

The QCD shockwave approach at NLO: towards precision physics in gluonic saturation



Laboratoire de Physique
des 2 Infinis



Samuel Wallon

Laboratoire de Physique des 2 Infinis Irène Joliot-Curie
IJCLab

CNRS / Université Paris Saclay

Orsay

and

Université Paris Saclay



Diffraction and Low-x 2022

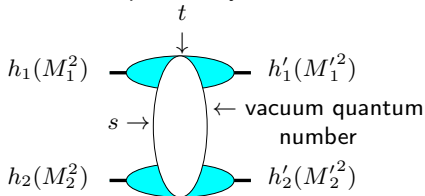
25 September 2022

Corigliano Calabro, Italy



QCD in the perturbative Regge limit

- One of the important longstanding theoretical questions raised by QCD is its behaviour in the perturbative Regge limit $s \gg -t$
- Based on theoretical grounds, one should identify and test suitable observables in order to test this peculiar dynamics



hard scales: $M_1^2, M_2^2 \gg \Lambda_{QCD}^2$ or $M_1'^2, M_2'^2 \gg \Lambda_{QCD}^2$ or $t \gg \Lambda_{QCD}^2$
 where the t -channel exchanged state is the so-called **hard Pomeron**

- Inclusive processes: the above picture applies at the level of **cross-sections** (optical theorem $\Rightarrow t = 0$)
- Diffractive processes: gap in rapidity between two clusters in the detector. The above picture applies at the level of **amplitudes**

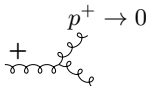
How to test QCD in the perturbative Regge limit?

What kind of observable?

- perturbation theory should be applicable:

selecting external or internal probes with transverse sizes $\ll 1/\Lambda_{QCD}$ (*hard* γ^* , *heavy meson* (J/Ψ , Υ), *energetic forward jets*) or by choosing large t in order to provide the hard scale.

- governed by the *rapidity divergences* of perturbative QCD



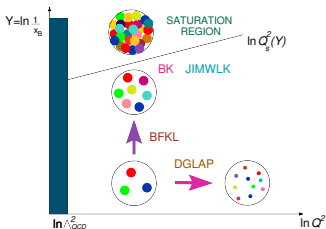
and *not* by its *collinear* dynamics

A Feynman diagram showing a gluon ladder. The top vertex is labeled $m=0$. The bottom vertex is labeled $m=0$. A dashed line from the bottom vertex to the right is labeled $Q/\theta \rightarrow 0$.

\implies select semi-hard processes with $s \gg p_{Ti}^2 \gg \Lambda_{QCD}^2$ where p_{Ti}^2 are typical transverse scale, **all of the same order**.

Quark and gluon content of proton

The various regimes governing the perturbative content of the proton



e.g.: DIS

- “usual” regime: x_B moderate ($x_B \gtrsim .01$):
Evolution in Q governed by the QCD renormalization group
(Dokshitzer, Gribov, Lipatov, Altarelli, Parisi equation)

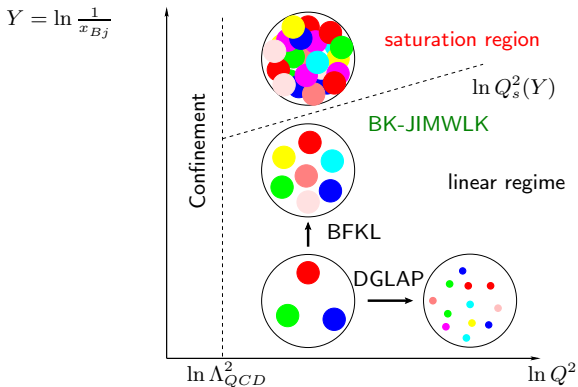
$$\sum_n (\alpha_s \ln Q^2)^n \quad \text{LLQ} \quad + \quad \alpha_s \sum_n (\alpha_s \ln Q^2)^n \quad \text{NLLQ} \quad + \dots$$

- perturbative Regge limit: $s_{\gamma^*p} \rightarrow \infty$ i.e. $x_B \sim Q^2/s_{\gamma^*p} \rightarrow 0$
in the perturbative regime (hard scale Q^2)
(Balitski Fadin Kuraev Lipatov equation)

$$\sum_n (\alpha_s \ln s)^n \quad \text{LLs} \quad + \quad \alpha_s \sum_n (\alpha_s \ln s)^n \quad \text{NLLs} \quad + \dots$$

