QCD factorization for $\gamma^*\gamma^* \to \rho_L^0 \rho_L^0$

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Blois 2007, DESY, Hamburg

based on:

Diffractive production of two ρ_L^0 mesons in e+e- collisions

M. Segond, L. Szymanowski, S. W. [hep-ph/0703166]

QCD factorizations in $\gamma^* \gamma^* \to \rho\rho$

B. Pire, M. Segond, L. Szymanowski, S. W. Phys.Lett.B639:642-651,2006 [hep-ph/0605320]

BFKL resummation effects in $\gamma^* \gamma^* \to \rho\rho$

R. Enberg, B. Pire, L. Szymanowski , S. W. Eur.Phys.J.C45:759-769,2006 [hep-ph/0508134]

Double diffractive rho-production in $\gamma^* \gamma^*$ collisions

B. Pire, L. Szymanowski, S. W. Eur.Phys.J.C44:545-558,2005 [hep-ph/0507038]

Outline

1. Introduction: Exclusive processes at high energy QCD
   - Motivation
   - GDA and TDA for $\gamma^*\gamma^* \to \rho_L^0\rho_L^0$
2. Computation in the collinear factorization with DA at fixed W^2
   - Direct calculation
   - Interpretation in terms of QCD Factorization
     - GDA for transverse photon in the limit $\Lambda_{QCD} \ll W^2 \ll Max(Q_1^2, Q_2^2)$
     - TDA for longitudinal photon in the limit $Q_1^2 \gg Q_2^2$ (or $Q_1^2 \ll Q_2^2$)
3. Computation at large W^2 : k_T factorization approach
   - Motivation and aims
   - k_T factorization
   - Non-forward Born order cross-section for $\gamma^*\gamma^* \to \rho_L^0\ \rho_L^0$
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Since a decade, there have been much developpements in hard exclusive processes.

- form factors $\rightarrow$ Distribution Amplitudes
- DVCS$\rightarrow$ Generalized Parton Distributions,
- $\cdots$

These tests are possible in fixed target experiments

- $e^{\pm}p$ HERA (HERMES), JLab, ...

as well as in colliders, mainly for fixed s

- $e^{\pm}p$ colliders: HERA (H1,ZEUS)
- $e^{+}e^{-}$ colliders: LEP, Belle, BaBar, BEPC

At the same time, the interest for phenomenological tests of hard Pomeron and related resummed approaches has become pretty wide:

- inclusive tests (total cross-section) and semi-inclusive tests (diffraction, forward jets, ...)
- exclusive tests (meson production)

These tests concern all type of collider experiments:

- $e^{\pm}p$ (HERA: H1, ZEUS)
- $p\bar{p}$ (TEVATRON: CDF, D0)
- $e^{+}e^{-}$ colliders (LEP, ILC)

We will focus on a specific exclusive process:

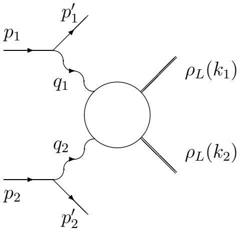
$$\boxed{\gamma^*\gamma^* \longrightarrow \rho_L^0 \rho_L^0} \qquad \text{with both } \gamma^* \text{ hard}$$

It is a beautiful theoretical laboratory for investigating different dynamics (collinear, multiregge) and factorization properties of high energy QCD:

- at low energy (fixed s) it provides an (almost) full perturbative laboratory for extended GPDs: GDA and TDA
- at high energy (asymptotic s) it provides an (almost) full perturbative laboratory for BFKL and related resummed effects, at amplitude level.

The corresponding experimental process is

$$e^+e^- \rightarrow e^+e^- \rho_L^0 \rho_L^0$$



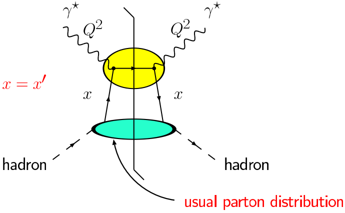
with double tagged outoing leptons.

Outline

- DIS: inclusive process $\to$ forward amplitude ($t = 0$)

Structure Function

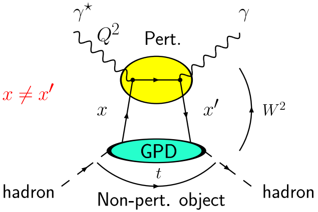
= Coefficient Function (hard) $\otimes$ Parton Distribution Function (soft)



- DVCS: exclusive process $\to$ non forward amplitude ($-t \ll s = W^2$)

Amplitude

= Coefficient Function (hard) $\otimes$ Generalized Parton Distribution (soft)

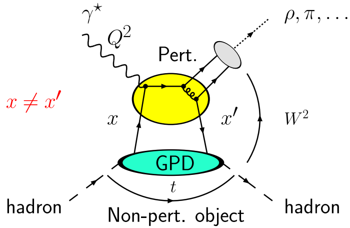


Extensions:

- Meson production: γ replaced by $\rho,\ \pi, \cdots$

Amplitude

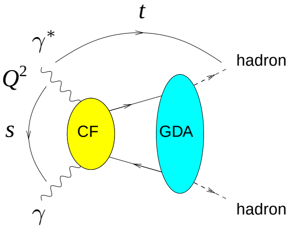
= GPD (soft) $\otimes$ CF (hard) $\otimes$ Distribution Amplitude (soft)



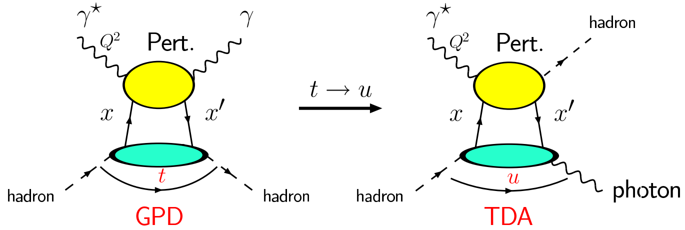
- Crossed process: $s \ll -t$

Amplitude

= Coefficient Function (hard) $\otimes$ Generalized Distribution Amplitude (soft)



- starting from usual DVCS, one allows initial hadron $\neq$ final hadron
  example:

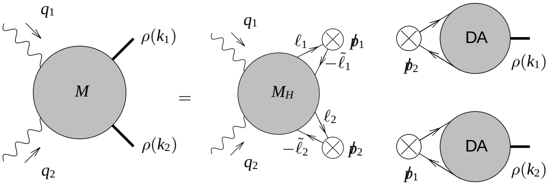


which can be further extended by replacing the outoing γ by any hadronic state

Amplitude = Transition Distribution Amplitude (soft) $\otimes$ CF (hard) $\otimes$ DA (soft)

