

# Double logarithm resummation for DVCS

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Deep Virtual Compton Scattering: from observables to GPDs  
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Bochum

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arXiv:1206.3115 [hep-ph]  
JHEP 1210 (2012) 049  
[arXiv:1207.4609 [hep-ph]]

# Extensions from DIS

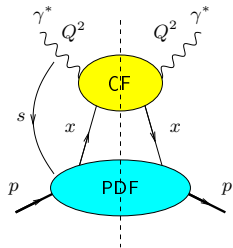
- DIS: inclusive process  $\rightarrow$  forward amplitude ( $t = 0$ ) (optical theorem)

(DIS: Deep Inelastic Scattering)

ex:  $e^\pm p \rightarrow e^\pm X$  at HERA

Structure Function

$$= \text{Coefficient Function (hard)} \otimes \text{Parton Distribution Function (soft)}$$

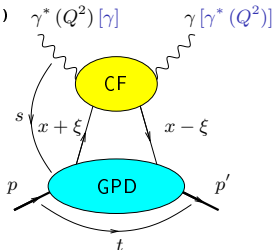


- DVCS (TCS): exclusive process  $\rightarrow$  non forward amplitude ( $-t \ll s = W^2$ )

(DVCS: Deep Virtual Compton Scattering; TCS: Timelike Compton Scattering)

Amplitude

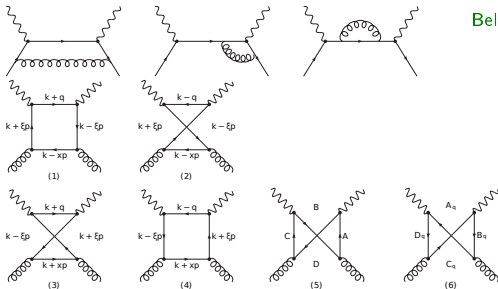
$$= \text{Coefficient Function (hard)} \otimes \text{Generalized Parton Distribution (soft)}$$



Müller et al. '91 - '94; Radyushkin '96; Ji '97

## DVCS and TCS at NLO

## One-loop contributions to the coefficient function



Belitsky, Mueller, Niedermeier, Schafer,  
 Phys.Lett.B474, 2000  
 Pire, Szymanowski, Wagner  
 Phys.Rev.D83, 2011

$$A^{\mu\nu} = g_T^{\mu\nu} \int_{-1}^1 dx \left[ \sum_q^{n_F} T^q(x) F^q(x) + T^g(x) F^g(x) \right]$$

(symmetric part of the factorised amplitude)

## Resummations effects are expected

- The renormalized quark **coefficient functions**  $T^q$  is

$$T^q = C_0^q + C_1^q + C_{coll}^q \log \frac{|Q^2|}{\mu_F^2}$$

$$C_0^q = e_q^2 \left( \frac{1}{x - \xi + i\varepsilon} - (x \rightarrow -x) \right)$$

$$C_1^q = \frac{e_q^2 \alpha_S C_F}{4\pi(x - \xi + i\varepsilon)} \left[ \log^2 \left( \frac{\xi - x}{2\xi} - i\varepsilon \right) + \dots \right] - (x \rightarrow -x)$$

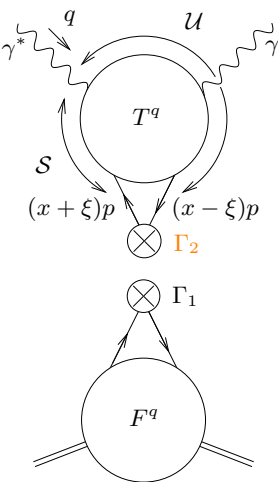
- Usual collinear approach: single-scale analysis w.r.t.  $Q^2$
- Consider the invariants  $S$  and  $U$ :

$$S = \frac{x - \xi}{2\xi} Q^2 \ll Q^2 \quad \text{when } x \rightarrow \xi$$

$$U = -\frac{x + \xi}{2\xi} Q^2 \ll Q^2 \quad \text{when } x \rightarrow -\xi$$

⇒ **two scales problem; threshold singularities to be resummed**

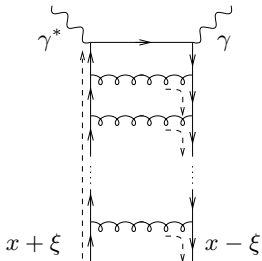
analogous to the  $\log(1 - x)$  resummation for DIS coefficient functions



## Resummation for Coefficient functions: our result

## Soft-collinear resummation effects for the coefficient function

- The resummation is made easier when using the axial gauge  $p_1 \cdot A = 0$  ( $p_\gamma \equiv p_1$ )
- The dominant diagram are **ladder-like**



resummed formula (for DVCS), for  $x \rightarrow \xi$  :

$$\begin{aligned}
 (T^q)^{\text{res}} = & \left( \frac{e_q^2}{x - \xi + i\epsilon} \left\{ \cosh \left[ D \log \left( \frac{\xi - x}{2\xi} - i\epsilon \right) \right] \right. \right. \\
 & \left. \left. - \frac{D^2}{2} \left[ 9 + 3 \frac{\xi - x}{x + \xi} \log \left( \frac{\xi - x}{2\xi} - i\epsilon \right) \right] \right\} \right) \\
 & + C_{\text{coll}}^q \log \left( \frac{Q^2}{\mu_F^2} \right) - (x \rightarrow -x) \quad \text{with} \quad D = \sqrt{\frac{\alpha_s C_F}{2\pi}}
 \end{aligned}$$

T. Altinoluk, B. Pire, L. Szymanowski, S. W.  
 JHEP 1210 (2012) 49; [arXiv:1206.3115]

## Kinematics, gauge, etc...

- We expand any momentum in the Sudakov basis  $p_1, p_2$  :

$$k = \alpha p_1 + \beta p_2 + k_\perp$$

- $p_2$  is the **light-cone direction** of the two incoming and outgoing partons

$$p_1^2 = p_2^2 = 0, \quad 2p_1 \cdot p_2 = s = \frac{Q^2}{2\xi}$$

- Momenta of the incoming and outgoing photons:

$$q_{\gamma^*} = p_1 - 2\xi p_2, \quad p_1 \equiv q_\gamma$$

- The extraction of soft-collinear singularities in the limit  $x \rightarrow \pm\xi$  is easier in the **light-like gauge**  $p_1 \cdot A = 0$ : in this gauge, gluon physical degrees of freedom are manifest and helicity conservation at each vertex implies that collinear singularities only arise in ladder-like diagrams

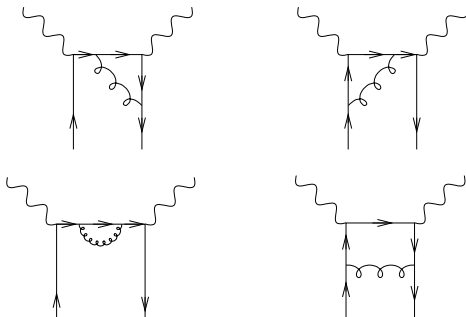
- $K_n$  is the contribution of a  $n$ -loop ladder to the CF :

$$K_n = -\frac{1}{4}e_q^2 \left( -i C_F \alpha_s \frac{1}{(2\pi)^2} \right)^n I_n$$

- The issue related to the  $i\epsilon$  prescription is solved by computing the CF in the unphysical region  $\xi > 1$ . After analytical continuation to the physical region  $0 \leq \xi \leq 1$ , the physical prescription is then obtained through the shift  $\xi \rightarrow \xi - i\epsilon$ .

