QCD factorization for $\gamma^*\gamma^* \to \rho_L^0 \rho_L^0$

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based on:

Diffractive production of two ρ_L^0 mesons in e+e- collisions

M. Segond, L. Szymanowski, S. W. Eur.Phys.J.C, 2007 (to appear) [hep-ph/0703166]

QCD factorizations in $\gamma^* \gamma^* \to \rho\rho$

B. Pire, M. Segond, L. Szymanowski, S. W. Phys.Lett.B639:642-651,2006 [hep-ph/0605320]

BFKL resummation effects in $\gamma^* \gamma^* \to \rho\rho$

R. Enberg, B. Pire, L. Szymanowski , S. W. Eur.Phys.J.C45:759-769,2006 [hep-ph/0508134]

Double diffractive rho-production in $\gamma^* \gamma^*$ collisions

B. Pire, L. Szymanowski, S. W. Eur.Phys.J.C44:545-558,2005 [hep-ph/0507038]

Outline

1. Introduction: Exclusive processes at high energy QCD
   - Motivation
   - GDA and TDA for $\gamma^*\gamma^* \to \rho_L^0\rho_L^0$
2. Computation in the collinear factorization with DA at fixed W^2
   - Direct calculation
   - Interpretation in terms of QCD Factorization
     - GDA for transverse photon in the limit $\Lambda_{QCD} \ll W^2 \ll Max(Q_1^2, Q_2^2)$
     - TDA for longitudinal photon in the limit $Q_1^2 \gg Q_2^2$ (or $Q_1^2 \ll Q_2^2$)
3. Computation at large W^2 : k_T factorization approach
   - Motivation and aims
   - k_T factorization
   - Non-forward Born order cross-section for $\gamma^*\gamma^* \to \rho_L^0\ \rho_L^0$
     - Analytical 2-dimensional integration through conformal transformations
     - Results
   - Non-forward Born order cross-section for $e^+e^- \to e^+e^-\rho_L^0\ \rho_L^0$
     - Equivalent photon approximation
     - Kinematical cuts
     - ILC collider and LDC detector
     - Born result
     - LL BFKL enhancement

Outline

Since a decade, there have been much developpements in hard exclusive processes.

- form factors $\longrightarrow$ Distribution Amplitudes
- DVCS$\longrightarrow$ Generalized Parton Distributions,
- $\cdots$

These tests are possible in fixed target experiments

- $e^{\pm}p$: HERA (HERMES), JLab, ...

as well as in colliders, mainly for fixed s

- $e^{\pm}p$ colliders: HERA (H1,ZEUS)
- $e^{+}e^{-}$ colliders: LEP, Belle, BaBar, BEPC

At the same time, the interest for phenomenological tests of hard Pomeron and related resummed approaches has become pretty wide:

- inclusive tests (total cross-section) and semi-inclusive tests (diffraction, forward jets, ...)
- exclusive tests (meson production, ...)

These tests concern all type of collider experiments:

- $e^{\pm}p$: (HERA: H1, ZEUS)
- $p\bar{p}$ (TEVATRON: CDF, D0)
- $e^{+}e^{-}$ colliders (LEP, ILC)

We will focus on a specific exclusive process:

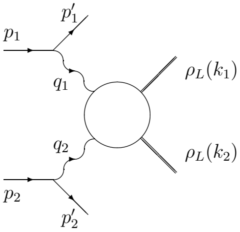
$$\boxed{\gamma^*\gamma^* \longrightarrow \rho_L^0 \rho_L^0}$$ with both γ^* hard

It is a beautiful theoretical laboratory for investigating different dynamics (collinear, multiregge) and factorization properties of high energy QCD:

- at low energy (fixed s) it provides an (almost) full perturbative laboratory for extended GPDs: GDA and TDA
- at high energy (asymptotic s) it provides an (almost) full perturbative laboratory for BFKL and related resummed effects, at amplitude level.

The corresponding experimental process is

$$e^+e^- \longrightarrow e^+e^- \rho_L^0 \rho_L^0$$



with double tagged outoing leptons.

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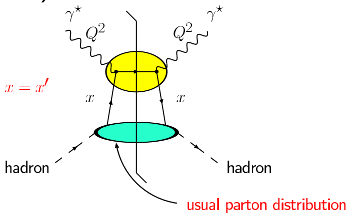
- DIS: inclusive process $\to$ forward amplitude ($t = 0$)

Structure Function

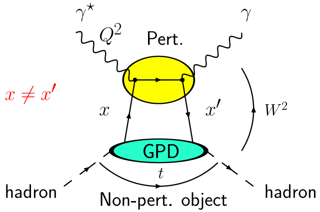
= Coeffi cient Function (hard) $\otimes$ Parton Distribution Function (soft)



- DVCS: exclusive process $\to$ non forward amplitude ($-t \ll s = W^2$)

Amplitude

= Coeffi cient Function (hard) $\otimes$ Generalized Parton Distribution (soft)

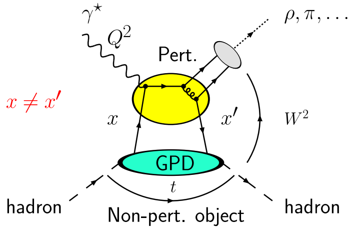


Extensions:

- Meson production: γ replaced by $\rho,\ \pi,\cdots$

Amplitude

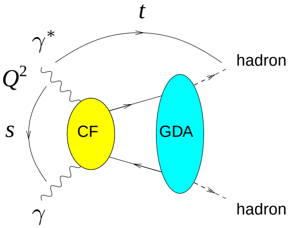
= GPD (soft) $\otimes$ CF (hard) $\otimes$ Distribution Amplitude (soft)



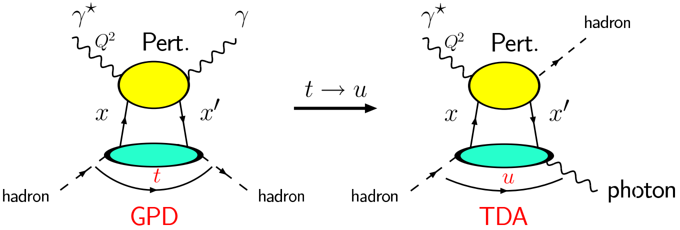
- Crossed process: $s \ll -t$

Amplitude

= Coeffi cient Function (hard) $\otimes$ Generalized Distribution Amplitude (soft)