Collinear factorization at $q\bar{q}\rho$ vertices

$Q^2_{1,2}$: hard scales $\Rightarrow$ collinear approximation at each $q\bar{q}\rho$ vertex



i.e. we neglect the transverse relative (anti-)quark momenta in the ρ mesons:

$$\ell_1 \sim z_1\, k_1 \qquad \ell_2 \sim z_2\, k_2$$
$$\tilde{\ell}_1 \sim \bar{z}_1\, k_1 \qquad \tilde{\ell}_2 \sim \bar{z}_2\, k_2$$

We limit ourselves to longitudinaly polarized mesons (to avoid potential end-point singularies due to higher twist contributions)

DA of the meson = matrix element of non local quarks fields correlator on the light cone

$$\langle 0|\bar{q}(x)\,\gamma^\mu\, q(-x)|\rho_L(p) = \bar{q}q\rangle = f_\rho\, p^\mu \int_0^1 dz\, e^{i(2z-1)(px)}\phi(z)$$

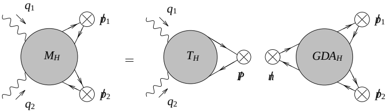
with $$\phi(z) = 6z(1-z)\,(1+\sum_{n=1}^{\infty} a_{2n} C^{3/2}_{2\,n}(2z-1))$$

Note: $p_1\,,\, p_2$ are light-like Sudakov vectors along the meson momenta.

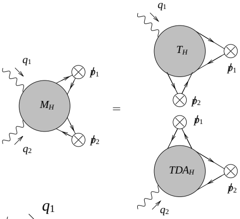
We will now consider two types of treatment for the hard part M_H

- at moderate W^2 $(\gg \Lambda^2_{QCD})$, we perform the direct calculation.
  We then show that it can be presented in a QCD factorized form involving

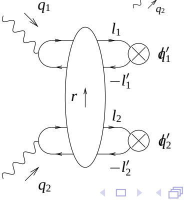
  - either a GDA for $W^2 \ll Max(Q_1^2, Q_2^2)$



  - or a TDA for $Q_1^2 \ll Q_2^2$ or $Q_1^2 \gg Q_2^2$



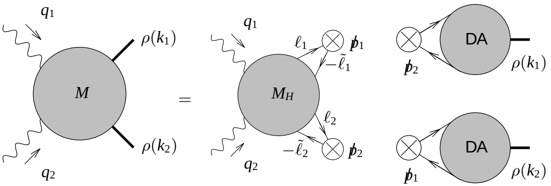
- at asymptotically large W^2,
  k_T factorization involving impact factors



Outline

Computation in the collinear factorization with DA, at fixed W^2

Direct calculation



- The computation follows the line of the Brodsky, Lepage approach.
- We consider the Born order, i.e. quark exchange.
- We restrict ourselves to the forward case
- We only consider longitudinally polarized mesons $\Rightarrow$ leading twist

The amplitude can be expressed as the sum of two tensors:

$$\mathcal{M} = T^{\mu\,\nu}\epsilon_\mu(q_1)\epsilon_\nu(q_2)$$

with

$$T^{\mu\,\nu} = \frac{1}{2}g_T^{\mu\,\nu}\,T^{\alpha\,\beta}g_{T\,\alpha\,\beta} + \left(p_1^\mu + \frac{Q_1^2}{s}p_2^\mu\right)\left(p_2^\nu + \frac{Q_2^2}{s}p_1^\nu\right)\frac{4}{s^2}T^{\alpha\,\beta}p_{2\,\alpha}\,p_{1\,\beta}$$

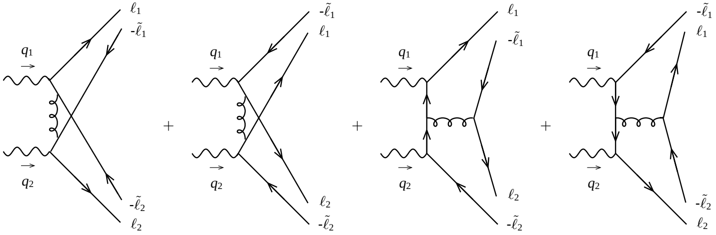
$$g_T^{\mu\,\nu} = g^{\mu\nu} - \frac{p_1^\mu p_2^\nu + p_1^\nu p_2^\mu}{p_1.p_2}$$

- The photons polarization vectors read

$$\epsilon_{\parallel}(q_1) = \frac{1}{Q_1} q_1 + \frac{2Q_1}{s} p_2 \quad \text{and} \quad \epsilon_{\parallel}(q_2) = \frac{1}{Q_2} q_2 + \frac{2Q_2}{s} p_1 \,.$$

- use QED gauge invariance
- remember that we only consider the forward kinematics

$\Rightarrow$ the number of diagrams reduces to 4



$$T^{\alpha\,\beta}p_{2\,\alpha}\,p_{1\,\beta} = -\frac{s^2 f_\rho^2 C_F e^2 g^2 (Q_u^2 + Q_d^2)}{8 N_c Q_1^2 Q_2^2} \int_0^1 dz_1\, dz_2\, \phi(z_1)\, \phi(z_2)$$

$$\times \left\{ \frac{(1-\frac{Q_1^2}{s})(1-\frac{Q_2^2}{s})}{(z_1+\bar{z}_1\frac{Q_2^2}{s})(z_2+\bar{z}_2\frac{Q_1^2}{s})} + \frac{(1-\frac{Q_1^2}{s})(1-\frac{Q_2^2}{s})}{(\bar{z}_1+z_1\frac{Q_2^2}{s})(\bar{z}_2+z_2\frac{Q_1^2}{s})} + \frac{1}{z_2\bar{z}_1} + \frac{1}{z_1\bar{z}_2} \right\}$$

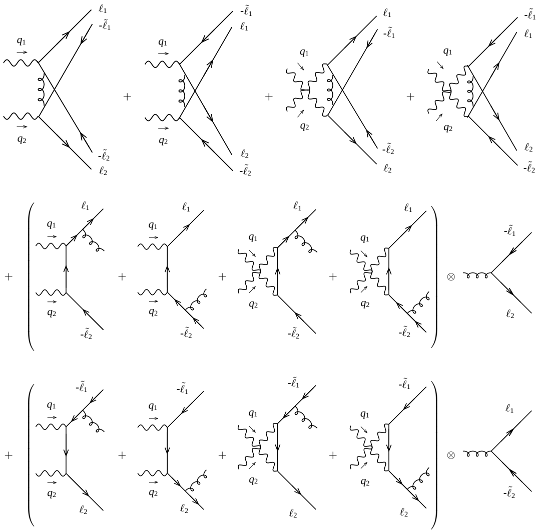
with $s = 2\,p_1 \cdot p_2$

Note:

Q_1^2 and Q_2^2 are non-zero and DA vanishes at $z_i = 0$

$\Rightarrow$no end-point singularity in the z_i integration

In this case no simplification occurs. One needs to compute 12 diagrams.



$$T^{\alpha\,\beta} g_{T\,\alpha\,\beta} \;=\; -\frac{e^2 (Q_u^2 + Q_d^2)\, g^2\, C_F\, f_\rho^2}{4\, N_c\, s} \int\limits_0^1 dz_1\, dz_2\, \phi(z_1)\, \phi(z_2)$$

$$\times \left\{ 2 \left(1 - \frac{Q_2^2}{s}\right) \left(1 - \frac{Q_1^2}{s}\right) \left[\frac{1}{(z_2 + \bar{z}_2 \frac{Q_1^2}{s})^2 (z_1 + \bar{z}_1 \frac{Q_2^2}{s})^2} + \frac{1}{(\bar{z}_2 + z_2 \frac{Q_1^2}{s})^2 (\bar{z}_1 + z_1 \frac{Q_2^2}{s})^2} \right] + \right.$$

$$\left. \left(\frac{1}{\bar{z}_2\, z_1} - \frac{1}{\bar{z}_1\, z_2} \right) \left[\frac{1}{1 - \frac{Q_2^2}{s}} \left(\frac{1}{\bar{z}_2 + z_2 \frac{Q_1^2}{s}} - \frac{1}{z_2 + \bar{z}_2 \frac{Q_1^2}{s}} \right) - \frac{1}{1 - \frac{Q_1^2}{s}} \left(\frac{1}{\bar{z}_1 + z_1 \frac{Q_2^2}{s}} - \frac{1}{z_1 + \bar{z}_1 \frac{Q_2^2}{s}} \right) \right] \right\}$$

Same remark:

Q_1^2 and Q_2^2 are non-zero and DA vanishes at $z_i = 0$

$\Rightarrow$ no end-point singularity in the z_i integration

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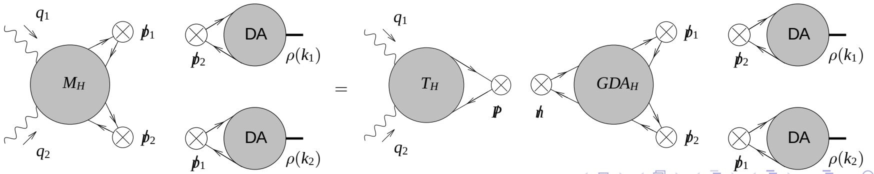
When W^2 is smaller then the highest photon virtuality

For example $\frac{W^2}{Q_1^2} = \frac{s}{Q_1^2}\left(1 - \frac{Q_1^2}{s}\right)\left(1 - \frac{Q_2^2}{s}\right) \approx 1 - \frac{Q_1^2}{s} \ll 1$

the result obtained from direct calculation simplifies into

$$T^{\alpha\beta} g_{T\,\alpha\beta} \approx \frac{e^2 (Q_u^2 + Q_d^2)\, g^2\, C_F\, f_\rho^2}{4\, N_c\, W^2} \times \int_0^1 dz_1\, dz_2 \left(\frac{1}{\bar{z}_1 + z_1 \frac{Q_2^2}{s}} - \frac{1}{z_1 + \bar{z}_1 \frac{Q_2^2}{s}} \right) \left(\frac{1}{\bar{z}_2\, z_1} - \frac{1}{\bar{z}_1\, z_2} \right) \phi(z_1)\, \phi(z_2)$$

which can be interpreted as ($P \sim p_1,\; n \sim p_2$)



Interpretation in terms of QCD Factorization

GDA for transverse photon in the limit $\Lambda^2_{QCD} \ll W^2 \ll Max(Q_1^2, Q_2^2)$: PROOF

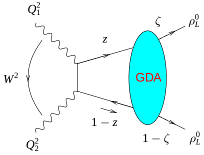
GDA computation

At leading twist, the GDA is calculated in the Born order of perturbation theory

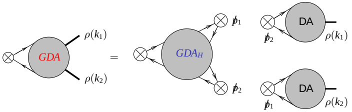
$$\langle \rho_L^0(k_1)\, \rho_L^0(k_2) | \bar{q}(-\alpha\, n/2)\, \not{n}\, \exp\left[ig \int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} dy\, n_\nu\, A^\nu(y) \right] q(\alpha\, n/2) | 0 \rangle$$

$$= \int_0^1 dz\, e^{-i(2z-1)\alpha(nP)/2}\, \Phi^{\rho_L^0 \rho_L^0}(z, \zeta, W^2)$$

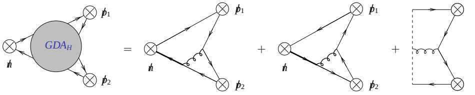
$(P \sim p_1$ and $n \sim p_2$ for $Q_1 > Q_2)$



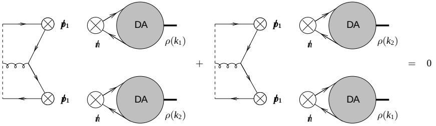
Since W^2 is hard, the GDA can be factorized:



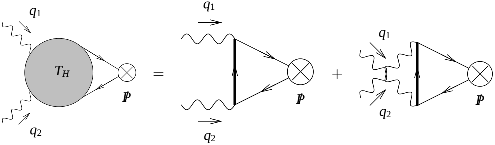
with



In our kinematics, the QCD Wilson line vanishes:



Hard Part computation at Born order



In the case of one flavored quark, it equals:

$$T_H(z) = -4\, e^2\, N_c\, Q_q^2 \left(\frac{1}{\bar{z} + z\, \frac{Q_2^2}{s}} - \frac{1}{z + \bar{z}\, \frac{Q_2^2}{s}} \right)$$