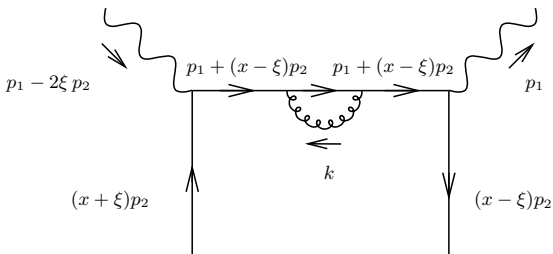
# Full one-loop analysis

- analyzing the one-loop diagrams



- no approximations!
- reduce the number of denominators in order to simplify the calculation.
- aims (we now assume  $x \rightarrow +\xi$ ):
  - to understand which diagrams give contribution at order  $\frac{\alpha_s \log^2(\xi - x)}{(x - \xi)}$
  - identify the part of the phase space that is responsible for this contribution

## Self energy diagram



- numerator for S.E. diagram :

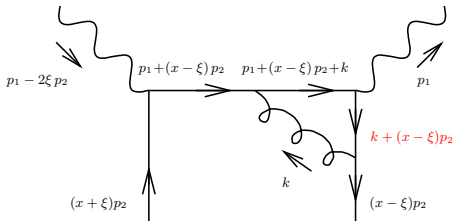
$$\begin{aligned}
 (\text{Num})_{\text{S.E.}} = & \text{tr} \left\{ \not{p}_2 \gamma_{\perp}^{\sigma} [\not{p}_1 + (x - \xi)\not{p}_2] \gamma^{\nu} [\not{p}_1 + (x - \xi)\not{p}_2 - \not{k}] \gamma^{\mu} [\not{p}_1 + (x - \xi)\not{p}_2] \gamma_{\perp\sigma} \right\} \\
 & \times \left\{ g_{\mu\nu} - \frac{k_{\mu} p_{1\nu} + k_{\nu} p_{1\mu}}{k \cdot p_1} \right\}.
 \end{aligned}$$

- a simple algebra shows that  $(\text{Num})_{\text{gauge}} = 0 \Rightarrow$  S.E. diagram is the same in Feynman gauge and in light-like gauge.
- In Feynman gauge S.E. diagram gives only single log's!  
[B. Pire, L. Szymanowski, J. Wagner, Phys.Rev. D83 (2011) 034009]
- S.E. diagram does not contribute to  $[\log^2(\xi - x)]/(x - \xi)$  terms!



# Right vertex, left vertex and box diagram

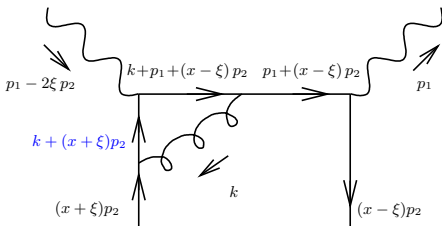
## • Right Vertex:



$$(\text{Num})_{\text{R.V.}} = 8s \frac{k_{\perp}^2}{\beta} (\beta + x - \xi)$$

$$I_{\text{R.V.}} = -\frac{s}{2} \int d\alpha d\beta d_2 \underline{k} 8s \frac{k_{\perp}^2}{\beta} (\beta + x - \xi) \frac{1}{s(x - \xi)} \frac{1}{[k + (x - \xi)p_2]^2} \frac{1}{k^2} \frac{1}{[k + p_1 + (x - \xi)p_2]^2}$$

## • Left Vertex:

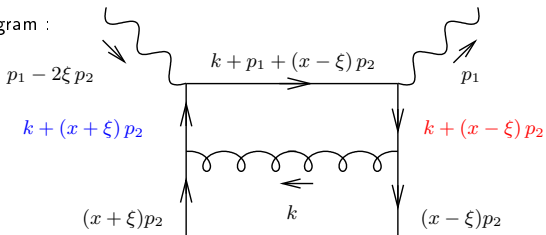


$$(\text{Num})_{\text{L.V.}} = 8s \frac{k_{\perp}^2}{\beta} (\beta + x + \xi)$$

$$I_{\text{L.V.}} = -\frac{s}{2} \int d\alpha d\beta d_2 \underline{k} 8s \frac{k_{\perp}^2}{\beta} (\beta + x + \xi) \frac{1}{s(x - \xi)} \frac{1}{[k + (x + \xi)p_2]^2} \frac{1}{k^2} \frac{1}{[k + p_1 + (x - \xi)p_2]^2}$$

# Right vertex, left vertex and box diagram

- Box diagram :



The  $g_{\mu\nu}$  part of the box diagram reads

$$(\text{Num})_{\text{box } g_{\mu\nu}} = -2 \text{tr} \left\{ [k + (x + \xi)p_2] \not{p}_2 [k + (x - \xi)p_2] \gamma_{\perp}^{\sigma} [k + p_1 + (x - \xi)p_2] \gamma_{\perp \sigma} \right\}$$

Noting that  $p_2$  can be written as (Ward identity)

$$p_2^{\mu} = \frac{1}{2\xi} \left( [k + (x + \xi)p_2] - [k + (x - \xi)p_2] \right)^{\mu}$$

one gets

$$(\text{Num})_{\text{box } g_{\mu\nu}} = -\frac{8}{\xi} [k + (x + \xi)p_2]^2 \left\{ k_{\perp}^2 - (\beta + x - \xi) \frac{s}{2} \right\} + \frac{8}{\xi} [k + (x - \xi)p_2]^2 \left\{ k_{\perp}^2 - (\beta + x + \xi) \frac{s}{2} + \xi \alpha s \right\}$$