Quark and gluon content of proton

The various regime governing the perturbative content of the proton



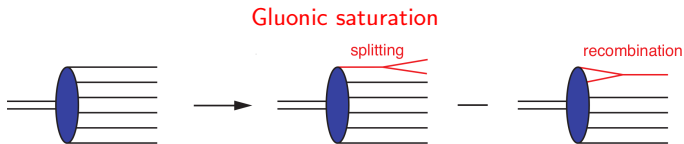
to handle with **saturation effects**:

one should resum powers of $\alpha_s^2 A^{1/3}$

typical order of magnitude of dipole-dipole scattering between a dipole probe and a dipole inside a large nucleus A through 2-gluon exchange

High energy: Regge limit

Non-linear perturbative regime and Color Glass Condensate



- $\alpha_s \ll 1$: weak coupling \Rightarrow perturbative approach
- very dense system: very high occupation numbers \Rightarrow gluons can recombine
- **characteristic scale**: saturation for $Q^2 \lesssim Q_s^2(x)$
 - number of gluons per surface unit:

$$\rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2}$$

- recombination cross-section:

$$\sigma_{gg \rightarrow g} \sim \frac{\alpha_s}{Q^2}$$

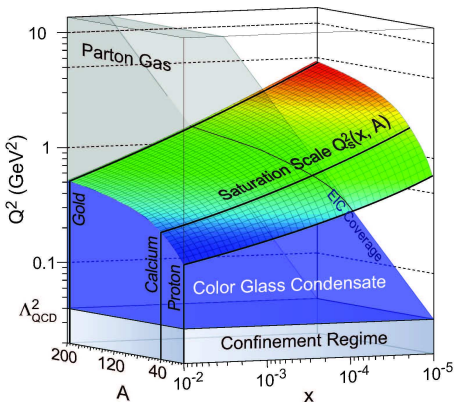
- effects are important when $\rho \sigma_{gg \rightarrow g} \gtrsim 1$

$$\text{i.e. } Q^2 \lesssim Q_s^2 \quad \text{with} \quad Q_s^2 \sim \frac{\alpha_s xG_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3} x^{-0.3}$$

Glueonic saturation

Experimental future

Glueonic saturation with a perturbative control



- At **EIC**, the saturation scale Q_s will be in the perturbative range

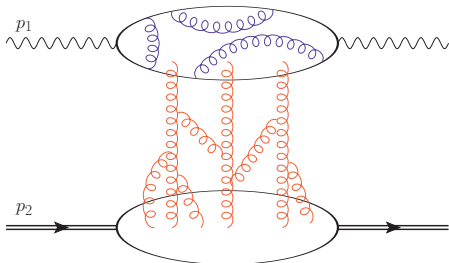
$$Q_s^2 \sim \left(\frac{A}{x} \right)^{1/3}$$

- Moderate center of mass energy
- Compensated by large A
- Large perturbative domain

$$\Lambda_{QCD}^2 \ll Q^2 \ll Q_s^2$$

in which saturation is under control

Kinematics



$$p_1 = p^+ n_1 - \frac{Q^2}{2p^+} n_2$$

$$p_2 = \frac{m_t^2}{2p_2^-} n_1 + p_2^- n_2$$

$$p^+ \sim p_2^- \sim \sqrt{\frac{s}{2}}$$

Lightcone **Sudakov** vectors

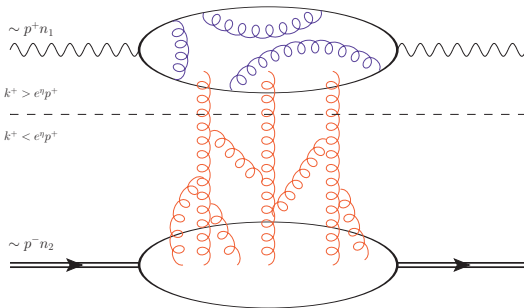
$$n_1 = \sqrt{\frac{1}{2}}(1, 0_\perp, 1), \quad n_2 = \sqrt{\frac{1}{2}}(1, 0_\perp, -1), \quad (n_1 \cdot n_2) = 1$$