We have thus shown that $T^{\alpha\,\beta} g_{T\,\alpha\,\beta}$ factorizes into Hard part $\otimes$ GDA:

$$T^{\alpha\,\beta} g_{T\,\alpha\,\beta} = \frac{e^2}{2}\left(Q_u^2 + Q_d^2\right)\int_0^1 dz\,\left(\frac{1}{\bar{z} + z\frac{Q_2^2}{s}} - \frac{1}{z + \bar{z}\frac{Q_2^2}{s}}\right)\Phi^{\rho_L\,\rho_L}(z, \zeta \approx 1, W^2)$$

with the GDA which itself factorizes into Hard part $\otimes$ DA DA:

$$\Phi^{\rho_L\,\rho_L}(z, \zeta \approx 1, W^2) = -\frac{f_\rho^2\, g^2\, C_F}{2\, N_c\, W^2}\int_0^1 dz_2\,\phi(z)\,\phi(z_2)\left[\frac{1}{z\bar{z}_2} - \frac{1}{\bar{z}z_2}\right]$$

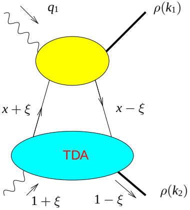
- This is a limiting case of the original equation obtained by D. Müller et al (2000)
- It extends the studies of $\gamma^*\gamma \to \pi\pi$ by M. Diehl et al (2000)
- We limited ourselves to the case of $t = t_{min}$

Interpretation in terms of QCD Factorization

TDA for longitudinal photon in the limit $Q_1^2 \gg Q_2^2$ (or $Q_1^2 \ll Q_2^2$)

The direct calculation of the amplitude $M = T^{\alpha\,\beta} p_{2\,\alpha}\, p_{1\,\beta}$ can be interpreted, in the limiting case $Q_1^2 \gg Q_2^2$ (or $Q_1^2 \ll Q_2^2$), as

$$M = TDA \otimes CF \otimes DA$$



TDA kinematics = GPD kinematics

$n_1 = (1+\xi)p_1$ and $n_2 = \frac{p_2}{1+\xi}$

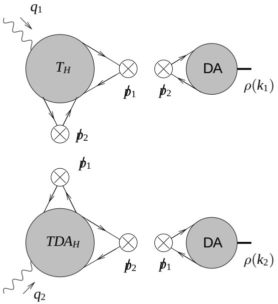
x, ξ are momentum fraction along $n_2 = \frac{p_2}{1+\xi}$

More precisely, we prove that M factorizes as

$$T^{\alpha\,\beta} p_{2\,\alpha} p_{1\,\beta}$$

$$= -if_\rho^2 e^2 (Q_u^2 + Q_d^2) g^2 \frac{C_F}{8N_C} \int_{-1}^{1} dx \int_0^1 dz_1 \left[\frac{1}{\bar{z}_1(x-\xi)} + \frac{1}{z_1(x+\xi)}\right] \phi(z_1)$$

$$\times N_c \left[\Theta(1 \geq x \geq \xi)\, \phi\left(\frac{x-\xi}{1-\xi}\right) - \Theta(-\xi \geq x \geq -1)\, \phi\left(\frac{1+x}{1-\xi}\right)\right]$$



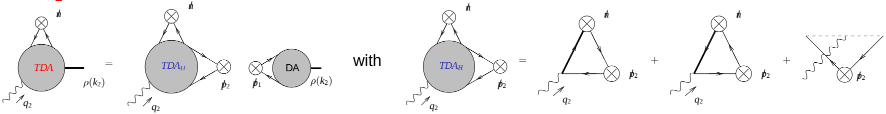
TDA computation at Born order

The TDA $\gamma^* \longrightarrow \rho_L^0$ is defined through ($n \sim n_1$)

$$\int \frac{dz^-}{2\pi}\, e^{ix(P.z)}\, \langle \rho_L^q(k_2)|\bar{q}(-z/2)\, \not{n}\, e^{-ieQ_q \int_{z/2}^{-z/2} dy_\mu\, A^\mu(y)}\, q(z/2)|\gamma^*(q_2)\rangle$$
$$= \frac{e\, Q_q\, f_\rho}{P^+}\, \frac{2}{Q_2^2}\, \epsilon_\nu(q_2)\left((1+\xi)n_2^\nu + \frac{Q_2^2}{s(1+\xi)}n_1^\nu\right)\, T(x, \xi, t_{min})\ ,$$

where the QED Wilson line is explicitly indicated (QCD Wilson line gives no contribution)

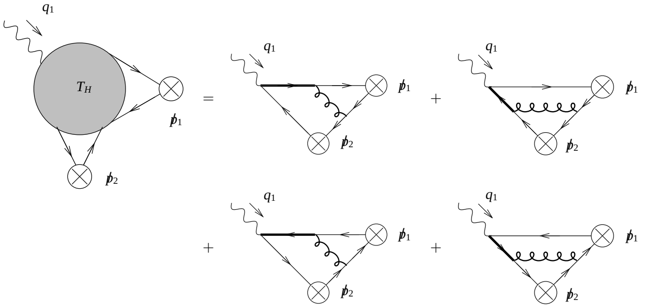
Since Q_2^2 is hard, the TDA can be factorized:



Explicit computation gives

$$T(x, \xi, t_{min}) \equiv N_c\left[\Theta(1 \geq x \geq \xi)\, \phi\left(\frac{x-\xi}{1-\xi}\right) - \Theta(-\xi \geq x \geq -1)\, \phi\left(\frac{1+x}{1-\xi}\right)\right]$$

Hard computation at Born order



$$T_H(z_1, x) = -i f_\rho \, g^2 \, e \, Q_q \, \frac{C_F \, \phi(z_1)}{2 \, N_c \, Q_1^2} \epsilon^\mu(q_1) \left(2\xi \, n_{2\,\mu} + \frac{1}{1+\xi} n_{1\,\mu} \right)$$
$$\times \left[\frac{1}{z_1(x+\xi-i\epsilon)} + \frac{1}{\bar{z}_1(x-\xi+i\epsilon)} \right],$$

We have shown, at Born order, that $T^{\alpha\,\beta} p_{2\,\alpha} p_{1\,\beta}$ factorizes into TDA $\otimes$ Hard part $\otimes$ DA:

$$T^{\alpha\,\beta} p_{2\,\alpha} p_{1\,\beta}$$

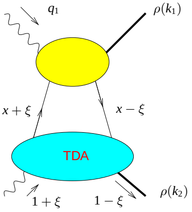
$$= \quad -i f_\rho^2 e^2 (Q_u^2 + Q_d^2) g^2 \, \frac{C_F}{8N_c} \int_{-1}^{1} dx \int_{0}^{1} dz_1 \, T(x,\xi,t_{min}) \left[\frac{1}{\bar{z}_1(x-\xi)} + \frac{1}{z_1(x+\xi)} \right] \phi(z_1)$$

with the TDA which itself factorizes into Hard part $\otimes$ DA:

$$T(x,\xi,t_{min}) \equiv N_c \left[\Theta(1 \geq x \geq \xi)\, \phi\left(\frac{x-\xi}{1-\xi}\right) - \Theta(-\xi \geq x \geq -1)\, \phi\left(\frac{1+x}{1-\xi}\right) \right]$$