# Right vertex, left vertex and box diagram

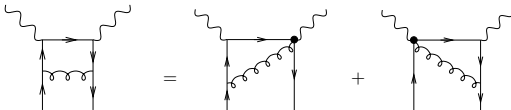
The gauge part of the numerator for the box diagram reads

$$\begin{aligned}
 (\text{Num})_{\text{box gauge}} &= -\frac{2}{\beta s} \text{tr} \left\{ [\not{k} + (x + \xi)\not{p}_2] \not{p}_1 \not{k} [\not{k} + (x - \xi)\not{p}_2] \gamma_{\perp}^{\sigma} [\not{k} + \not{p}_1 + (x - \xi)\not{p}_2] \gamma_{\perp\sigma} \right\} \\
 &\quad - \frac{2}{\beta s} \text{tr} \left\{ [\not{k} + (x + \xi)\not{p}_2] \not{k} \not{p}_2 \not{p}_1 [\not{k} + (x - \xi)\not{p}_2] \gamma_{\perp}^{\sigma} [\not{k} + \not{p}_1 + (x - \xi)\not{p}_2] \gamma_{\perp\sigma} \right\}
 \end{aligned}$$

Using the fact that  $p_2^2 = 0$ , then one can write  $k \rightarrow k + (x \pm \xi)p_2$  inside the trace and gets

$$\begin{aligned}
 (\text{Num})_{\text{box}} &= 8 [k + (x - \xi)p_2]^2 \left\{ \frac{1}{\xi} \left[ k_{\perp}^2 - (\beta + x + \xi) \frac{s}{2} + \xi \alpha s \right] + \frac{s}{\beta} (1 + \alpha)(\beta + x + \xi) \right\} \\
 &\quad - 8 [k + (x + \xi)p_2]^2 \left\{ \frac{1}{\xi} \left[ k_{\perp}^2 - (\beta + x - \xi) \frac{s}{2} \right] - \frac{s}{\beta} (1 + \alpha)(\beta + x - \xi) \right\}
 \end{aligned}$$

$\Rightarrow$  box diagram = right + left vertices:



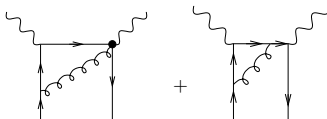
# Right vertex, left vertex and box diagram

Combining right vertex, left vertex and box diagram

$$I_{\text{box} + \text{L.V.} + \text{R.V.}} = I_{\text{E.L.V.}} + I_{\text{E.R.V.}}$$

Effective Left Vertex:

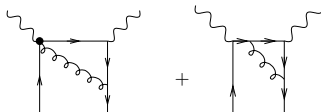
$$I_{\text{E.L.V.}} =$$



$$I_{\text{E.L.V.}} = -\frac{s}{2} \int d\alpha d\beta d_2 \underline{k} \ 8 \left\{ \frac{1}{\xi} \left[ \underline{k}^2 + (\beta + x + \xi) \frac{s}{2} - \xi \alpha s \right] - \frac{s}{\beta} (1 + \alpha) (\beta + x + \xi) + \frac{k^2}{\beta} \frac{(\beta + x + \xi)}{(x - \xi)} \right\} \\ \times \frac{1}{k^2} \frac{1}{[k + (x + \xi)p_2]^2} \frac{1}{[k + p_1 + (x - \xi)p_2]^2}$$

Effective Right Vertex:

$$I_{\text{E.R.V.}} =$$



$$I_{\text{E.R.V.}} = -\frac{s}{2} \int d\alpha d\beta d_2 \underline{k} \ (-8) \left\{ \frac{1}{\xi} \left[ \underline{k}^2 + (\beta + x - \xi) \frac{s}{2} \right] + \frac{s}{\beta} (1 + \alpha) (\beta + x - \xi) - \frac{k^2}{\beta} \frac{(\beta + x - \xi)}{(x - \xi)} \right\} \\ \times \frac{1}{k^2} \frac{1}{[k + p_1 + (x - \xi)p_2]^2} \frac{1}{[k + (x - \xi)p_2]^2}$$

## Loop integration

 $I_{\text{E.L.V.}}$ 

- Write  $d^4k = \frac{s}{2} d\alpha d\beta d^2 k_\perp$  ( $k_\perp^2 = -\underline{k}^2$ )
- We use Cauchy integration to integrate over  $\alpha$
- There are two contributions :
  - cutting the gluonic line  $\rightarrow \alpha_g = \frac{k^2}{s\beta}$
  - cutting the fermionic line  $\rightarrow \alpha_f = \frac{k^2}{s(\beta+x+\xi)}$

- distribution of the poles in  $\alpha$  sets the integration region of  $\beta$ :

$$I_{\text{E.L.V.}} = -2\pi i \left[ \int_0^{\xi-x} d\beta \int_0^\infty d_N \underline{k} \text{Res}_{\alpha_g} + \int_{-\xi-x}^{\xi-x} d\beta \int_0^\infty d_N \underline{k} \text{Res}_{\alpha_f} \right]$$

- integration over  $\underline{k}$  is performed by using dimensional regularization:  
 $N = 2 - \epsilon_{UV} = 2 + \epsilon_{IR}$
- the ultraviolet divergence in  $\underline{k}$  integral is taken into account by renormalization
- the IR divergent part is absorbed by the **DGLAP-ERBL** evolution kernel
- We are only interested in the finite part, which is reminiscent of the IR soft and collinear divergencies

# Loop integration

$I_{E.L.V.}$ : the gluonic pole contribution [dominates]

$$I_{E.L.V.,g} = \text{[Diagram 1]} + \text{[Diagram 2]}$$

The integration over  $\underline{k}$  gives

$$I_{E.L.V.,g} = 4 \frac{2\pi i}{x-\xi} \int_0^{\xi-x} d\beta \left[ \frac{\beta}{\xi(x+\xi)} - \frac{1}{(x+\xi)} + \frac{(\beta+x+\xi)}{(x+\xi)(x-\xi)} - \frac{(\beta+x+\xi)}{2\xi(\beta+x-\xi)} \right] \Gamma(\epsilon_{UV}) \times \left[ \frac{s\beta(\beta+x-\xi)}{x-\xi} \right]^{\epsilon_{IR}}$$

- We are only interested in terms that contribute to  $\frac{\log^2(\xi-x)}{(x-\xi)}$  terms
- These corresponds to most singular terms, at the limits of  $\beta$  integration.
- For  $I_{E.L.V.}$ 
  - $\frac{1}{\beta}$  terms that are singular at 0
  - $\frac{1}{\beta+x-\xi}$  terms that are singular at  $\xi-x$
- There are no  $\frac{1}{\beta}$  terms in  $I_{E.L.V.,g}$