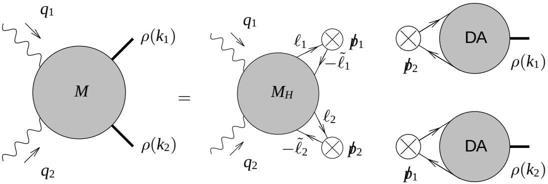
- starting from usual DVCS, one allows initial hadron $\neq$ final hadron
  example:



which can be further extended by replacing the outoing γ by any hadronic state

Amplitude	=	Transition Distribution Amplitude (soft)	$\otimes$	CF (hard)	$\otimes$	DA (soft)

$Q^2_{1,2}$: hard scales $\Rightarrow$ collinear approximation at each $q\bar{q}\rho$ vertex



i.e. we neglect the transverse relative (anti-)quark momenta in the ρ mesons:

$$\ell_1 \sim z_1\, k_1 \qquad \ell_2 \sim z_2\, k_2$$
$$\tilde{\ell}_1 \sim \bar{z}_1\, k_1 \qquad \tilde{\ell}_2 \sim \bar{z}_2\, k_2$$

We limit ourselves to longitudinaly polarized mesons (to avoid potential end-point singularies due to higher twist contributions)

DA of the meson = matrix element of non local quarks fields correlator on the light cone

$$\langle 0|\bar{q}(x)\,\gamma^\mu\, q(-x)|\rho_L(p) = \bar{q}q\rangle = f_\rho\, p^\mu \int_0^1 dz\, e^{i(2z-1)(px)}\phi(z)$$

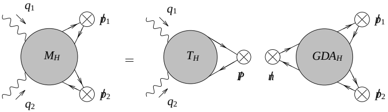
with $$\phi(z) = 6z(1-z)\,(1 + \sum_{n=1}^{\infty} a_{2n} C_{2\,n}^{3/2}(2z-1))$$

Note: p_1 , p_2 are light-like Sudakov vectors along the meson momenta.

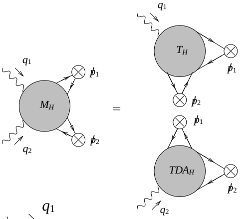
We will now consider two types of treatment for the hard part M_H

- at moderate W^2 ($\gg \Lambda^2_{QCD}$), we perform the direct calculation.
  We then show that it can be presented in a QCD factorized form involving

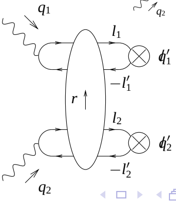
  - either a GDA for $W^2 \ll Max(Q_1^2, Q_2^2)$



  - or a TDA for $Q_1^2 \ll Q_2^2$ or $Q_1^2 \gg Q_2^2$



- at asymptotically large W^2, we rely on k_T factorization involving impact factors



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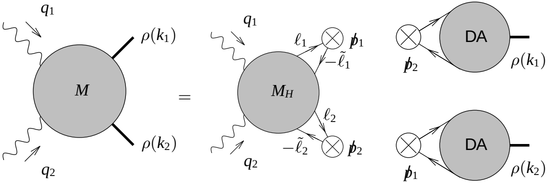


- The computation follows the line of the Brodsky, Lepage approach.
- We consider the Born order, i.e. quark exchange.
- We restrict ourselves to the forward case
- We only consider longitudinally polarized mesons $\Rightarrow$ leading twist

The amplitude can be expressed as the sum of two tensors:

$$\mathcal{M} = T^{\mu\,\nu} \epsilon_\mu(q_1)\epsilon_\nu(q_2)$$

with

$$T^{\mu\,\nu} = \frac{1}{2} g_T^{\mu\,\nu}\; T^{\alpha\,\beta} g_{T\,\alpha\,\beta} + \left(p_1^\mu + \frac{Q_1^2}{s} p_2^\mu\right)\left(p_2^\nu + \frac{Q_2^2}{s} p_1^\nu\right) \frac{4}{s^2} T^{\alpha\,\beta} p_{2\,\alpha}\, p_{1\,\beta}$$

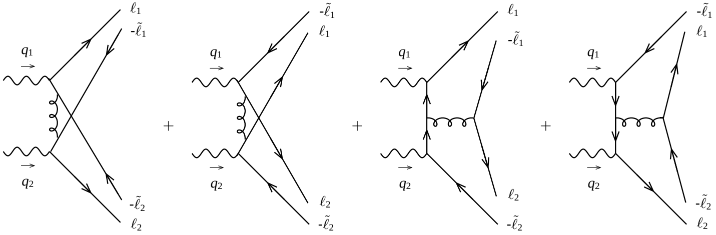
$$g_T^{\mu\,\nu} = g^{\mu\nu} - \frac{p_1^\mu p_2^\nu + p_1^\nu p_2^\mu}{p_1 . p_2}$$

- The photons polarization vectors read

$$\epsilon_{\parallel}(q_1) = \frac{1}{Q_1} q_1 + \frac{2Q_1}{s} p_2 \quad \text{and} \quad \epsilon_{\parallel}(q_2) = \frac{1}{Q_2} q_2 + \frac{2Q_2}{s} p_1 \,.$$

- use QED gauge invariance
- remember that we only consider the forward kinematics

$\Rightarrow$ the number of diagrams reduces to 4



$$T^{\alpha\,\beta} p_{2\,\alpha}\, p_{1\,\beta} = -\frac{s^2 f_\rho^2 C_F e^2 g^2 (Q_u^2 + Q_d^2)}{8 N_c Q_1^2 Q_2^2} \int_0^1 dz_1\, dz_2\, \phi(z_1)\, \phi(z_2)$$

$$\times \left\{ \frac{(1-\frac{Q_1^2}{s})(1-\frac{Q_2^2}{s})}{(z_1+\bar{z}_1\frac{Q_2^2}{s})(z_2+\bar{z}_2\frac{Q_1^2}{s})} + \frac{(1-\frac{Q_1^2}{s})(1-\frac{Q_2^2}{s})}{(\bar{z}_1+z_1\frac{Q_2^2}{s})(\bar{z}_2+z_2\frac{Q_1^2}{s})} + \frac{1}{z_2\bar{z}_1} + \frac{1}{z_1\bar{z}_2} \right\}$$

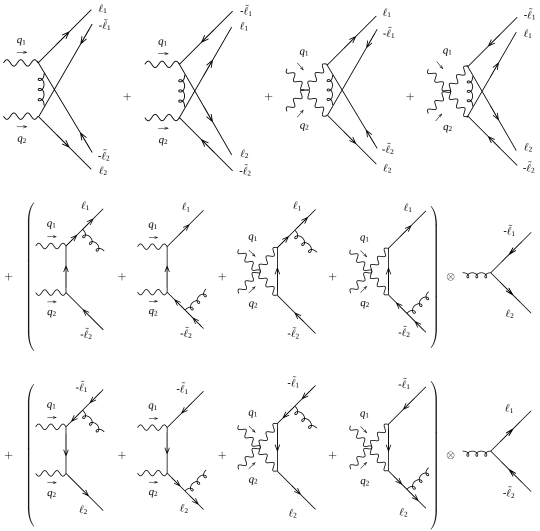
with $s = 2\, p_1 \cdot p_2$

Note:

Q_1^2 and Q_2^2 are non-zero and DA vanishes at $z_i = 0$

$\Rightarrow$ no end-point singularity in the z_i integration

In this case no simplifi cation occurs. One needs to compute 12 diagrams.



$$T^{\alpha\,\beta} g_{T\,\alpha\,\beta} \quad = \quad -\frac{e^2 (Q_u^2 + Q_d^2)\, g^2\, C_F\, f_\rho^2}{4\, N_c\, s} \int_0^1 dz_1\, dz_2\, \phi(z_1)\, \phi(z_2)$$

$$\times \left\{ 2 \left(1 - \frac{Q_2^2}{s}\right) \left(1 - \frac{Q_1^2}{s}\right) \left[\frac{1}{(z_2 + \bar{z}_2 \frac{Q_1^2}{s})^2 (z_1 + \bar{z}_1 \frac{Q_2^2}{s})^2} + \frac{1}{(\bar{z}_2 + z_2 \frac{Q_1^2}{s})^2 (\bar{z}_1 + z_1 \frac{Q_2^2}{s})^2} \right] + \right.$$

$$\left. \left(\frac{1}{\bar{z}_2\, z_1} - \frac{1}{\bar{z}_1\, z_2} \right) \left[\frac{1}{1 - \frac{Q_2^2}{s}} \left(\frac{1}{\bar{z}_2 + z_2 \frac{Q_1^2}{s}} - \frac{1}{z_2 + \bar{z}_2 \frac{Q_1^2}{s}} \right) - \frac{1}{1 - \frac{Q_1^2}{s}} \left(\frac{1}{\bar{z}_1 + z_1 \frac{Q_2^2}{s}} - \frac{1}{z_1 + \bar{z}_1 \frac{Q_2^2}{s}} \right) \right] \right\}$$

Same remark:

Q_1^2 and Q_2^2 are non-zero and DA vanishes at $z_i = 0$

$\Rightarrow$ no end-point singularity in the z_i integration

Outline

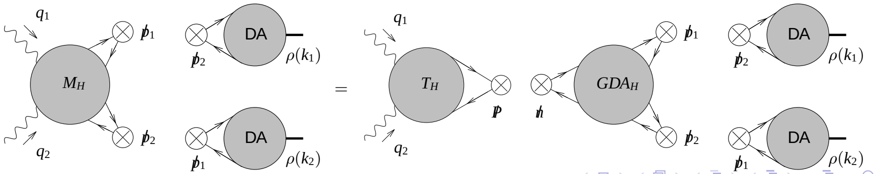
When W^2 is smaller then the highest photon virtuality

For example $\frac{W^2}{Q_1^2} = \frac{s}{Q_1^2}\left(1 - \frac{Q_1^2}{s}\right)\left(1 - \frac{Q_2^2}{s}\right) \approx 1 - \frac{Q_1^2}{s} \ll 1$ $\qquad s \equiv 2\, p_1 \cdot p_2$

the result obtained from direct calculation simplifies into

$$T^{\alpha\beta} g_{T\,\alpha\beta} \approx \frac{e^2 (Q_u^2 + Q_d^2)\, g^2\, C_F\, f_\rho^2}{4\, N_c\, W^2} \times \int_0^1 dz_1\, dz_2 \left(\frac{1}{\bar{z}_1 + z_1 \frac{Q_2^2}{s}} - \frac{1}{z_1 + \bar{z}_1 \frac{Q_2^2}{s}} \right) \left(\frac{1}{\bar{z}_2\, z_1} - \frac{1}{\bar{z}_1\, z_2} \right) \phi(z_1)\, \phi(z_2)$$

which can be interpreted as ($P \sim p_1,\ n \sim p_2$)



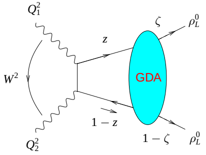
GDA computation

At leading twist, the GDA is calculated in the Born order of perturbation theory

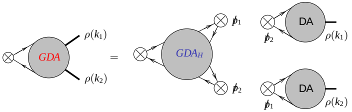
$$\langle \rho_L^0(k_1)\,\rho_L^0(k_2)|\bar q(-\alpha\, n/2)\,\not n\,\exp\left[ig\int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} dy\, n_\nu\, A^\nu(y)\right] q(\alpha\, n/2)|0\rangle$$

$$= \int_0^1 dz\, e^{-i(2z-1)\alpha\,(nP)/2}\,\Phi^{\rho_L^0\rho_L^0}(z,\zeta,W^2)$$

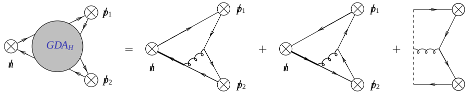
$(P \sim p_1$ and $n \sim p_2$ for $Q_1 > Q_2)$



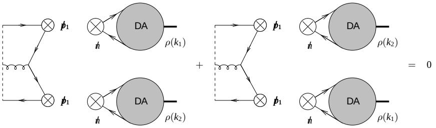
Since W^2 is hard, the GDA can be factorized:



with



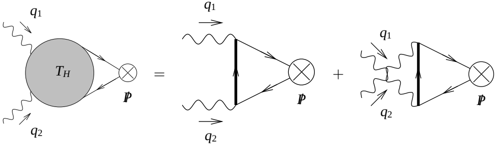
In our kinematics, the QCD Wilson line vanishes:



Interpretation in terms of QCD Factorization

GDA for transverse photon in the limit $\Lambda_{QCD}^2 \ll W^2 \ll Max(Q_1^2, Q_2^2)$: PROOF

Hard Part computation at Born order



In the case of one flavored quark, it equals:

$$T_H(z) = -4\, e^2\, N_c\, Q_q^2 \left(\frac{1}{\bar{z} + z\, \frac{Q_2^2}{s}} - \frac{1}{z + \bar{z}\, \frac{Q_2^2}{s}} \right)$$

We have thus shown that $T^{\alpha\,\beta} g_{T\,\alpha\,\beta}$ factorizes into Hard part $\otimes$ GDA:

$$T^{\alpha\,\beta} g_{T\,\alpha\,\beta} = \frac{e^2}{2}\left(Q_u^2 + Q_d^2\right)\int_0^1 dz \left(\frac{1}{\bar{z} + z\frac{Q_2^2}{s}} - \frac{1}{z + \bar{z}\frac{Q_2^2}{s}}\right)\Phi^{\rho_L\,\rho_L}(z, \zeta \approx 1, W^2)$$