Lightcone coordinates:

$$x = (x^0, x^1, x^2, x^3) \rightarrow (x^+, x^-, \vec{x})$$

$$x^+ = x_- = (x \cdot n_2) \quad x^- = x_+ = (x \cdot n_1)$$

Rapidity separation



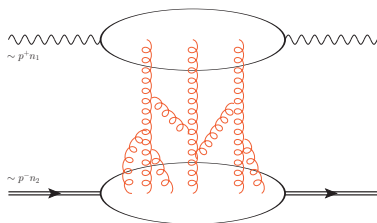
Let us split the gluonic field between "fast" and "slow" gluons

$$\mathcal{A}^{\mu a}(k^+, k^-, \vec{k}) = A_{\eta}^{\mu a}(|k^+| > e^{\eta} p^+, k^-, \vec{k}) \quad \text{quantum part} \\ + b_{\eta}^{\mu a}(|k^+| < e^{\eta} p^+, k^-, \vec{k}) \quad \text{classical part}$$

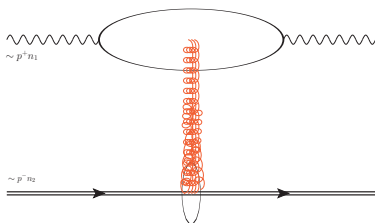
$$e^{\eta} \ll 1$$

\Rightarrow effective field theory I. Balitsky 1996

Large longitudinal boost to the projectile frame

Large longitudinal **boost**: $\Lambda \propto \sqrt{s}$ 

$$b^\mu(x)$$

just like usual boost of (\vec{E}, \vec{B}) or A^μ in electrodynamicsboost
→

$$b^-(x) n_2^\mu \simeq \delta(x^+) \mathbf{B}(\vec{x}) n_2^\mu$$

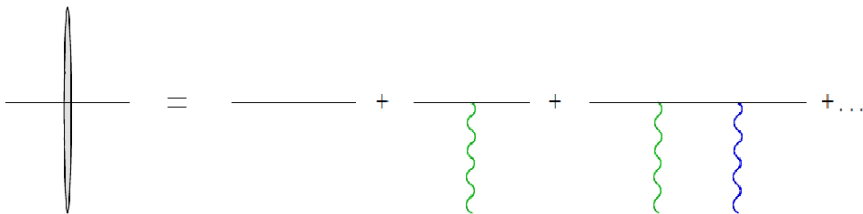
Shockwave approximation $b^-(x) n_2^\mu$: background fieldLight-cone gauge: $n_2 \cdot A = 0$ $\Rightarrow b \cdot A = 0$ which leads to simple Feynman rules in this effective field theory.

Multiple interactions with the target = Propagator in the shockwave field

Multiple interactions with the target can be resummed into **path-ordered Wilson lines** attached to each parton crossing lightcone time 0:

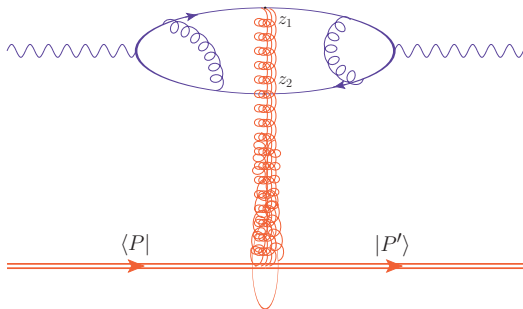
$$U_i = U_{\vec{z}_i} = U(\vec{z}_i, \eta) = P \exp \left[ig \int_{-\infty}^{+\infty} b_{\eta}^{-}(z_i^+, \vec{z}_i) dz_i^+ \right]$$

$$U_i = 1 + ig \int_{-\infty}^{+\infty} b_{\eta}^{-}(z_i^+, \vec{z}_i) dz_i^+ + (ig)^2 \int_{-\infty}^{+\infty} b_{\eta}^{-}(z_i^+, \vec{z}_i) b_{\eta}^{-}(z_j^+, \vec{z}_j) \theta(z_j^+) dz_i^+ dz_j^+ + \dots$$



Factorized picture in the projectile frame

Factorized amplitude



$$\mathcal{A}^\eta = \int d^{D-2} \vec{z}_1 d^{D-2} \vec{z}_2 \Phi^\eta(\vec{z}_1, \vec{z}_2) \langle P' | [\text{Tr}(U_{\vec{z}_1}^\eta U_{\vec{z}_2}^{\eta\dagger}) - N_c] | P \rangle$$

$$\text{Dipole operator } \mathcal{U}_{ij}^\eta = \frac{1}{N_c} \text{Tr}(U_{\vec{z}_i}^\eta U_{\vec{z}_j}^{\eta\dagger}) - 1$$