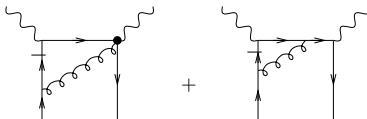
For  $\frac{1}{\beta+x-\xi}$  type of singularity, the contribution is

$$I_{E.L.V.,g} = -4 \frac{2\pi i}{x-\xi} \frac{1}{2!} \log^2(\xi-x)$$

Actually, this contributions originates from the box diagram term

# Loop integration

$I_{E.L.V.}$ : the fermionic pole contribution [suppressed]

$$I_{E.L.V.,f} =$$


$$I_{E.L.V.,f} \sim 4 \frac{2\pi i}{x - \xi} \frac{1}{2!} \log^2(2\xi)$$

[backup]

this term is less singular than the term we are looking for

$I_{E.R.V.}$  [suppressed]

$$I_{E.R.V.} =$$


no contribution from  $I_{E.R.V.}$

[backup]

## Full one-loop analysis: summary

The only contribution to  $[\log^2(\xi - x)]/x - \xi$  terms comes from the **box diagram** in the case of **cutting the gluonic line** around  $\beta + x - \xi \approx 0$  in the phase space

The precision of our calculation does not permit us to fix the multiplicative coefficient  $a$  of  $(\xi - x)$  under logarithm, i.e. our result can be equivalently written as

$$I_{\text{one loop}}^{\text{dominant}} \approx -4 \frac{2\pi i}{x - \xi} \frac{1}{2!} \log^2[a(\xi - x)]$$

- The coefficient  $a$  is fixed to  $\frac{1}{2\xi}$  by comparing the  $\log^2(\xi - x)$  terms in the exact NLO result.
- The shift  $\xi \rightarrow \xi - i\epsilon$  correctly takes into account the imaginary part.

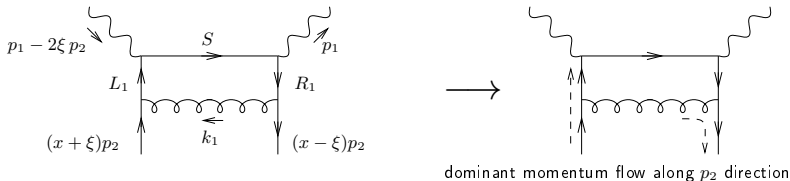
our final formula reads:

$$I_{\text{one loop}}^{\text{dominant}} \approx -4 \frac{2\pi i}{x - \xi + i\epsilon} \frac{1}{2!} \log^2 \left[ \frac{\xi - x}{2\xi} - i\epsilon \right]$$



# One-loop in semi-eikonal approximation

Aim : *obtain the same result by using eikonal techniques on the left fermionic line of the box diagram*



The corresponding integral  $\rightarrow I_1 = \frac{s}{2} \int d\alpha_1 d\beta_1 d_2 \underline{k}_1 (\text{Num})_1 \frac{1}{L_1^2} \frac{1}{S^2} \frac{1}{R_1^2} \frac{1}{k_1^2}$

with  $(\text{Num})_1 = \text{tr} \{ \not{p}_2 \gamma_\mu [k_1 + (x - \xi) \not{p}_2] \theta [k_1 + (x + \xi) \not{p}_2] \gamma_\nu \} d^{\mu\nu}$  ( $\theta \equiv$  Born term numerator)

and  $L_1^2 = [k_1 + (x + \xi)p_2]^2$ ,  $S^2 = [k_1 + p_1 + (x - \xi)p_2]^2$ ,  $R_1^2 = [k_1 + (x - \xi)p_2]^2$

- use eikonal coupling on the left quark line and treat the gluon as soft with respect to this quark  $\Rightarrow$  in the quark numerator  $L_1$ :

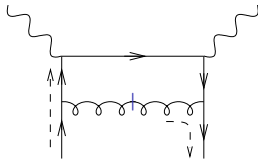
$$[k_1 + (x + \xi)p_2] \rightarrow (x + \xi)p_2$$

- gluon is soft w.r.t.  $s$ -channel fermionic line  $\Rightarrow \alpha_1 \ll 1$ .

$$\theta = \gamma_\perp^\sigma [\not{k}_1 + \not{p}_1 + (x - \xi)\not{p}_2] \gamma_{\sigma\perp} \rightarrow -2\not{p}_1$$

# One-loop in semi-eikonal approximation (1)

- The dominant contribution comes from the gluon pol.



on mass shell: 
$$d^{\mu\nu} = - \sum_{\lambda} \epsilon_{(\lambda)}^{\mu} \epsilon_{(\lambda)}^{\nu}$$

- The numerator becomes

$$(\text{Num})_1 = -2(x + \xi) \sum_{\lambda} \text{tr} \{ \not{p}_2 \gamma_{\mu} [ \not{k}_1 + (x - \xi) \not{p}_2 ] \not{p}_1 \not{p}_2 \not{\epsilon}_{(\lambda)} \} (-\epsilon_{(\lambda)}^{\mu})$$

- Sudakov decomposition of  $\epsilon_{(\lambda)}^{\mu}$  in  $p_1$  gauge  $\rightarrow \epsilon_{(\lambda)}^{\mu} = \epsilon_{\perp(\lambda)}^{\mu} - 2 \frac{\epsilon_{\perp(\lambda)} \cdot k_{\perp 1}}{\beta_1 s} p_1^{\mu}$
- Summing over the polarizations

$$\sum_{\lambda} \epsilon_{\perp(\lambda)} \cdot k_{\perp 1} \epsilon_{(\lambda)}^{\mu} = \left( -k_{\perp 1}^{\mu} + 2 \frac{k_{\perp 1}^2}{\beta_1 s} p_1^{\mu} \right)$$

Then 
$$(\text{Num})_1 = \frac{2(x + \xi)}{\beta_1} \left[ \frac{2(x - \xi)}{\beta_1} + 1 \right] 4s \underline{k}_1^2$$

left eikonal coupling

right eikonal coupling

non eikonal correction

# One-loop in semi-eikonal approximation (1)

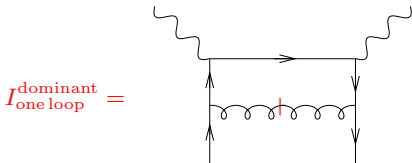
- After Cauchy integration over  $\alpha_1$  and considering only the  $1/(\beta + x - \xi)$  type of singularities one gets

$$I_1 = -4 \frac{2\pi i}{x - \xi} \int_0^{\xi - x} d\beta_1 \int_0^\infty d_N \underline{k}_1 \frac{1}{(\beta_1 + x - \xi)} \frac{1}{\underline{k}_1^2 - (\beta_1 + x - \xi)s}$$