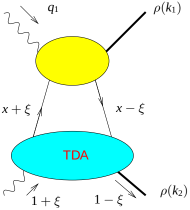
with the GDA which itself factorizes into Hard part $\otimes$ DA DA:

$$\Phi^{\rho_L\,\rho_L}(z, \zeta \approx 1, W^2) = -\frac{f_\rho^2\, g^2\, C_F}{2\, N_c\, W^2}\int_0^1 dz_2\, \phi(z)\, \phi(z_2)\left[\frac{1}{z\bar{z}_2} - \frac{1}{\bar{z} z_2}\right]$$

- This is a limiting case of the original equation obtained by D. Müller et al (2000)
- It extends the studies of $\gamma^*\gamma \to \pi\pi$ by M. Diehl et al (2000)
- We limited ourselves to the case of $t = t_{min}$

The direct calculation of the amplitude $M = T^{\alpha\,\beta} p_{2\,\alpha}\, p_{1\,\beta}$ can be interpreted, in the limiting case $Q_1^2 \gg Q_2^2$ (or $Q_1^2 \ll Q_2^2$), as

$$M = TDA \otimes CF \otimes DA$$



TDA kinematics = GPD kinematics

$n_1 = (1+\xi)p_1$ and $n_2 = \frac{p_2}{1+\xi}$

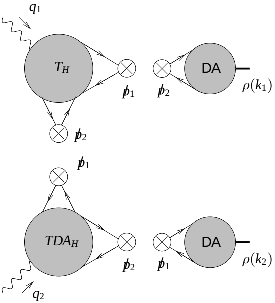
x, ξ are momentum fraction along $n_2 = \frac{p_2}{1+\xi}$

More precisely, we prove that M factorizes as

$$T^{\alpha\,\beta} p_{2\,\alpha} p_{1\,\beta}$$

$$= -i f_\rho^2 e^2 (Q_u^2 + Q_d^2) g^2 \, \frac{C_F}{8N_C} \int_{-1}^{1} dx \int_0^1 dz_1 \left[\frac{1}{\bar{z}_1\,(x-\xi)} + \frac{1}{z_1\,(x+\xi)} \right] \phi(z_1)$$

$$\times N_c \left[\Theta(1 \geq x \geq \xi)\, \phi\left(\frac{x-\xi}{1-\xi}\right) - \Theta(-\xi \geq x \geq -1)\, \phi\left(\frac{1+x}{1-\xi}\right) \right]$$



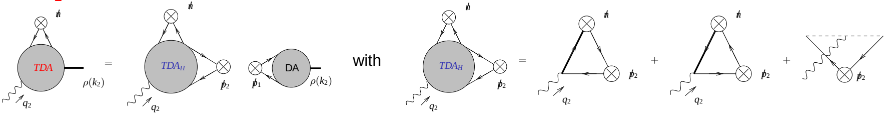
TDA computation at Born order

The TDA $\gamma^* \to \rho_L^0$ is defined through $(n \sim n_1)$

$$\int \frac{dz^-}{2\pi} e^{ix(P.z)} \langle \rho_L^q(k_2)|\bar{q}(-z/2)\, \not{n}\, e^{-ieQ_q \int_{z/2}^{-z/2} dy_\mu A^\mu(y)} q(z/2)|\gamma^*(q_2)\rangle$$
$$= \frac{e\,Q_q\,f_\rho}{P^+}\,\frac{2}{Q_2^2}\,\epsilon_\nu(q_2)\left((1+\xi)n_2^\nu + \frac{Q_2^2}{s(1+\xi)}n_1^\nu\right) T(x,\xi,t_{min}) ,$$

where the QED Wilson line is explicitly indicated (QCD Wilson line gives no contribution)

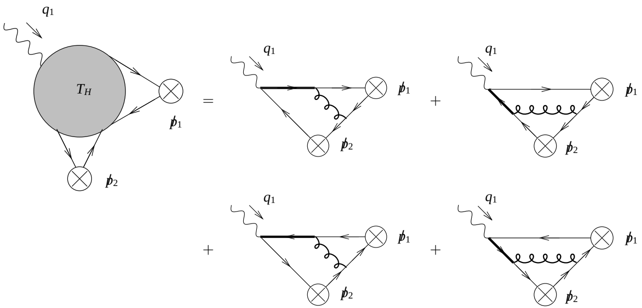
Since Q_2^2 is hard, the TDA can be factorized:



Explicit computation gives

$$T(x,\xi,t_{min}) \equiv N_c\left[\Theta(1\geq x\geq \xi)\,\phi\left(\frac{x-\xi}{1-\xi}\right) - \Theta(-\xi\geq x\geq -1)\,\phi\left(\frac{1+x}{1-\xi}\right)\right]$$

Hard computation at Born order



$$T_H(z_1, x) = -i f_\rho g^2 e Q_q \frac{C_F \phi(z_1)}{2 N_c Q_1^2} \epsilon^\mu(q_1) \left(2\xi n_{2\,\mu} + \frac{1}{1+\xi} n_{1\,\mu} \right) \times \left[\frac{1}{z_1(x+\xi-i\epsilon)} + \frac{1}{\bar{z}_1(x-\xi+i\epsilon)} \right],$$

We have shown, at Born order, that $T^{\alpha\,\beta} p_{2\,\alpha} p_{1\,\beta}$ factorizes into TDA $\otimes$ Hard part $\otimes$ DA:

$$T^{\alpha\,\beta} p_{2\,\alpha} p_{1\,\beta}$$

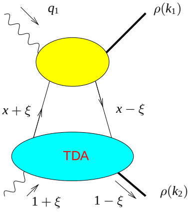
$$= -i f_\rho^2 e^2 (Q_u^2 + Q_d^2) g^2 \frac{C_F}{8N_c} \int_{-1}^{1} dx \int_{0}^{1} dz_1\, T(x, \xi, t_{min}) \left[\frac{1}{\bar{z}_1 (x-\xi)} + \frac{1}{z_1 (x+\xi)}\right] \phi(z_1)$$

with the TDA which itself factorizes into Hard part $\otimes$ DA:

$$T(x, \xi, t_{min}) \equiv N_c \left[\Theta(1 \geq x \geq \xi)\, \phi\left(\frac{x-\xi}{1-\xi}\right) - \Theta(-\xi \geq x \geq -1)\, \phi\left(\frac{1+x}{1-\xi}\right)\right]$$

Note:

Only the DGLAP part of the TDA contributes because of support properties of the ρ meson DA



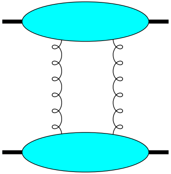
DGLAP(1)	$-1 \leq x \leq -\xi$
ERBL	$-\xi \leq x \leq \xi$
DGLAP(2)	$\xi \leq x \leq 1$

Outline

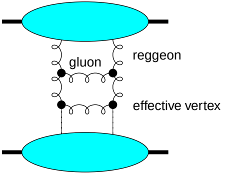
QCD in perturbative Regge limit

- In this limit, the dynamics is dominated by gluons (dominance of spin 1 exchange in t channel)
- BFKL (and extensions: NLL, saturations effects, ...) is expected to dominates with respect to Born order at large relative rapidity.