Written similarly for any number of Wilson lines in any color representation

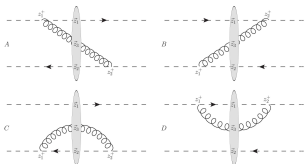
Evolution for the dipole operator

B-JIMWLK hierarchy of equations

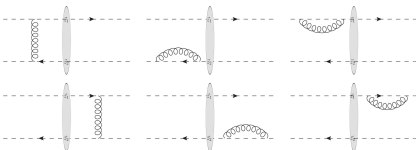
[I. Balitsky, J. Jalilian-Marian, E. Iancu, L. McLerran, H. Weigert, A. Leonidov, A. Kovner]

$$\frac{\partial \mathcal{U}_{12}^n}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^2}{\vec{z}_{13}^2 \vec{z}_{23}^2} [\mathcal{U}_{13}^n + \mathcal{U}_{32}^n - \mathcal{U}_{12}^n + \mathcal{U}_{13}^n \mathcal{U}_{32}^n]$$

$$\frac{\partial \mathcal{U}_{13}^n \mathcal{U}_{32}^n}{\partial \eta} = \dots$$



double dipole contribution



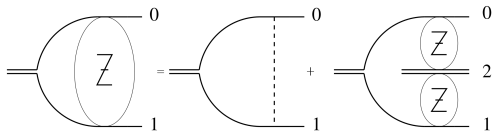
dipole contribution

Evolution for the dipole operator

Mean field approximation

Mean field approximation

↔ close connection with **Mueller** dipole's model **1994-1995** (large N_C)
(obtained using light-front quantization)



⇒ **BK equation** [I. Balitsky, 1995] [Y. Kovchegov, 1999]

$$\frac{\partial \langle \mathcal{U}_{12}^\eta \rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^2}{\vec{z}_{13}^2 \vec{z}_{23}^2} [\langle \mathcal{U}_{13}^\eta \rangle + \langle \mathcal{U}_{32}^\eta \rangle - \langle \mathcal{U}_{12}^\eta \rangle - \langle \mathcal{U}_{13}^\eta \rangle \langle \mathcal{U}_{32}^\eta \rangle]$$

Non-linear term : **saturation**

The JIMWLK Hamiltonian

Hamiltonian formulation of the hierarchy of equations

For an operator built from n Wilson lines, the JIMWLK evolution is given at LO accuracy by

$$\frac{\partial}{\partial \eta} \left[U_{\vec{z}_1}^\eta \dots U_{\vec{z}_n}^\eta \right] = \sum_{i,j=1}^n H_{ij} \cdot \left[U_{\vec{z}_1}^\eta \dots U_{\vec{z}_n}^\eta \right],$$

JIMWLK Hamiltonian

$$H_{ij} = \frac{\alpha_s}{2\pi^2} \int d\vec{z}_k \frac{\vec{z}_{ik} \cdot \vec{z}_{kj}}{\vec{z}_{ik}^2 \vec{z}_{kj}^2} [T_{i,L}^a T_{j,L}^a + T_{i,R}^a T_{j,R}^a - U_{\vec{z}_k}^{ab} (T_{i,L}^a T_{j,R}^b + T_{j,L}^a T_{i,R}^b)]$$

Theoretical status

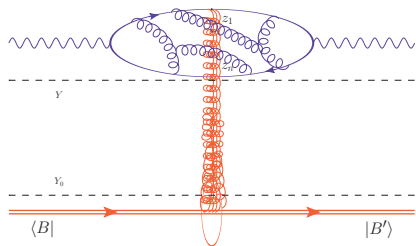
Evolution equations

Known pieces of the evolution beyond leading accuracy

- Explicit NLO dipole operator evolution [I. Balitsky, G. Chirilli 2007]
- Explicit NLO 3-point operator evolution [I. Balitsky, Grabovsky 2014]
- Explicit NLO 4-point operator evolution [A. Grabovsky 2015]
- Complete NLO JIMWLK Hamiltonian [A. Kovner, M. Lublinsky, Y. Mulian 2013]
- Additional resummation of collinear logarithms [E. Iancu, J. Madrigal, A. Mueller, G. Soyez, D. Triantafyllopoulos 2015], improved kinematics [Beuf 2015]
- Progress towards a more moderate- x extension [I. Balitsky, B. Tarasov 2015]
- Progress towards "Next-to-Eikonal" and "Next-to-Next-to-Eikonal" corrections for nucleus targets [T. Altinoluk, N. Armesto, G. Beuf, M. Martinez, A. Moscoso, C. Salgado 2014]
- Extensions to spin dependent distributions [F. Cougoulic, Y. Kovchegov, B. Tarasov, Tawaburst 2022]