- Dominant phase space  $\beta \rightarrow \xi - x$

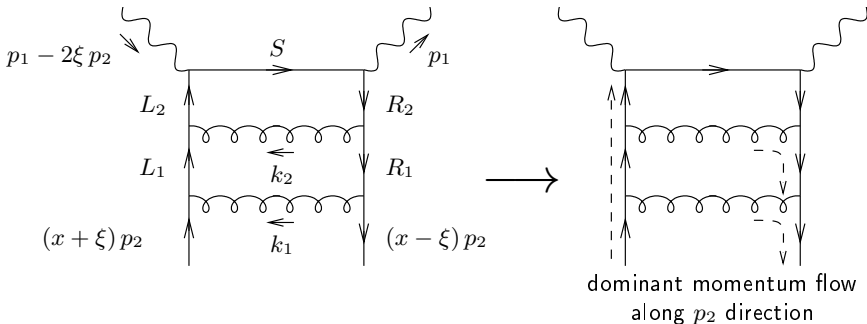
$$\Rightarrow \frac{2(x - \xi)}{\beta_1} + 1 \sim -1 \quad (\text{instead of } -2 \text{ in a pure eikonal limit})$$

- The integration over  $\underline{k}$  and  $\beta$  leads to



$$= -4 \frac{2\pi i}{x - \xi + i\epsilon} \frac{1}{2!} \log^2 \left[ \frac{\xi - x - i\epsilon}{2\xi} \right]$$

## Two-loop in semi-eikonal approximation



- The  $\log^2$  terms we are resumming arise from **soft-collinear** singularities :
  - Dominance of **on-shell gluons** contributions
  - Strong ordering in  $|\underline{k}_i|$  and  $\beta_i$

$$|\underline{k}_2| \gg |\underline{k}_1| \quad \text{and} \quad x \sim \xi \gg |\beta_1| \sim |x - \xi| \gg |x - \xi + \beta_1| \sim |\beta_2|$$

$$\text{and} \quad 1 \gg |\alpha_2| \gg |\alpha_1|$$

- Other diagrams which are not ladder-like or do not respect this strong ordering are suppressed [backup]

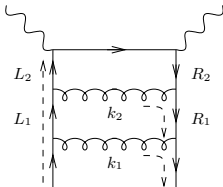
# Two-loop in semi-eikonal approximation

The integral for the 2-loop case is

$$I_2 = \left(\frac{s}{2}\right)^2 \int d\alpha_1 d\beta_1 d_2 \underline{k}_1 \int d\alpha_2 d\beta_2 d_2 \underline{k}_2 (\text{Num})_2 \frac{1}{L_1^2} \frac{1}{R_1^2} \frac{1}{S^2} \frac{1}{L_2^2} \frac{1}{R_2^2} \frac{1}{k_1^2} \frac{1}{k_2^2}$$

Using eikonal coupling on the left fermionic line, the numerator is given as

$$(\text{Num})_2 = -4s \underbrace{\frac{-2\underline{k}_1^2(x+\xi)}{\beta_1} \left[1 + \frac{2(x-\xi)}{\beta_1}\right]}_{\text{gluon 1}} \underbrace{\frac{-2\underline{k}_2^2(x+\xi)}{\beta_2} \left[1 + \frac{2(\beta_1+x-\xi)}{\beta_2}\right]}_{\text{gluon 2}}$$



and the propagators

$$\begin{aligned} L_1^2 &= \alpha_1(x+\xi)s \quad , \quad R_1^2 = -\underline{k}_1^2 + \alpha_1(\beta_1+x-\xi)s \quad , \quad S^2 = -\underline{k}_2^2 + (\beta_1+\beta_2+x-\xi)s \\ L_2^2 &= \alpha_2(x+\xi)s \quad , \quad R_2^2 = -\underline{k}_2^2 + \alpha_2(\beta_1+\beta_2+x-\xi)s \end{aligned}$$

After integrating over  $\alpha_1$  and  $\alpha_2$  and using the properties of dimensional regularization

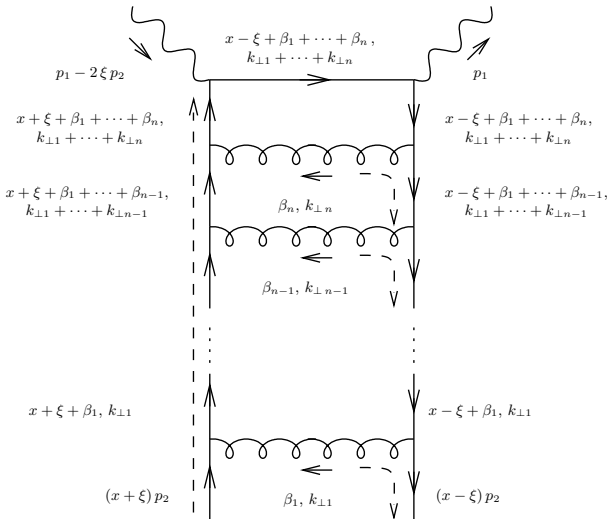
$$\begin{aligned} I_2 &= -4 \frac{(2\pi i)^2}{x-\xi} \int_0^{\xi-x} d\beta_1 \int_0^{\xi-x-\beta_1} d\beta_2 \frac{1}{\beta_1+x-\xi} \frac{1}{\beta_1+\beta_2+x-\xi} \\ &\quad \times \int_0^\infty d_N \underline{k}_2 \int_{\underline{k}_2^2}^\infty d_N \underline{k}_1 \frac{1}{\underline{k}_1^2} \frac{1}{\underline{k}_2^2 - (\beta_1+\beta_2+x-\xi)s} \end{aligned}$$

Integrating over  $\beta_i$  and  $\underline{k}_i$  and using the matching condition, the final result is

$$I_2^{\text{fin.}} = -4 \frac{(2\pi i)^2}{x-\xi+i\epsilon} \frac{1}{4!} \log^4 \left[ \frac{\xi-x}{2\xi} - i\epsilon \right]$$

# Computation of the $n$ -loop ladder-like diagram

## Generalisation of the 1- and 2-loop diagrams



- All gluons are assumed to be on mass shell.
- Strong ordering in  $k_i$ ,  $\alpha_i$  and  $\beta_i$ .
- The dominant momentum flows along  $p_2$  are indicated

Computation of the  $n$ -loop ladder-like diagram

- Strong ordering is given as :

$$|\underline{k}_n| \gg |\underline{k}_{n-1}| \gg \dots \gg |\underline{k}_1| \quad , \quad 1 \gg |\alpha_n| \gg |\alpha_{n-1}| \gg \dots \gg |\alpha_1|$$

$$x \sim \xi \gg |\beta_1| \sim |x - \xi| \gg |x - \xi + \beta_1| \sim |\beta_2| \gg \dots \gg |x - \xi + \beta_1 + \beta_2 - \dots + \beta_{n-1}| \sim |\beta_n|$$