Born order:



BFKL ladder:



$e^+e^- \to e^+e^- \rho_L^0 \, \rho_L^0$ is a good observable in order to test this limit:

- IR-safe probes: double tagging of the final leptons and cut-off over soft photons $\Rightarrow$ the hard virtual photons give the hard scales on both sides of the t-channel exchanged state $\Rightarrow$ fully perturbative process (except for DAs of ρ).
- observable dominated by the "soft" (but still perturbative) dynamics of QCD (BFKL and extensions) and not by its collinear dynamics (DGLAP, ERBL): we impose $Q_1^2 \sim Q_2^2$
- gives access to the interplay between collinear and soft dynamics by getting away from $Q_1^2 \sim Q_2^2$ domain and by playing with the relative rapidity
- one can control the spread in k_T of the partons: transition from linear to non-linear (saturated regime), when increasing $s_{\gamma^*\gamma^*}$ for given values Q_1^2 and Q_2^2. Experimentally feasible by increasing $s_{e^+e^-}$
- it gives access to non-forward dynamics
  - can reveal Pomeron structure apart from the forward limit
  - for saturation studies, it is important to get a full impact parameter picture of the process (Froissart bound is for each impact parameter)
  - Note that for $t = 0$, the simplest model for non-linearity is the Balitskii Kovchegov equation
- cross-section are expected to be peaked in the forward limit $\Rightarrow$ the forward differential cross-section gives the general trends

- Compute the scattering amplitude for $\gamma^*\gamma^* \to \rho_L^0\ \rho_L^0$ with gluon exchange, in the range $s_{\gamma^*\gamma^*} \gg -t, Q_1^2, Q_2^2$ for every photons polarizations and check dominance with respect to quarks exchange at ILC energies
- We focus on $Q_1^2 \sim Q_2^2 \Rightarrow$ no DGLAP evolution (this is practically imposed by the small range in both Q_i^2 due to the lower perturbative cut-off and by the fast decreasing amplitude as powers of Q_i^2)
- We prove the experimental feasability at ILC, with LDC detector project
- Study linear and non linear dynamical effects, and the expected enhancement at large rapidity

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1. Introduction: Exclusive processes at high energy QCD
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   - Direct calculation
   - Interpretation in terms of QCD Factorization
     - GDA for transverse photon in the limit $\Lambda_{QCD} \ll W^2 \ll Max(Q_1^2, Q_2^2)$
     - TDA for longitudinal photon in the limit $Q_1^2 \gg Q_2^2$ (or $Q_1^2 \ll Q_2^2$)
3. Computation at large W^2 : k_T factorization approach
   - Motivation and aims
   - k_T factorization
   - Non-forward Born order cross-section for $\gamma^*\gamma^* \to \rho_L^0\ \rho_L^0$
     - Analytical 2-dimensional integration through conformal transformations
     - Results
   - Non-forward Born order cross-section for $e^+e^- \to e^+e^- \rho_L^0\ \rho_L^0$
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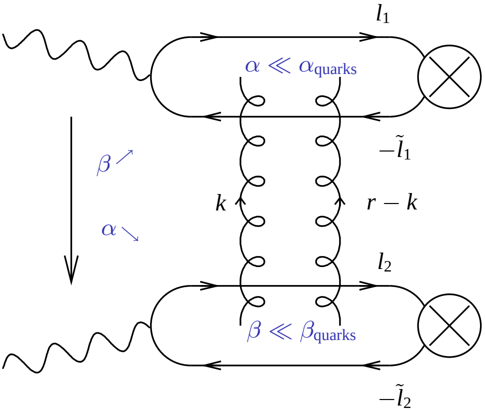
     - Born result
     - LL BFKL enhancement

- Use Sudakov decomposition $k = \alpha p_1 + \beta p_2 + k_\perp$
- write

$$d^4k = \frac{s}{2}\, d\alpha\, d\beta\, d^2k_\perp$$

and rearrange integrations in the large s limit:



$\Rightarrow$ set $\alpha = 0$ and $\int d\beta$

$\Rightarrow$ set $\beta = 0$ and $\int d\alpha$

$\Rightarrow$ impact representation (written here for the whole process) note: $\underline{k}$ = Eucl. $\leftrightarrow k_\perp$ = Mink.

$$\mathcal{M} = is \int \frac{d^2 \underline{k}}{(2\pi)^4 \underline{k}^2 (\underline{r} - \underline{k})^2} \mathcal{J}^{\gamma^*_{L,T}(q_1) \to \rho^0_L(k_1)}(\underline{k}, \underline{r} - \underline{k}) \, \mathcal{J}^{\gamma^*_{L,T}(q_2) \to \rho^0_L(k_2)}(-\underline{k}, -\underline{r} + \underline{k})$$

- For longitudinally polarized photons the impact factor reads

$$\mathcal{J}^{\gamma^*_L(q_i) \to \rho_L(k_i)}(\underline{k}, \underline{r} - \underline{k}) = 8\pi^2 \alpha_s \frac{e}{\sqrt{2}} \frac{\delta^{ab}}{2N_c} Q_i f_\rho \, \alpha(k_i) \int_0^1 dz_i z_i \, \bar{z}_i \, \phi(z_i) \mathrm{P_P}(z_i, \underline{k}, \underline{r}, \mu_i)$$

where

$$\mathrm{P_P}(z_i, \underline{k}, \underline{r}, \mu_i) = \frac{1}{z_i^2 \underline{r}^2 + \mu_i^2} + \frac{1}{\bar{z}_i^2 \underline{r}^2 + \mu_i^2} - \frac{1}{(z_i \underline{r} - \underline{k})^2 + \mu_i^2} - \frac{1}{(\bar{z}_i \underline{r} - \underline{k})^2 + \mu_i^2} \propto \mathcal{J}^{\gamma^*_L(q_i) \to q\,\bar{q}}$$