Note:

Only the DGLAP part of the TDA contributes because of support properties of the ρ meson DA



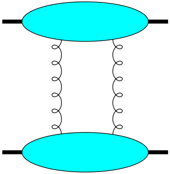
DGLAP(1)	$-1 \leq x \leq -\xi$
ERBL	$-\xi \leq x \leq \xi$
DGLAP(2)	$\xi \leq x \leq 1$

Outline

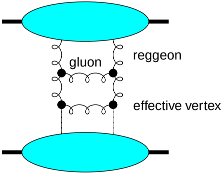
QCD in perturbative Regge limit

- In this limit, the dynamics is dominated by gluons (dominance of spin 1 exchange in t channel)
- BFKL (and extensions: NLL, saturations effects, ...) is expected to dominates with respect to Born order at large relative rapidity.

Born order:



BFKL ladder:



Computation at large W^2 : k_T factorization approach

Theoretical motivations

$e^+e^- \to e^+e^- \rho_L^0\ \rho_L^0$ is a good observable in order to test this limit:

- IR-safe probes: double tagging of the final leptons and cut-off over soft photons $\Rightarrow$ the hard virtual photons give the hard scales on both sides of the t-channel exchanged state $\Rightarrow$ fully perturbative process (except for DAs of ρ).
- observable dominated by the "soft" (but still perturbative) dynamics of QCD (BFKL and extensions) and not by its collinear dynamics (DGLAP, ERBL): we impose $Q_1^2 \sim Q_2^2$
- gives access to the interplay between collinear and soft dynamics by getting away from $Q_1^2 \sim Q_2^2$ domain and by playing with the relative rapidity
- one can control the spread in k_T of the partons: transition from linear to non-linear (saturated regime), when increasing $s_{\gamma^*\gamma^*}$ for given values Q_1^2 and Q_2^2. Experimentally feasible by increasing $s_{e^+e^-}$
- it gives access to non-forward dynamics
  - can reveal Pomeron structure apart from the forward limit
  - for saturation studies, it is important to get a full impact parameter picture of the process (Froissart bound is for each impact parameter)
  - Note that for $t = 0$, the simplest model for non-linearity is the Balitskii Kovchegov equation
- cross-section are expected to be peaked in the forward limit $\Rightarrow$ the forward differential cross-section gives the general trends

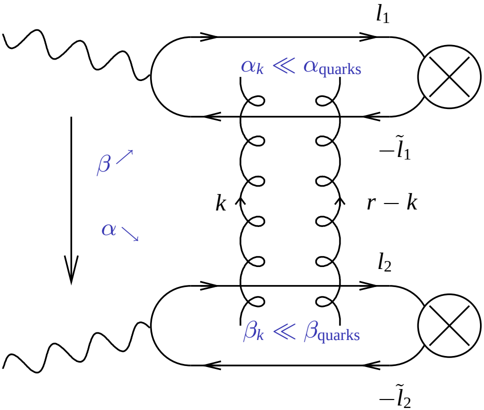
- Compute the scattering amplitude for $\gamma^*\gamma^* \to \rho_L^0\ \rho_L^0$ with gluon exchange, in the range $s_{\gamma^*\gamma^*} \gg -t, Q_1^2, Q_2^2$ for every photons polarizations and check dominance with respect to quarks exchange at ILC energies
- We focus on $Q_1^2 \sim Q_2^2 \Rightarrow$ no DGLAP evolution (this is practically imposed by the small range in both Q_i^2 due to the lower perturbative cut-off and by the fast decreasing amplitude as powers of Q_i^2)
- We prove the experimental feasability at ILC, with LDC detector project
- Study linear and non linear dynamical effects, and the expected enhancement at large rapidity

Outline

- Use Sudakov decomposition $k = \alpha p_1 + \beta p_2 + k_\perp$
- write

$$d^4k = \frac{s}{2}\, d\alpha\, d\beta\, d^2k_\perp$$

and rearrange integrations in the large s limit:



$\Rightarrow$ set $\alpha_k = 0$ and $\int d\beta$

$\Rightarrow$ set $\beta_k = 0$ and $\int d\alpha$

$\Rightarrow$ impact representation (written here for the whole process) note: $\underline{k}$ = Eucl. $\leftrightarrow$ $k_\perp$ = Mink.

$$\mathcal{M} = is \int \frac{d^2\,\underline{k}}{(2\pi)^4 \underline{k}^2\,(\underline{r}-\underline{k})^2}\,\mathcal{J}^{\gamma^*_{L,T}(q_1)\to\rho^0_L(k_1)}(\underline{k},\underline{r}-\underline{k})\;\mathcal{J}^{\gamma^*_{L,T}(q_2)\to\rho^0_L(k_2)}(-\underline{k},-\underline{r}+\underline{k})$$

- For longitudinally polarized photons the impact factor reads

$$\mathcal{J}^{\gamma^*_L(q_i)\to\rho_L(k_i)}(\underline{k},\underline{r}-\underline{k}) = 8\pi^2\alpha_s\frac{e}{\sqrt{2}}\frac{\delta^{ab}}{2N_c}Q_i\,f_\rho\,\alpha(k_i)\int_0^1 dz_i z_i\,\bar{z}_i\,\phi(z_i)\mathrm{P_P}(z_i,\underline{k},\underline{r},\mu_i)$$

where

$$\mathrm{P_P}(z_i,\underline{k},\underline{r},\mu_i) = \frac{1}{z_i^2\underline{r}^2+\mu_i^2}+\frac{1}{\bar{z}_i^2\underline{r}^2+\mu_i^2}-\frac{1}{(z_i\underline{r}-\underline{k})^2+\mu_i^2}-\frac{1}{(\bar{z}_i\underline{r}-\underline{k})^2+\mu_i^2}\propto\mathcal{J}^{\gamma^*_L(q_i)\to q\,\bar{q}}$$