Practical use of the formalism

- Compute the upper impact factor using the effective Feynman rules
- Build **non-perturbative models** for the matrix elements of the Wilson line operators acting on the target states
- Solve the **B-JIMWLK** evolution for these matrix elements with such non-perturbative initial conditions at a **typical target rapidity** Y_0 .
- Evaluate the solution at a **typical projectile rapidity** Y , or at the rapidity of the slowest gluon
- **Convolute** the solution and the impact factor



$$\mathcal{A} = \int d\vec{z}_1 \dots d\vec{z}_n \Phi(\vec{z}_1, \dots, \vec{z}_n) \times \langle P' | U_{\vec{z}_1} \dots U_{\vec{z}_n} | P \rangle$$

Exclusive diffraction allows one to probe the b_{\perp} -dependence of the non-perturbative scattering amplitude

Probing QCD in the Regge limit and towards saturation

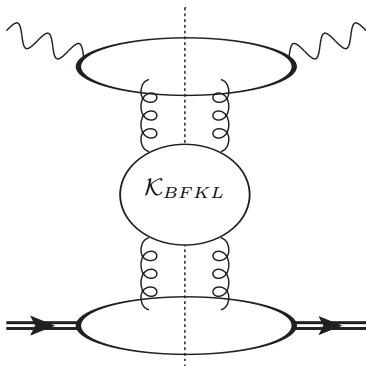
Observables to probe small- x QCD and saturation physics

- Perturbation theory should apply : a **hard scale** Q^2 is required
- One needs **semihard kinematics** : $s \gg p_T^2 \gg \Lambda_{QCD}^2$ where all the typical transverse scales p_T are of the same order
- Saturation is reached when $Q^2 \sim Q_s^2 \propto \left(\frac{A}{x}\right)^{\frac{1}{3}}$: **the smaller** $x \sim \frac{Q^2}{s}$ is and **the heavier the target ion**, the easier saturation is reached.

Known NLO impact factors

Known NLO **BFKL** impact factors

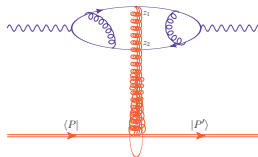
- $\gamma^* \rightarrow \gamma^*$ [J. Bartels, D. Colferai, S. Gieseke, A. Kyrielis, C. Qiao 2001]
- Forward jet production [J. Bartels, D. Colferai, G. Vacca 2003; F. Caporale, D. Ivanov, B. Murdaca, A. Papa, A. Perri 2011; G. Chachamis, M. Hentschinski, J. Madrigal, A. Sabio Vera 2012]
- Inclusive production of a pair of hadrons separated by a large interval of rapidity [D. Ivanov, A. Papa 2012]
- Diffractive $\gamma_L^* \rightarrow V_L$ in the forward limit [D. Ivanov, I. Kotsky, A. Papa 2004]
- Higgs production [F. G. Celiberto, D. Ivanov, M. Fucilla, M. Mohammed, A. Papa 2022]



Known NLO impact factors

Known NLO CGC impact factors

- $\gamma^* \rightarrow \gamma^*$ [I. Balitsky, G. Chirilli, 2011]; in the wave function approach [G. Beuf 2016]
 - Single inclusive particle production [G. Chirilli, B.-W. Xiao, F. Yuan 2012]
 - Exclusive diffractive electro- and photo- production of a forward dijet [R. Boussarie, A. Grabovsky, L. Szymanowski, S.W. 2016]
 - $\gamma_{L,T}^{(*)} \rightarrow V_L$ [R. Boussarie, A. Grabovsky, D. Ivanov, L. Szymanowski, S.W. 2017]
 - inclusive photon+dijet production in e+A DIS [K. Roy, R. Venugopalan 2019]
 - Dijet impact factor in DIS [R. Venugopalan, F. Salazar, P. Caucal 2021]
 - $\gamma^* \rightarrow \gamma^*$ with massive quarks in the wave function approach [G. Beuf, T. Lappi, R. Paatelainen 2021]
 - Dijet impact factor in DIS [R. Venugopalan, F. Salazar, P. Caucal 2021]
 - Semi-inclusive diffractive electro- and photo- production of a pair of hadrons at large p_T [M. Fucilla, A. Grabovsky, E. Li, L. Szymanowski, S.W. 2022]
- see talk of **Emilie Li** just after this talk