- For transversally polarized photons, one obtains

$$\mathcal{J}^{\gamma^*_T(q_i) \to \rho_L(k_i)}(\underline{k}, \underline{r} - \underline{k}) = 4\pi^2 \alpha_s \frac{e}{\sqrt{2}} \frac{\delta^{ab}}{2N_c} f_\rho \alpha(k_i) \int_0^1 dz_i \, (z_i - \bar{z}_i) \, \phi(z_i) \, \underline{\epsilon} \cdot \underline{\mathrm{Q}}(z_i, \underline{k}, \underline{r}, \mu_i)$$

where

$$\underline{\mathrm{Q}}(z_i, \underline{k}, \underline{r}, \mu_i) = \frac{z_i \, \underline{r}}{z_i^2 \underline{r}^2 + \mu_i^2} - \frac{\bar{z}_i \, \underline{r}}{\bar{z}_i^2 \underline{r}^2 + \mu_i^2} + \frac{\underline{k} - z_i \, \underline{r}}{(z_i \underline{r} - \underline{k})^2 + \mu_i^2} - \frac{\underline{k} - \bar{z}_i \underline{r}}{(\bar{z}_i \, \underline{r} - \underline{k})^2 + \mu_i^2} \propto \mathcal{J}^{\gamma^*_T(q_i) \to q\,\bar{q}}$$

we denote $\mu_i^2 = Q_i^2 \, z_i \, \bar{z}_i + m^2$, where m is the quark mass (set to zero in practice)

- due to QCD gauge invariance (probes are colorless), both impact factor vanishes when $\underline{k} \to 0$ or $\underline{r} - \underline{k} \to 0$

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- All the 2-d integrations with respect to $\underline{k}$ are treated analytically
- The method relies on conformal transformation in the transverse momentum plane (method inspired by Vassiliev in 2-d coordinate space)
- The idea is to reduce the number of propagators, in order to be able to perform standard Feynman parameter integration
- the whole computation involves integrals with up to 4 propagators (2 massive, with different masses) which we would have been enable to compute without this method

The integral ($\bar{a} \equiv 1 - a$)

$$J_{3\mu}(a) = \int \frac{d^2\underline{k}}{\underline{k}^2(\underline{k}-\underline{r})^2}\left[\frac{1}{(\underline{k}-\underline{r}a)^2+\mu^2} - \frac{1}{a^2\underline{r}^2+\mu^2} + (a \leftrightarrow \bar{a})\right]$$

has 3 propagators (1 massive)

- perform the inversion on integration variable and parameters:

$$\underline{k} \longrightarrow \frac{\underline{K}}{\underline{K}^2}, \quad \underline{r} \longrightarrow \frac{\underline{R}}{\underline{R}^2}, \quad m \longrightarrow \frac{1}{M}$$

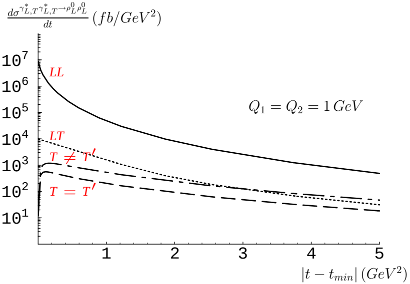
- perform a shift of variable: $\underline{K} = \underline{R} + \underline{k}'$
- perform another inversion
- one then obtains an integral with 2 propagators (1 massive)

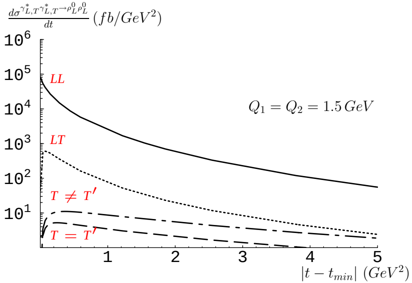
$$J_{3m} = \frac{1}{r^2}\int \frac{d^2\underline{k}}{\underline{k}^2}\left[\frac{(\underline{r}+\underline{k})^2}{(r^2a^2+m^2)\left(\left(\underline{k}-\underline{r}\frac{r^2a\,\bar{a}-m^2}{r^2\bar{a}^2+m^2}\right)^2+\frac{m^2r^4}{(r^2\bar{a}^2+m^2)^2}\right)} - \frac{1}{a^2r^2+m^2} + (a \leftrightarrow \bar{a})\right]$$

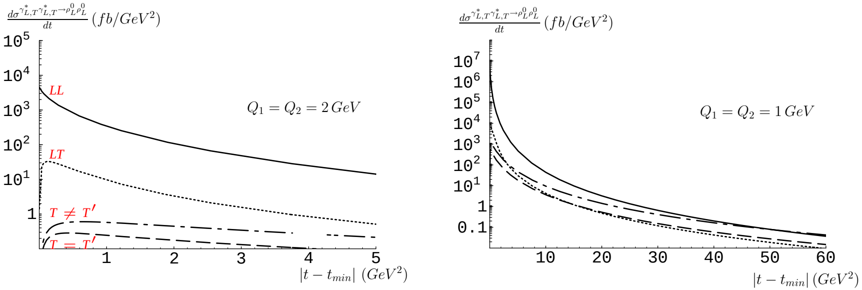
which is easily computed.

- The integration over momentum fractions z_1 and z_2 are performed numerically
- we use Q_1Q_2 as a scale for α_S (3 loops)

differential cross-sections for $\gamma_i^*\gamma_j^* \to \rho_L^0\ \rho_L^0$







- the cross-sections strongly decrease with Q^2 (as $1/Q^8$ for LL)
- any cross-section with at least one tranverse photon vanishes at $t = 0$ (due to s-channel helicity conservation): remember that ρ is longitudinal
- at large t, $\gamma_T^*\gamma_{T'}^*$ dominates (photon are then almost on-shell with respect to t)

Non-forward Born order cross-section for $e^+e^- \to e^+e^- \rho_L^0\ \rho_L^0$

Equivalent photon approximation

$$\gamma^*\gamma^* \to \rho_L^0\ \rho_L^0 \qquad \longrightarrow \qquad e^+e^- \to e^+e^- \rho_L^0\ \rho_L^0$$

using equivalent photon approximation

$$\frac{d\sigma(e^+e^- \to e^+e^- \rho_L^0\rho_L^0)}{dy_1\, dy_2\, dQ_1^2\, dQ_2^2}$$
$$= \frac{1}{y_1 y_2\, Q_1^2 Q_2^2} \left(\frac{\alpha}{\pi}\right)^2 \Big[l(y_1)\, l(y_2)\, \sigma(\gamma_L^*\gamma_L^* \to \rho_L^0\rho_L^0) + t(y_1)\, l(y_2)\, \sigma(\gamma_T^*\gamma_L^* \to \rho_L^0\rho_L^0)$$
$$+ l(y_1)\, t(y_2)\, \sigma(\gamma_L^*\gamma_T^* \to \rho_L^0\rho_L^0) + t(y_1)\, t(y_2)\, \sigma(\gamma_T^*\gamma_T^* \to \rho_L^0\rho_L^0) \Big]$$

with the usual flux factors given by

$$t(y_i) = \frac{1 + (1-y_i)^2}{2}, \quad l(y_i) = 1 - y_i\,,$$

y_i $(i = 1, 2)$ are the longitudinal momentum fractions of the bremsstrahlung photons with respect to the incoming leptons

