0 \text{ GeV}^2$ ,  $s_{e\gamma} = 10 \text{ GeV}^2$ ,  $\alpha = 0$ . The solid line corresponds to  $K = 0.8$ , the short-dashed line to  $K = 1.0$ , the long-dashed line to  $K = 0.5$

# Conclusion

- Hybrid mesons  $H$  are a key stone for our understanding of QCD
- There are now strong candidates for  $J^{PC} = 1^{-+}$
- As a first step, one should determine their mass, width and quantum numbers, as well as their decay modes
- A second step should be to determine their partonic content
- These questions can be addressed in **hard processes**
- $\Rightarrow$  Access to their light-cone wave function (Distribution Amplitudes)
- **Hard hybrid production is governed by twist 2 operators**
- The non-perturbative coupling  $f_H$  can be evaluated from QCD sum-rules
- The rates for electroproduction (or muoproduction!) are very sizable:

$$\frac{d\sigma^H(Q^2, x_B, t)}{d\sigma^\rho(Q^2, x_B, t)} \approx \left(\frac{5f_H}{3f_\rho}\right)^2 \approx 15\% \rightarrow \text{JLab, COMPASS}$$

- The **DA** can be replaced by the **GDA** of the decay modes
- $\Rightarrow$  Framework for angular asymmetry with the dominant background (e.g.  $\pi_1(1400)(1^{-+}) + a_2(1329)(2^{++})$  interference within the  $C = (+)$   $\pi\eta$  GDA)
- $\gamma^*\gamma \rightarrow H^0$  at  $e^+e^-$  colliders is also very promising

$$\frac{d\sigma^H}{d\sigma^{\pi^0}} \approx 38\%$$

- $H$  subleading twist content accessible using SSA with **polarized lepton**