- eikonal coupling on the left
- coupling on the right goes beyond eikonal
- Integral for  $n$ -loop:

$$I_n = \left(\frac{s}{2}\right)^n \int d\alpha_1 d\beta_1 d_2\underline{k}_1 \dots \int d\alpha_n d\beta_n d_2\underline{k}_n (\text{Num})_n \frac{1}{L_1^2} \dots \frac{1}{L_n^2} \frac{1}{S^2} \frac{1}{R_1^2} \dots \frac{1}{R_n^2} \frac{1}{k_1^2} \dots \frac{1}{k_n^2}$$

- Numerator:

$$(\text{Num})_2 = -4s \underbrace{\frac{-2\underline{k}_1^2(x+\xi)}{\beta_1} \left[1 + \frac{2(x-\xi)}{\beta_1}\right]}_{\text{gluon 1}} \underbrace{\frac{-2\underline{k}_2^2(x+\xi)}{\beta_2} \left[1 + \frac{2(\beta_1+x-\xi)}{\beta_2}\right]}_{\text{gluon 2}} \dots \underbrace{\frac{-2\underline{k}_n^2(x+\xi)}{\beta_n} \left[1 + \frac{2(\beta_{n-1}+\dots+\beta_1+x-\xi)}{\beta_n}\right]}_{\text{gluon n}}$$

- Propagators:

$$L_1^2 = \alpha_1(x+\xi)s, \quad R_1^2 = -\underline{k}_1^2 + \alpha_1(\beta_1+x-\xi)s,$$

$$L_2^2 = \alpha_2(x+\xi)s, \quad R_2^2 = -\underline{k}_2^2 + \alpha_2(\beta_1+\beta_2+x-\xi)s,$$

$$\vdots$$

$$L_n^2 = \alpha_n(x+\xi)s, \quad R_n^2 = -\underline{k}_n^2 + \alpha_n(\beta_1+\dots+\beta_n+x-\xi)s,$$

Computation of the  $n$ -loop ladder-like diagram

## Final step

$$I_n = -4 \frac{(2\pi i)^n}{x - \xi} \int_0^{\xi - x} d\beta_1 \cdots \int_0^{\xi - x - \beta_1 - \cdots - \beta_{n-1}} d\beta_n \frac{1}{\beta_1 + x - \xi} \cdots \frac{1}{\beta_1 + \cdots + \beta_n + x - \xi} \\ \times \int_0^\infty d_N \underline{k}_n \cdots \int_{\underline{k}_2}^\infty d_N \underline{k}_1 \frac{1}{\underline{k}_1^2} \cdots \frac{1}{\underline{k}_{n-1}^2} \frac{1}{\underline{k}_n^2 - (\beta_1 + \cdots + \beta_n + x - \xi)s}$$

integration over  $\underline{k}_i$  and  $\beta_i$  leads to our final result :

$$I_n^{\text{fin.}} = -4 \frac{(2\pi i)^n}{x - \xi + i\epsilon} \frac{1}{(2n)!} \log^{2n} \left[ \frac{\xi - x}{2\xi} - i\epsilon \right]$$

Resummation :

remember that  $K_n = -\frac{1}{4} e_q^2 \left( -i C_F \alpha_s \frac{1}{(2\pi)^2} \right)^n I_n$

$$\left( \sum_{n=0}^{\infty} K_n \right) - (x \rightarrow -x) = \frac{e_q^2}{x - \xi + i\epsilon} \cosh \left[ D \log \left( \frac{\xi - x}{2\xi} - i\epsilon \right) \right] - (x \rightarrow -x)$$

where  $D = \sqrt{\frac{\alpha_s C_F}{2\pi}}$



## Resummed formula

## Inclusion of our resummed formula into the NLO coefficient function

The inclusion procedure is not unique and it is natural to propose two choices:

- modifying only the Born term and the  $\log^2$  part of the  $C_1^q$  and keeping the rest of the terms untouched :

$$(T^q)^{\text{res1}} = \left( \frac{e_q^2}{x-\xi+i\epsilon} \left\{ \cosh \left[ D \log \left( \frac{\xi-x}{2\xi} - i\epsilon \right) \right] - \frac{D^2}{2} \left[ 9 + 3 \frac{\xi-x}{x+\xi} \log \left( \frac{\xi-x}{2\xi} - i\epsilon \right) \right] \right\} + C_{coll}^q \log \frac{Q^2}{\mu_F^2} \right) - (x \rightarrow -x)$$

- the resummation effects are accounted for in a multiplicative way for  $C_0^q$  and  $C_1^q$  :

$$(T^q)^{\text{res2}} = \left( \frac{e_q^2}{x-\xi+i\epsilon} \cosh \left[ D \log \left( \frac{\xi-x}{2\xi} - i\epsilon \right) \right] \left[ 1 - \frac{D^2}{2} \left\{ 9 + 3 \frac{\xi-x}{x+\xi} \log \left( \frac{\xi-x}{2\xi} - i\epsilon \right) \right\} \right] + C_{coll}^q \log \frac{Q^2}{\mu_F^2} \right) - (x \rightarrow -x)$$

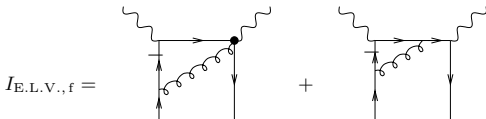
These resummed formulas differ through logarithmic contributions which are beyond the precision of our study.

# Conclusions

- The resummation of soft-collinear gluon radiation effects allowed us to get a close all-order formula that modifies significantly the coefficient function in the specific region  $x$  near  $\pm\xi$ .
- Our analysis can be used for the gluon coefficient function [In progress].
- The measurement of the phenomenological impact of this procedure on the data analysis needs further analysis with the implementation of modeled generalized parton distributions [backup].
- Our analysis could and should be applied to other processes: TCS [done], exclusive meson production, form factors... [In progress].
- A formulation of resummation in our exclusive case in terms of (conformal) moments is not yet available. This would generalize analogous resummation of inclusive DIS cross-section which were performed in terms of Mellin moments.
- Our one-loop treatment involves a non-symmetric treatment for gluon emission. This whole result can presumably be obtained based on the Low theorem (known for the Bremsstrahlung in QED) [F. Low 1958 PRD]: the classical radiation should be fully extracted from the elastic amplitude (in our case the Born order hand-bag diagram) [In progress]

# Loop integration

$I_{E.L.V.,f}$ : the fermionic pole contribution



The integration over  $\underline{k}$  gives

$$I_{E.L.V.,f} = 4 \frac{2\pi i}{(x+\xi)2\xi} \int_{-\xi-x}^{\xi-x} d\beta \left\{ (\beta+x+\xi) \left[ \frac{1}{\xi} + \frac{1}{x-\xi} + \frac{(x+\xi)}{(x-\xi)} \frac{1}{(\beta+x-\xi)} \right] - 1 \right\} \Gamma(\epsilon_{UV}) \times \left[ \frac{s(\beta+x+\xi)(\beta+x-\xi)}{2\xi} \right]^{\epsilon_{IR}}$$

- There are no  $\frac{1}{\beta+x+\xi}$  type of terms!