$s_{\gamma^*\gamma^*} \sim y_1\, y_2\, s_{e^+e^-}$

$\Rightarrow$ $\sigma^{e^+e^- \to e^+e^- \rho_L \rho_L}$ is peaked in the low y and Q^2 region

- photon momentum fractions: (in the laboratory frame = center of mass system (cms) for an e^+e^- collider)

$$y_i = \frac{E - E'_i \cos^2(\theta_i/2)}{E}$$

- virtualities:

$$Q_i^2 = 4EE'_i \sin^2(\theta_i/2)$$

- cross-section peaked at small Q_i^2 and y_i
  $\Rightarrow$ one needs to get access to the (very) forward region
- kinematical constraints:
  - minimal detection angle (detector constraint)
  - conditions on the energies of outgoing leptons (detector constraint)
  - Regge condition

$$y_{i\,max} = 1 - \frac{E_{min}}{E}$$

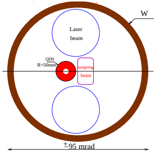
$$y_{1\,min} = \max\left(f(Q_1), 1 - \frac{E_{max}}{E}\right)$$

$$y_{2\,min} = \max\left(f(Q_2), 1 - \frac{E_{max}}{E}, \frac{c\,Q_1\,Q_2}{s\,y_1}\right)$$

with $f(Q_i) = 1 - \frac{Q_i^2}{s\,\tan^2(\theta_{min}/2)}$

Reference Design Report for International Linear Collider

- $\sqrt{s_{e^+e^-}} = 2E_{lepton}$: nominal value of $500\,\mathrm{GeV}$
- high luminosity, with $125\,\mathrm{fb}^{-1}$ per year within 4 years of running at $500\,\mathrm{GeV}$
- possible scan in energy between $200\,\mathrm{GeV}$ and $500\,\mathrm{GeV}$.
- upgrade at $1\,\mathrm{TeV}$, with a luminosity of $1\,\mathrm{ab}^{-1}$ within 3 to 4 years
- two interaction regions are highly desirable: one which could be at low crossing-angle, and one compatible with $e\gamma$ and $\gamma\gamma$ physics (through single or double laser Compton backscattering)
    - at the moment, 3 options are considered: 2 mrad, 14 mrad and 20 mrad
    - in $e\gamma$ and $\gamma\gamma$ modes, for which $\alpha_c > 25\,\mathrm{mrad}$:
        - no BeamCal can be placed around the beampipe in a cone of 12 mrad (angular size of the disrupted outgoing beam after laser Compton backscattering)
        - tiny space for any forward detector in a cone of 95 mrad



Layout of the quad and electron and laser beams at the distance of 4 m from the interaction point

- it thus means that if a single detector would be used at the same interaction point (to reduce the budget devoted to $\gamma\gamma$ mode, this solution without displacement of the detector has been suggested: Telnov), no forward calorimeter like BeamCal could be installed

In the case of e^+e^- mode

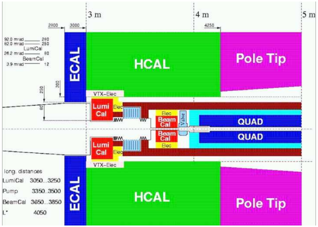
- Each design of detector for ILC project involves a very forward electromagnetic calorimeter for luminosity measurement, with tagging angle for outgoing leptons down to 5 mrad (design 10 years ago were considering 20 mrad as almost impossible!)
- This is an ideal tool for diffractive physics: cross-section are sharply peaked in the very forward region
- luminosity is enough to give high statistics, even with exclusive events
- there are 4 concepts of detectors at the moment:
  - GLD
  - Large Detector Concept (LDC)
  - Silicon Design Detector Study (Sid)
  - 4th

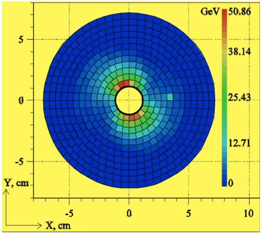
Non-forward Born order cross-section for $e^+e^- \to e^+e^- \rho_L^0\ \rho_L^0$

LDC detector

We focus specifically on the LDC project

- The BeamCal is an electromagnetic calorimeter devoted to luminosity measurement, located at 3.65 m from the vertex

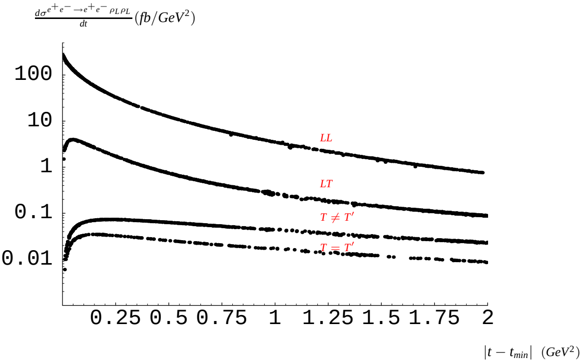




- it can be used for diffractive physics
- the main background is due to beamstrahlung photons, which leads to energy deposit in cells close from the beampipe
  $\Rightarrow$ in practice we cut-off the cells for lepton tagging with

$$E_{min} = 100 \text{ GeV}$$
$$\theta_{min} = 4 \text{ mrad}$$

$$\frac{d\sigma^{e^+e^- \to e^+e^- \rho_L \rho_L}}{dt} = \int_{Q^2_{1min}}^{Q^2_{1max}} dQ_1^2 \int_{Q^2_{2min}}^{Q^2_{2max}} dQ_2^2 \int_{\epsilon}^{y_{max}} dy_1 \int_{\frac{Q_1 Q_2}{s y_1}}^{y_{max}} dy_2 \, \frac{d\sigma^{e^+e^- \to e^+e^- \rho_L \rho_L}}{dt\, dy_1\, dy_2\, dQ_1^2\, dQ_2^2},$$