- For transversally polarized photons, one obtains

$$\mathcal{J}^{\gamma^*_T(q_i)\to\rho_L(k_i)}(\underline{k},\underline{r}-\underline{k}) = 4\pi^2\alpha_s\frac{e}{\sqrt{2}}\frac{\delta^{ab}}{2N_c}f_\rho\alpha(k_i)\int_0^1 dz_i\,(z_i-\bar{z}_i)\,\phi(z_i)\,\underline{\epsilon}\cdot\underline{\mathrm{Q}}(z_i,\underline{k},\underline{r},\mu_i)$$

where

$$\underline{\mathrm{Q}}(z_i,\underline{k},\underline{r},\mu_i) = \frac{z_i\,\underline{r}}{z_i^2\underline{r}^2+\mu_i^2}-\frac{\bar{z}_i\,\underline{r}}{\bar{z}_i^2\underline{r}^2+\mu_i^2}+\frac{\underline{k}-z_i\,\underline{r}}{(z_i\underline{r}-\underline{k})^2+\mu_i^2}-\frac{\underline{k}-\bar{z}_i\underline{r}}{(\bar{z}_i\,\underline{r}-\underline{k})^2+\mu_i^2}\propto\mathcal{J}^{\gamma^*_T(q_i)\to q\,\bar{q}}$$

we denote $\mu_i^2 = Q_i^2\;z_i\;\bar{z}_i + m^2$, where m is the quark mass (set to zero in practice)

- due to QCD gauge invariance (probes are colorless), both impact factor vanishes when $\underline{k}\to 0$ or $\underline{r}-\underline{k}\to 0$

Outline

1. Introduction: Exclusive processes at high energy QCD
   - Motivation
   - GDA and TDA for $\gamma^*\gamma^* \to \rho_L^0\rho_L^0$
2. Computation in the collinear factorization with DA at fixed W^2
   - Direct calculation
   - Interpretation in terms of QCD Factorization
     - GDA for transverse photon in the limit $\Lambda_{QCD} \ll W^2 \ll Max(Q_1^2, Q_2^2)$
     - TDA for longitudinal photon in the limit $Q_1^2 \gg Q_2^2$ (or $Q_1^2 \ll Q_2^2$)
3. Computation at large W^2 : k_T factorization approach
   - Motivation and aims
   - k_T factorization
   - Non-forward Born order cross-section for $\gamma^*\gamma^* \to \rho_L^0\ \rho_L^0$
     - Analytical 2-dimensional integration through conformal transformations
     - Results
   - Non-forward Born order cross-section for $e^+e^- \to e^+e^-\rho_L^0\ \rho_L^0$
     - Equivalent photon approximation
     - Kinematical cuts
     - ILC collider and LDC detector
     - Born result
     - LL BFKL enhancement

- All the 2-d integrations with respect to $\underline{k}$ are treated analytically
- The method relies on conformal transformation in the transverse momentum plane (method inspired by Vassiliev in 2-d coordinate space)
- The idea is to reduce the number of propagators, in order to be able to perform standard Feynman parameter integration
- the whole computation involves integrals with up to 4 propagators (2 massive, with different masses) which we would have been enable to compute without this method

The integral ($\bar{a} \equiv 1 - a$)

$$J_{3\mu}(a) = \int \frac{d^2\underline{k}}{\underline{k}^2(\underline{k}-\underline{r})^2}\left[\frac{1}{(\underline{k}-\underline{r}a)^2+\mu^2} - \frac{1}{a^2\underline{r}^2+\mu^2} + (a \leftrightarrow \bar{a})\right]$$

has 3 propagators (1 massive)

- perform the inversion on integration variable and parameters:

$$\underline{k} \longrightarrow \frac{\underline{K}}{\underline{K}^2}, \quad \underline{r} \longrightarrow \frac{\underline{R}}{\underline{R}^2}, \quad m \longrightarrow \frac{1}{M}$$

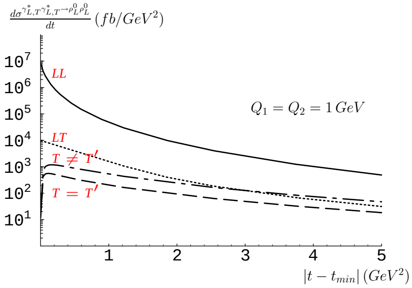
- perform a shift of variable: $\underline{K} = \underline{R} + \underline{k}'$
- perform another inversion
- one then obtains an integral with 2 propagators (1 massive)

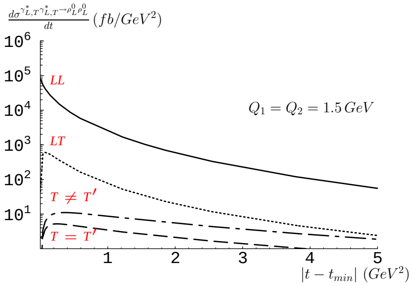
$$J_{3m} = \frac{1}{r^2}\int \frac{d^2\underline{k}}{\underline{k}^2}\left[\frac{(\underline{r}+\underline{k})^2}{(r^2a^2+m^2)\left(\left(\underline{k}-\underline{r}\frac{r^2a\,\bar{a}-m^2}{r^2\bar{a}^2+m^2}\right)^2+\frac{m^2r^4}{(r^2\bar{a}^2+m^2)^2}\right)} - \frac{1}{a^2r^2+m^2} + (a \leftrightarrow \bar{a})\right]$$

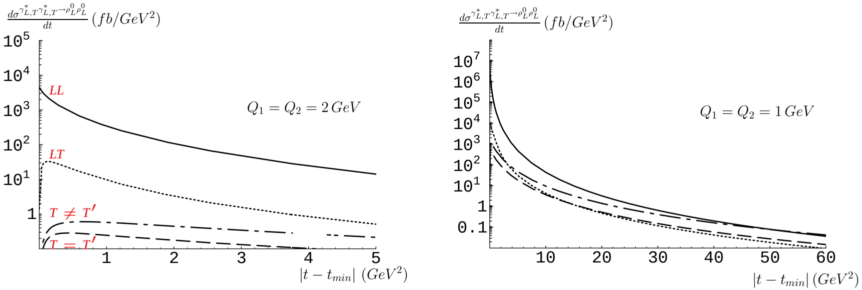
which is easily computed.