Practical implementation for diffractive processes

Framework

We are using the following framework:

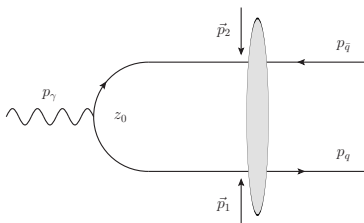
- **Regge-Gribov** limit : $s \gg (\text{hard scale})^2 \gg \Lambda_{QCD}^2$

hard scale:

- Q^2
 - t
 - Diffractive mass of a dijet system
 - p_T of produced hadrons
- Otherwise **completely general kinematics** \Rightarrow connection with **Wigner** distributions
 - **Shockwave (CGC)** Wilson line approach
 - Longitudinal cutoff: $|p_g^+| > \alpha p_\gamma^+$
 - Transverse dimensional regularization: $d = 2 + 2\varepsilon$

Various diffractive processes in the shockwave approach

LO open $q\bar{q}$ production



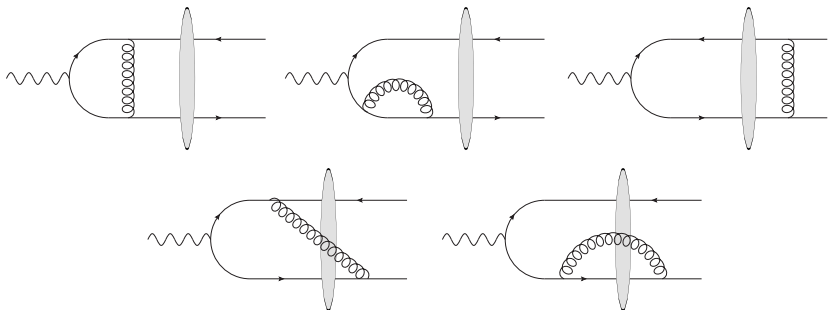
$$\mathcal{A} = \delta(p_q^+ + p_{\bar{q}}^+ - p_\gamma^+) \int d^d \vec{p}_1 d^d \vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2) \Phi_0(\vec{p}_1, \vec{p}_2) \\ \times C_F \langle P' | \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) | P \rangle$$

$$\tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) = \int d^d \vec{z}_1 d^d \vec{z}_2 e^{-i(\vec{p}_1 \cdot \vec{z}_1) - i(\vec{p}_2 \cdot \vec{z}_2)} \left[\frac{1}{N_c} \text{Tr}(U_{\vec{z}_1}^\alpha U_{\vec{z}_2}^{\alpha\dagger}) - 1 \right] \quad \text{Target}$$

Various diffractive processes in the shockwave approach

NLO open $q\bar{q}$ production

Virtual corrections

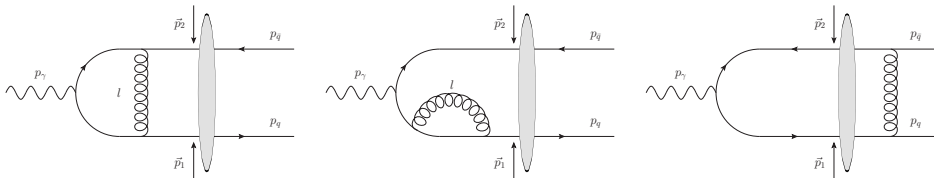


Diagrams contributing to the NLO correction

Various diffractive processes in the shockwave approach

First kind of virtual corrections

Virtual corrections (1)

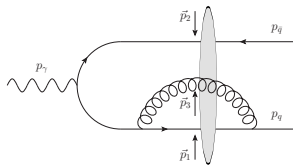
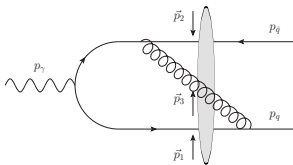


$$\mathcal{A}_{NLO}^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d \vec{p}_1 d^d \vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2) \Phi_{V1}(\vec{p}_1, \vec{p}_2) \times C_F \langle P' | \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) | P \rangle$$