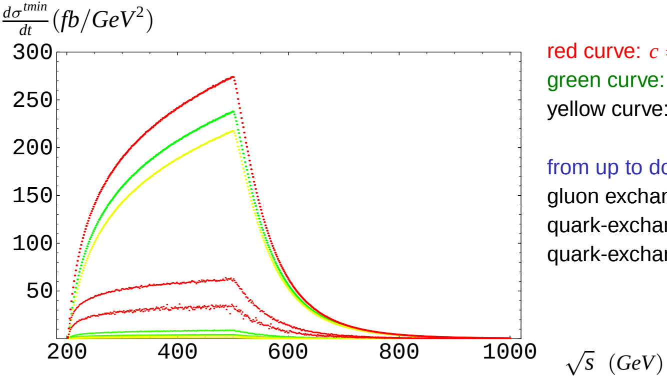


We obtain, at $\sqrt{s_{e^+e^-}} = 500$ GeV (and $c = 1$)

$$\begin{aligned}
\sigma^{LL} &= 32.4 \text{ fb} \\
\sigma^{LT} &= 1.5 \text{ fb} \\
\sigma^{TT} &= 0.2 \text{ fb} \\
\sigma^{tot} &= 34.1 \text{ fb}
\end{aligned}$$

which leads to $4.3\,10^3$ events per year with foreseen luminosity

- the background (dominated by γ which would be misidentified in BeamCal as e^+ or e^-) is completely negligible at $\sqrt{s_{e^+e^-}} = 500$ GeV
- quarks contribution are indeed negligible. This is related to c through $s_{\gamma^*\gamma^*} > c\,Q_1\,Q_2$
- more drastic Regge constraint by performing $c = 1 \to c = 10$ reduces the cross-section by 40% ⇒still statistically measurable
- changing order of loop for α_S only has a few % effect



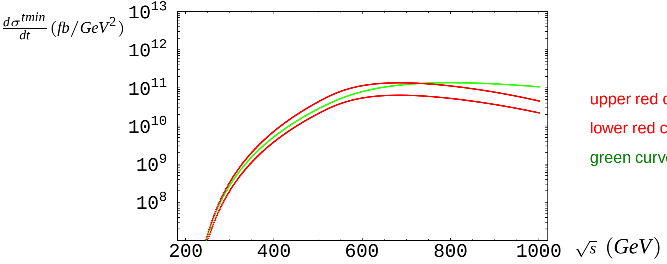
red curve: $c = 1$
green curve: $c = 2$
yellow curve: $c = 3$

from up to down:
gluon exchange
quark-exchange with γ_L^*
quark-exchange with γ_T^*

Non-forward Born order cross-section for $e^+e^- \to e^+e^- \rho_L^0\ \rho_L^0$

BFKL enhancement

LL BFKL differential cross-section at $t = t_{min}$



upper red curve: $\alpha_S(\sqrt{Q_1\,Q_2})$ at one loop

lower red curve: $\alpha_S(\sqrt{Q_1\,Q_2})$ at three loop

green curve: $\alpha_S = 0.46$ fixed

- Enhancement is enormous but not trustable: it is well known that NLL BFKL is far below LL BFKL and almost always above Born (cf HERA, LEP) .
- At the level of $\gamma^*\gamma^*$, corrections to LL BFKL have been studied earlier
  - resummed BFKL à la Khoze, Martin, Ryskin, Stirling (based on Salam): Enberg, Pire, Szymanowski, S.W with LL impact factor and BLM scale fixing
  - NLL BFKL with NLL impact factor: Ivanov, Papa.
  - Both approaches are compatible within a few %
- Work to implement this resummed BFKL effects at e^+e^- level is in progress. Trends:
  - enhancement less dramatic (~ 5) but still visible
  - due to detector constraint, the expected increase of the cross-section with $\sqrt{s_{e^+e^-}}$ is washed-out for $\sqrt{s_{e^+e^-}} > 500$ GeV: sharked curves, with Born level clearly below resummed BFKL

Summary (1)

- $\gamma^* \, \gamma^* \to \rho^0_L \rho^0_L$ is a very nice process for studying QCD dynamics in its perturbative regime, with a minimal onset of non-perturbative physics
- At low energy, it is dominated by quark exchange
  - Its perturbative analysis in the Born approximation, in the forward case, leads to two different types of QCD factorization
  - We have shown that the polarization states of the photons dictate either the factorization involving a GDA or involving a TDA.
  - Usually these two types of factorizations are applied to two different kinematical regimes.
  - The arbitrariness in choosing values of photon virtualities Q_i^2 shows that there may exist an intersection region where both types of factorization are simultaneously valid.
  - the obtained TDA contains a perturbative part which could give a hint for modelling in non perturbative cases
  - further generalizations:
    - non-forward kinematics (rather easy)
    - transverse photon (hard: higher twist contributes)
    - charged meson pair (hard: non-trivial QED gauge invariance)
  - the measure could be done at Babar, Belle, BEPC-II,...,ILC

Summary (2)

- At high energy, it is dominated by gluon exchange
  - we gave a precise estimation of the Born order cross-section for arbitrary photon polarizations
  - we have demonstrated the feasability of the measurement at the level of $e^+e^- \rightarrow e^+e^- \rho_L^0\ \rho_L^0$ with double tagged outgoing leptons, within ILC collider and LDC detector with a forward electromagnetic calorimeter
  - this evaluation can be considered as the background for any resummation à la BFKL
  - we have made a fi rst estimate of BFKL evolution at LL, to be dramatically modifi ed by higher order corrections
  - we are now implementing our previous estimate of resummed BFKL evolution for $\gamma^*\ \gamma^* \rightarrow \rho_L^0 \rho_L^0$ at $e^+e^- \rightarrow e^+e^- \rho_L^0\ \rho_L^0$ level: enhancement with respect to Born is still there, but moderate (~ 5) (results to come soon)
  - there is a potential very interesting possibility of entering smoothly into the non-linear saturation regime when the machine would be upgraded up to 1 TeV:
    - at $\sqrt{s_{e^+e^-}} = 500$ GeV, $Q_{sat} \sim 1.1$ GeV saturation is at the border, almost negligible
    - at $\sqrt{s_{e^+e^-}} = 1$ TeV, $Q_{sat} \sim 1.4$ GeV saturation effects should start to be rather important (but still in the almost linear regime)
    - $\gamma^*\ \gamma^*$ total cross-section as well as $\gamma^*\ \gamma^*$ exclusive processes are very symmetrical; usual saturation studies are made in typically non-symmetrical situation ($e^\pm - p$ and $e^\pm - A$ DIS) $\Rightarrow$ further formal developments are required