- The integration over momentum fractions z_1 and z_2 are performed numerically
- we use $Q_1 Q_2$ as a scale for α_S (3 loops)

differential cross-sections for $\gamma_i^*\gamma_j^* \to \rho_L^0\ \rho_L^0$







- the cross-sections strongly decrease with Q^2 (as $1/Q^4$ for LL)
- any cross-section with at least one tranverse photon vanishes at $t = 0$ (due to s-channel helicity conservation): remember that ρ is transverse
- at large t, $\gamma_T^*\gamma_{T'}^*$ dominates (photon are then almost on-shell with respect to t)

Outline

$$\gamma^*\gamma^* \to \rho_L^0\ \rho_L^0 \qquad \longrightarrow \qquad e^+e^- \to e^+e^- \rho_L^0\ \rho_L^0$$

using equivalent photon approximation

$$\frac{d\sigma(e^+e^- \to e^+e^- \rho_L^0\rho_L^0)}{dy_1 dy_2 dQ_1^2 dQ_2^2}$$
$$= \frac{1}{y_1 y_2\, Q_1^2 Q_2^2}\left(\frac{\alpha}{\pi}\right)^2 \Big[l(y_1)\, l(y_2)\, \sigma(\gamma_L^*\gamma_L^* \to \rho_L^0\rho_L^0) + t(y_1)\, l(y_2)\, \sigma(\gamma_T^*\gamma_L^* \to \rho_L^0\rho_L^0)$$
$$+\ l(y_1)\, t(y_2)\, \sigma(\gamma_L^*\gamma_T^* \to \rho_L^0\rho_L^0) + t(y_1)\, t(y_2)\, \sigma(\gamma_T^*\gamma_T^* \to \rho_L^0\rho_L^0)\Big]$$

with the usual flux factors given by

$$t(y_i) = \frac{1+(1-y_i)^2}{2}, \quad l(y_i) = 1 - y_i\,,$$

$y_i\ (i = 1, 2)$ are the longitudinal momentum fractions of the bremsstrahlung photons with respect to the incoming leptons

$s_{\gamma^*\gamma^*} \sim y_1\, y_2\, s_{e^+e^-}$

$\Rightarrow\ \sigma^{e^+e^- \to e^+e^- \rho_L \rho_L}$ is peaked in the low y and Q^2 region

- photon momentum fractions: (in the laboratory frame = center of mass system (cms) for an e^+e^- collider)

$$y_i = \frac{E - E_i' \cos^2(\theta_i/2)}{E}$$

- virtualities:

$$Q_i^2 = 4EE_i' \sin^2(\theta_i/2)$$

- cross-section peaked at small Q_i^2 and y_i
  $\Rightarrow$one needs to get access to the (very) forward region
- kinematical constraints:
  - minimal detection angle (detector constraint)
  - conditions on the energies of outgoing leptons (detector constraint)
  - Regge condition

$$y_{i\,max} = 1 - \frac{E_{min}}{E}$$

$$y_{1\,min} = \max\left(f(Q_1), 1 - \frac{E_{max}}{E}\right)$$

$$y_{2\,min} = \max\left(f(Q_2), 1 - \frac{E_{max}}{E}, \frac{c\,Q_1\,Q_2}{s\,y_1}\right)$$

with $f(Q_i) = 1 - \frac{Q_i^2}{s\tan^2(\theta_{min}/2)}$

Reference Design Report for International Linear Collider

- $\sqrt{s_{e^+e^-}} = 2E_{lepton}$: nominal value of $500\,\mathrm{GeV}$
- high luminosity, with $125\,\mathrm{fb}^{-1}$ per year within 4 years of running at $500\,\mathrm{GeV}$
- possible scan in energy between $200\,\mathrm{GeV}$ and $500\,\mathrm{GeV}$.
- upgrade at $1\,\mathrm{TeV}$, with a luminosity of $1\,\mathrm{ab}^{-1}$ within 3 to 4 years
- two interaction regions are highly desirable: one which could be at low crossing-angle, and one compatible with $e\gamma$ and $\gamma\gamma$ physics (through single or double laser Compton backscattering)
  - at the moment, 3 options are considered: 2 mrad, 14 mrad and 20 mrad
  - note that in $e\gamma$ and $\gamma\gamma$ modes, for which $\alpha_c > 25\,\mathrm{mrad}$, no BeamCal can be placed around the beampipe, at least at $\alpha < 12\,\mathrm{mrad}$ (angular size of the disrupred outgoing beam after laser Compton backscattering)
  - it thus means that if a single detector would be used at the same interaction point (in order to reduce the budget devoted to $\gamma\gamma$ mode, this solution without displacement of the detector has been suggested: Telnov), no forward calorimeter like BeamCal could be installed

In the case of e^+e^- mode

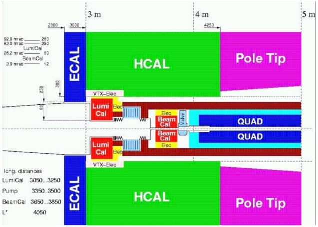
- Each design of detector for ILC project involves a very forward electromagnetic calorimeter for luminosity measurement, with tagging angle for outgoing leptons down to 5 mrad (design 10 years ago were considering 20 mrad as almost impossible!)
- This is an ideal tool for diffractive physics: cross-section are sharply peaked in the very forward region
- luminosity is enough to give high statistics, even with exclusive events
- there are 4 concepts of detectors at the moment:
  - GLD
  - Large Detector Concept (LDC)
  - Silicon Design Detector Study (Sid)
  - 4th

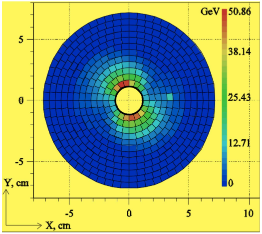
Non-forward Born order cross-section for $e^+e^- \to e^+e^- \rho_L^0\ \rho_L^0$

LDC detector

We focus specifically on the LDC project

- The BeamCal is an electromagnetic calorimeter devoted to luminosity measurement, located at 3.65 m from the vertex

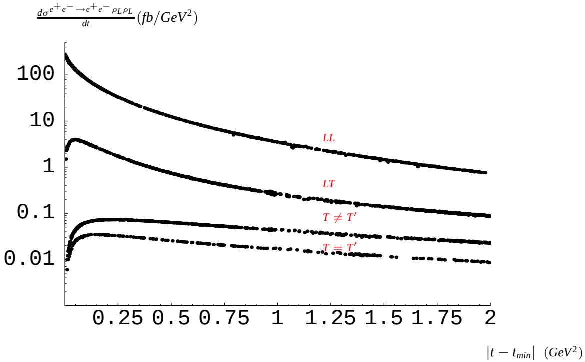




- it can be used for diffractive physics
- the main background is due to beamstrahlung photons, which leads to energy deposit in cells close from the beampipe
  $\Rightarrow$ in practice we cut-off the cells for lepton tagging with

$$E_{min} = 100 \text{ GeV}$$
$$\theta_{min} = 4 \text{ mrad}$$

$$\frac{d\sigma^{e^+e^-\to e^+e^-\rho_L\rho_L}}{dt} = \int_{Q^2_{1min}}^{Q^2_{1max}} dQ_1^2 \int_{Q^2_{2min}}^{Q^2_{2max}} dQ_2^2 \int_{\epsilon}^{y_{max}} dy_1 \int_{\frac{Q_1Q_2}{sy_1}}^{y_{max}} dy_2 \, \frac{d\sigma^{e^+e^-\to e^+e^-\rho_L\rho_L}}{dt\, dy_1\, dy_2\, dQ_1^2\, dQ_2^2},$$