Various diffractive processes in the shockwave approach

Second kind of virtual corrections

Virtual corrections (2)

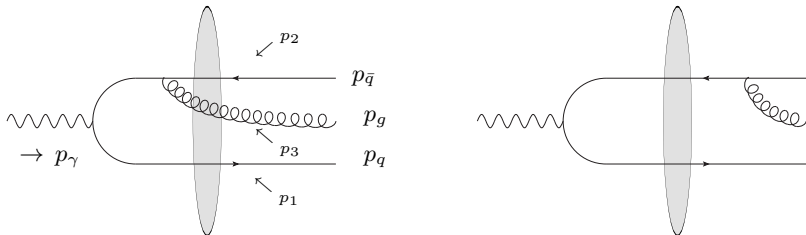


$$\begin{aligned}
 \mathcal{A}_{NLO}^{(2)} &\propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d \vec{p}_1 d^d \vec{p}_2 d^d \vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3) \\
 &\times [\Phi'_{V1}(\vec{p}_1, \vec{p}_2) C_F \langle P' | \tilde{U}^\alpha(\vec{p}_1, \vec{p}_2) | P \rangle \quad \text{dipole contribution} \\
 &+ \Phi_{V2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \langle P' | \tilde{W}(\vec{p}_1, \vec{p}_2, \vec{p}_3) | P \rangle] \quad \text{double dipole contribution}
 \end{aligned}$$

Various diffractive processes in the shockwave approach

LO open $q\bar{q}g$ production

Real corrections



$$\begin{aligned}
 \mathcal{A}_R^{(2)} &\propto \delta(p_q^+ + p_{\bar{q}}^+ + p_g^+ - p_\gamma^+) \int d^d \vec{p}_1 d^d \vec{p}_2 d^d \vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3) \\
 &\times [\Phi'_{R1}(\vec{p}_1, \vec{p}_2) C_F \langle P' | \tilde{U}^\alpha(\vec{p}_1, \vec{p}_2) | P \rangle \\
 &+ \Phi_{R2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \langle P' | \tilde{W}(\vec{p}_1, \vec{p}_2, \vec{p}_3) | P \rangle]
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{A}_R^{(1)} &\propto \delta(p_q^+ + p_{\bar{q}}^+ + p_g^+ - p_\gamma^+) \int d^d \vec{p}_1 d^d \vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2) \\
 &\times \Phi_{R1}(\vec{p}_1, \vec{p}_2) C_F \langle P' | \tilde{U}^\alpha(\vec{p}_1, \vec{p}_2) | P \rangle
 \end{aligned}$$

Various diffractive processes in the shockwave approach

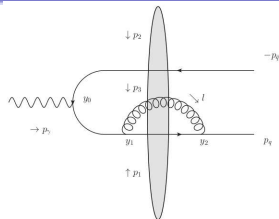
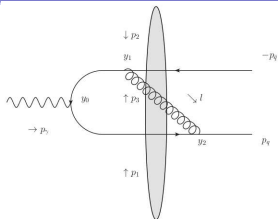
Various type of divergences

Divergences

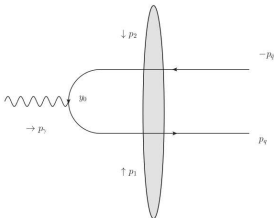
- Rapidity divergence $p_g^+ \rightarrow 0$ $\Phi_{V2}\Phi_0^* + \Phi_0\Phi_{V2}^*$
- UV divergence $\vec{p}_g^2 \rightarrow +\infty$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \rightarrow 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_q^+}{p_q^+}p_q$ or $\frac{p_{\bar{q}}^+}{p_{\bar{q}}^+}p_{\bar{q}}, p_g^+ \rightarrow 0$ $\Phi_{R1}\Phi_{R1}^*$