For  $\frac{1}{\beta+x-\xi}$  type of singularity, the contribution is

$$I_{E.L.V.,f} = 4 \frac{2\pi i}{x-\xi} \frac{1}{2!} \log^2(2\xi)$$

this term is less singular than the term we are looking for

## Loop integration

 $I_{E.R.V.}$ 

$$I_{E.R.V.} = \text{[Diagram 1]} + \text{[Diagram 2]}$$

- gluonic contribution  $\rightarrow \alpha_g = \frac{k^2}{s\beta}$     fermionic contribution  $\rightarrow \alpha_f = \frac{k^2}{s(\beta+x-\xi)}$

$$I_{E.R.V.} = -2\pi i \left[ \int_0^{\xi-x} d\beta \int_0^\infty d_N \underline{k} \text{Res}_{\alpha_g} + \int_{-\xi-x}^{\xi-x} d\beta \int_0^\infty d_N \underline{k} \text{Res}_{\alpha_f} \right]$$

with

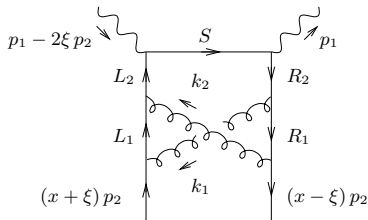
$$\text{Res}_{\alpha_g} = 4 \frac{1}{(x-\xi)^2} \left[ \frac{\beta}{\xi} + 1 - \frac{(\beta+x-\xi)}{x-\xi} - \frac{(x-\xi)}{2\xi} \right] \frac{1}{\underline{k}^2 + \frac{\beta(\beta+x+\xi)s}{x-\xi}} + 4 \frac{1}{(x-\xi)} \left[ \frac{1}{2\xi} + \frac{1}{\beta} \right] \frac{1}{\underline{k}^2}$$

$$\text{Res}_{\alpha_f} = -4 \frac{1}{(x-\xi)} \left\{ \left[ \frac{1}{\xi(\beta+x-\xi)} + \frac{1}{\beta(\beta+x-\xi)} - \frac{1}{\beta(x-\xi)} \right] + s \left( \frac{1}{2\xi} + \frac{1}{\beta} \right) \frac{1}{\underline{k}^2} \right\}$$

 $\Rightarrow$  fermionic contribution vanishes $\Rightarrow$  no  $1/\beta$  or  $1/(\beta+x-\xi)$  type of singularity in gluonic contributionno contribution from  $I_{E.R.V.}$

# Suppressed 2-loop diagrams

## Cross diagram



- The dominant contribution is provided by a strong ordering
  - of transverse momenta
  - of collinear momenta

$$|\underline{k}_2| \gg |\underline{k}_1| \quad \text{and} \quad x \sim \xi \gg |\beta_1| \gg |\beta_2|$$

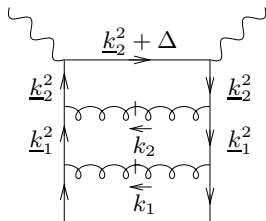
- Within this ordering:

$$I = 4s(2\pi i)^2 \int_0^{\xi-x} d\beta_1 \int_0^{\xi-x-\beta_1} d\beta_2 \int_0^\infty d_2 \underline{k}_2 \int_0^{\underline{k}_2^2} d_2 \underline{k}_1 \frac{1}{x-\xi} \frac{1}{\underline{k}_2^2(x-\xi)} \frac{1}{\underline{k}_2^2} \frac{1}{\underline{k}_2^2 - (\beta_1 + \beta_2 + x - \xi)s}$$

- no  $\underline{k}_1$  dependence!  $\Rightarrow$  one less power of  $\log(\xi - x)$
- this cross diagram does not generate maximal collinear singularity!

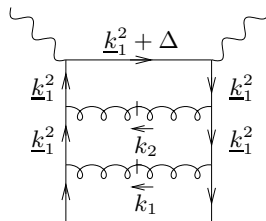
# Suppressed 2-loop diagrams

## Ladder diagram with reverse ordering



$$|\underline{k}_1| \ll |\underline{k}_2|$$

dominates



$$|\underline{k}_2| \ll |\underline{k}_1|$$

suppressed

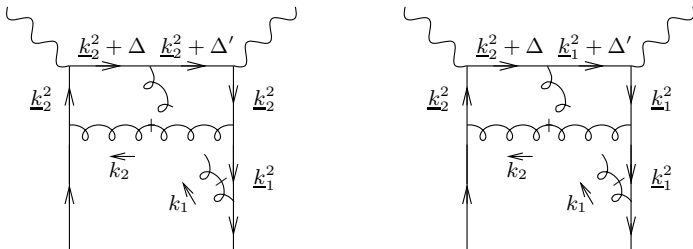
- Left : natural ordering gives  $\log^4(\xi - x)$ . Maximal number of  $\underline{k}_i$  for each  $i$
- Right : reverse ordering gives less powers of  $\log^4(\xi - x)$ . No  $\underline{k}_2$ !

⇒ Second rule:

(ii) Each loop should involve a maximal number of collinear singularities, which manifest themselves as maximal powers of  $1/\underline{k}_i^2$  for each  $i$ , after the  $\alpha_i$  integration.

# Suppressed 2-loop diagrams

Diagram with gluon coupled to the  $s$ -channel quark



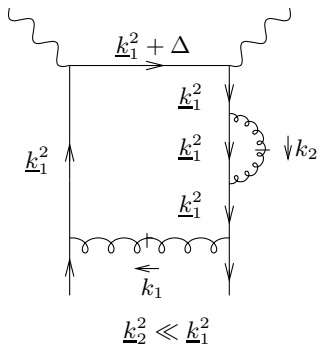
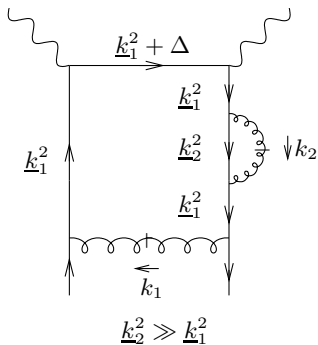
- Left:  $\underline{k}_2^2 \gg \underline{k}_1^2$  : the number of collinear singularities originating from  $k_1$  is not maximal  $\Rightarrow$  violates rule (ii)!
- Right:  $\underline{k}_1 \gg \underline{k}_2$  : the virtuality of the upper left fermionic propagator is  $\underline{k}_2^2 + \Delta$  where  $\Delta = -(x - \xi + \beta_2)s$ . This lowers the level of singularity, again leading to a suppressed contribution.