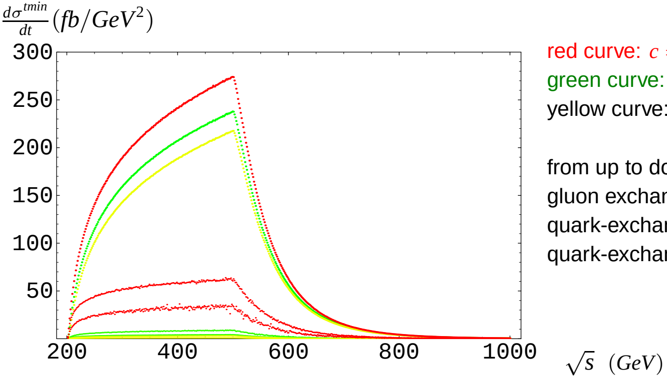


We obtain, at $\sqrt{s_{e^+e^-}} = 500$ GeV (and $c = 1$)

$$\begin{aligned} \sigma^{LL} &= 32.4 \text{ fb} \\ \sigma^{LT} &= 1.5 \text{ fb} \\ \sigma^{TT} &= 0.2 \text{ fb} \\ \sigma^{tot} &= 34.1 \text{ fb} \end{aligned}$$

which leads to $4.3\, 10^3$ events per year with foreseen luminosity

- the background (dominated by γ which would be misidentified in BeamCal as e^+ or e^-) is completely negligible at $\sqrt{s_{e^+e^-}} = 500$ GeV
- quarks contribution are indeed negligible. This is related to c through $s_{\gamma^*\gamma^*} > c\,Q_1\,Q_2$
- more drastic Regge constraint by performing $c = 1 \to c = 10$ reduces the cross-section by 40% $\Rightarrow$still statistically measurable
- changing order of loop for α_S only has a few % effect



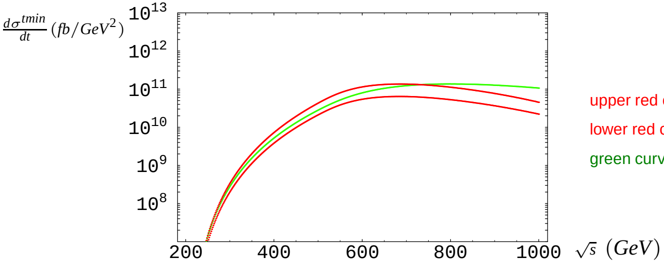
red curve: $c = 1$
green curve: $c = 2$
yellow curve: $c = 3$

from up to down:
gluon exchange
quark-exchange with γ_L^*
quark-exchange with γ_T^*

Non-forward Born order cross-section for $e^+e^- \to e^+e^- \rho_L^0\ \rho_L^0$

BFKL enhancement

LL BFKL differential cross-section at $t = t_{min}$



upper red curve: $\alpha_S(\sqrt{Q_1\,Q_2})$ at one loop

lower red curve: $\alpha_S(\sqrt{Q_1\,Q_2})$ at three loop

green curve: $\alpha_S = 0.46$ fixed

- Enhancement is enormous but it is well known that NLL BFKL is bellow LL BFKL and almost always above Born. This latter issue is true except at small rapidity in certain perculiar scale fixing scheme: see Ivanov Papa
- At the level of $\gamma^*\gamma^*$ this has been studied earlier
  - resummed BFKL à la Khoze, Martin, Ryskin, Stirling:
    Enberg, Pire, Szymanowski, S.W with LL impact factor and BLM scale fixing
  - NLL BFKL with NLL impact factor: Ivanov Papa.
  - Work to implement this at e^+e^- level is in progress.
- due to detector constraint, the expected increase of the cross-section with $\sqrt{s_{e^+e^-}}$ is washed-out for $\sqrt{s_{e^+e^-}} > 500$ GeV: flattish curve to be compared with sharked curve at Born level

Summary (1)

- $\gamma^* \gamma^* \to \rho_L^0 \rho_L^0$ is a very nice process for studying QCD dynamics in its perturbative regime, with a minimal onset of non-perturbative physics
- At low energy, it is dominated by quark exchange
  - Its perturbative analysis in the Born approximation, in the forward case, leads to two different types of QCD factorization
  - We have shown that the polarization states of the photons dictate either the factorization involving a GDA or involving a TDA.
  - Usually these two types of factorizations are applied to two different kinematical regimes.
  - The arbitrariness in choosing values of photon virtualities Q_i^2 shows that there may exist an intersection region where both types of factorization are simultaneously valid.
  - the obtained TDA contains a perturbative part which could give a hint for modelling in non perturbative cases
  - further generalizations:
    - non-forward kinematics (rather easy)
    - transverse photon (hard: higher twist contributes)
    - charged meson pair (hard: non-trivial QED gauge invariance)
  - the measure could be done at Babar, Belle, BEPC-II,...,ILC

Summary (2)

- At high energy, it is dominated by gluon exchange
  - we gave a precise estimation of the Born order cross-section for arbitrary photon polarization
  - we have demonstrated the feasability of the measurement at the level of $e^+e^- \to e^+e^- \rho_L^0\ \rho_L^0$ with double tagged outgoing leptons, within ILC collider and LDC detector with a forward electromagnetic calorimeter
  - this evaluation can be considered as the background for any resummation à la BFKL
  - we have made a first estimate of BFKL evolution at LL.
  - our previous estimate of resummed BFKL evolution for $\gamma^*\ \gamma^* \to \rho_L^0 \rho_L^0$ should now be implemented at $e^+e^- \to e^+e^- \rho_L^0\ \rho_L^0$ level
  - there is a potential very interesting possibility of entering smoothly into the non-linear saturation regime when the machine would be upgraded up to 1 TeV:
    - at $\sqrt{s_{e^+e^-}} = 500$ GeV, $Q_{sat} \sim 1.1$ GeV
      saturation is at the border, almost negligible
    - at $\sqrt{s_{e^+e^-}} = 1$ TeV, $Q_{sat} \sim 1.4$ GeV
      saturation effects should start to be rather important (but still in the almost linear regime)