$\Rightarrow$  Third rule :

*(iii) Any coupling of a gluon to the  $s$ -channel fermionic line leads to a suppressed contribution.*

# Suppressed 2-loop diagrams

## Fermion self-energy diagrams



key point :  $s$ -channel fermion virtuality =  $\underline{k}_1^2 + \Delta$ , where  $\Delta = -(x - \xi + \beta_1)s$ .

$\Delta$  does not involve  $\beta_2 \Rightarrow$  reduces the power of  $\log(\xi - x)$  after  $\beta_2$  integration

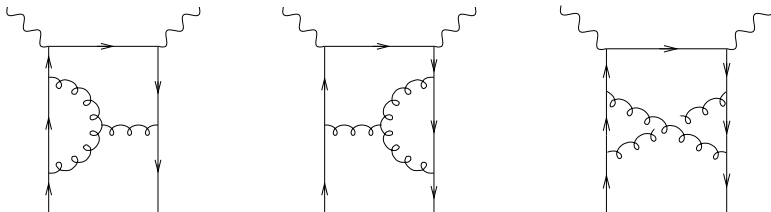
$\Rightarrow$  Fourth rule :

*(iv) The diagram should be sufficiently non-local in order that the  $s$ -channel fermionic line involves the whole  $p_2$  flux.*



# Suppressed 2-loop diagrams

## Other suppressed diagrams (rule (ii))

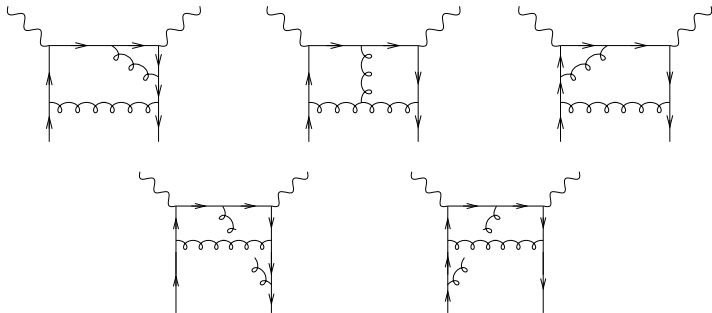


violate the rule:

*(ii) Each loop should involve a maximal number of collinear singularities, which manifest themselves as maximal powers of  $1/\underline{k}_i^2$  for each  $i$ , after the  $\alpha_i$  integration.*

# Suppressed 2-loop diagrams

## Other suppressed diagrams (rule (iii))

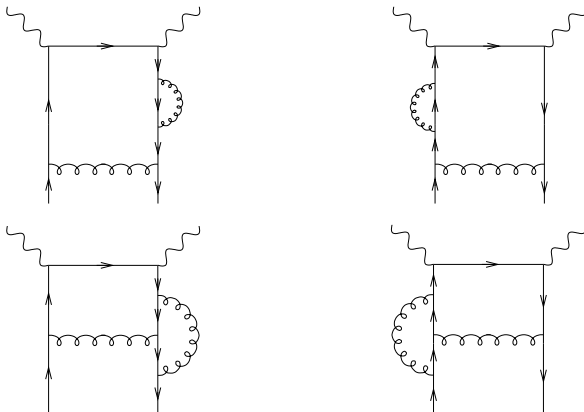


violate the rule:

*(iii) Any coupling of a gluon to the s-channel fermionic line leads to a suppressed contribution.*

# Suppressed 2-loop diagrams

## Other suppressed diagrams (rule (iv))



violate the rule:

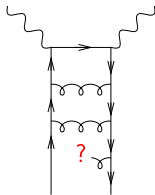
*(iv) The diagram should be sufficiently non-local in order that the  $s$ -channel fermionic line involves the whole  $p_2$  flux.*

# Beyond the 2-loop level

## Dominance of the ladder-like diagrams

The two-loop analysis showed that only ladder-like diagrams give contribution to  $\alpha_s^2 \frac{\log^4(\xi-x)}{x-\xi}$  terms.

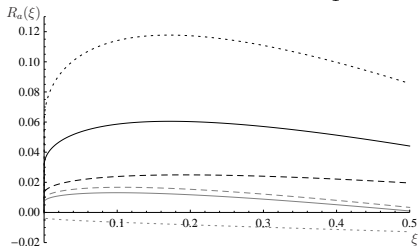
- Beyond the 2-loop level : recursive argument.
    - at 3-loop level the only missing building block is the four-gluon vertex
    - four-gluon vertex = contraction of two 3-gluon (subleading) diagrams with one less propagator.
      - ⇒ this kind of diagrams are also subleading
  - Dress a 2-loop (or n loop) ladder diagram from the right fermionic line :
    - only abelian-like diagrams are allowed
    - can not end on the right fermionic line → (local) violates rule (iv)
    - can not end on the s-channel fermionic line → violates rule (iii)
    - crossing of any gluon line is not permitted → violates rule (ii)
- ⇒ Only ladder-like diagrams are allowed



## Phenomenological implications

- We use a Double Distribution based model
  - S. V. Goloskokov and P. Kroll, *Eur. Phys. J. C* **50**, 829 (2007)
- Blind integral in the whole  $x$ -range: amplitude = NLO result  $\pm 1\%$
- To respect the domain of applicability of our resummation procedure:
  - restrict the use of our formula to  $\xi - a\gamma < |x| < \xi + a\gamma$
  - width  $a\gamma$  defined through  $|D \log(\gamma/(2\xi))| = 1$
  - theoretical uncertainty evaluated by varying  $a$
  - a more precise treatment is beyond the leading logarithmic approximation

$$R_a(\xi) = \frac{[\int_{\xi-a\gamma}^{\xi+a\gamma} + \int_{-\xi-a\gamma}^{-\xi+a\gamma}] dx (C^{\text{res}} - C_0 - C_1) H(x, \xi, 0)}{|\int_{-1}^1 dx (C_0 + C_1) H(x, \xi, 0)|}$$



$Re[R_a(\xi)]$  : black upper curves

$Im[R_a(\xi)]$  : grey lower curves

$a = 1$  (solid)

$a = 1/2$  (dotted)

$a = 2$  (dashed)