Various diffractive processes in the shockwave approach

Rapidity divergence



Double dipole virtual correction Φ_{V2}



B-JIMWLK evolution of the LO term : $\Phi_0 \otimes \mathcal{K}_{BK}$

Various diffractive processes in the shockwave approach

Rapidity divergence

B-JIMWLK equation for the dipole operator

$$\frac{\partial \tilde{U}_{12}^\alpha}{\partial \ln \alpha} = 2\alpha_s N_c \mu^{2-d} \int \frac{d^d \vec{k}_1 d^d \vec{k}_2 d^d \vec{k}_3}{(2\pi)^{2d}} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1 - \vec{p}_2) \left(\tilde{u}_{13}^\alpha \tilde{u}_{32}^\alpha + \tilde{u}_{13}^\alpha + \tilde{u}_{32}^\alpha - \tilde{u}_{12}^\alpha \right) \\ \times \left[2 \frac{(\vec{k}_1 - \vec{p}_1) \cdot (\vec{k}_2 - \vec{p}_2)}{(\vec{k}_1 - \vec{p}_1)^2 (\vec{k}_2 - \vec{p}_2)^2} + \frac{\pi^{\frac{d}{2}} \Gamma(1 - \frac{d}{2}) \Gamma^2(\frac{d}{2})}{\Gamma(d-1)} \left(\frac{\delta(\vec{k}_2 - \vec{p}_2)}{[(\vec{k}_1 - \vec{p}_1)^2]^{1-\frac{d}{2}}} + \frac{\delta(\vec{k}_1 - \vec{p}_1)}{[(\vec{k}_2 - \vec{p}_2)^2]^{1-\frac{d}{2}}} \right) \right]$$

η **rapidity divide**, which separates the upper and the lower impact factors

$$\Phi_0 \tilde{U}_{12}^\alpha \rightarrow \Phi_0 \tilde{U}_{12}^\eta + 2 \ln \left(\frac{e^\eta}{\alpha} \right) \mathcal{K}_{BK} \Phi_0 \tilde{W}_{123}$$

Provides a counterterm to the $\ln \alpha$ divergence in the virtual double dipole impact factor:

$$\Phi_0 \tilde{U}_{12}^\alpha + \Phi_{V2} \tilde{W}_{123}^\alpha \text{ is finite and independent of } \alpha$$

Various diffractive processes in the shockwave approach

Various type of divergences

- Rapidity divergence

- UV divergence $\vec{p}_g^2 \rightarrow +\infty$

$$\Phi_{V1} \Phi_0^* + \Phi_0 \Phi_{V1}^*$$

- Soft divergence $p_g \rightarrow 0$

$$\Phi_{V1} \Phi_0^* + \Phi_0 \Phi_{V1}^*, \Phi_{R1} \Phi_{R1}^*$$

- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$

$$\Phi_{R1} \Phi_{R1}^*$$

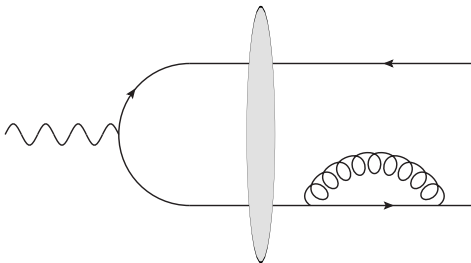
- Soft and collinear divergence $p_g = \frac{p_q^+}{p_q^+} p_q$ or $\frac{p_{\bar{q}}^+}{p_{\bar{q}}^+} p_{\bar{q}}, p_g^+ \rightarrow 0$

$$\Phi_{R1} \Phi_{R1}^*$$

Various diffractive processes in the shockwave approach

UV divergence

Dressing of the external lines



Some null diagrams just contribute to turning UV divergences into IR divergences

$$\Phi = 0 \propto \left(\frac{1}{2\epsilon_{IR}} - \frac{1}{2\epsilon_{UV}} \right)$$

Various diffractive processes in the shockwave approach

Various type of divergences

- Rapidity divergence
- UV divergence
- Soft divergence $p_g \rightarrow 0$ $\Phi_{V1} \Phi_0^* + \Phi_0 \Phi_{V1}^*, \Phi_{R1} \Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1} \Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_q^+}{p_q^-} p_q$ or $\frac{p_{\bar{q}}^+}{p_{\bar{q}}^-} p_{\bar{q}}, p_g^+ \rightarrow 0$ $\Phi_{R1} \Phi_{R1}^*$

At this stage, the treatment of collinear and soft and collinear divergence are process dependent

3 examples

- **exclusive dijet diffractive production:** jet algorithm [RB, Grabovsky, Szymanowski, Wallon ; JHEP 1611 (2016)]
- **exclusive meson diffractive production:** renormalisation of the meson distribution amplitude [R. Boussarie, A. Grabovsky, D. Ivanov, L. Szymanowski, S.W. ; PRL 119 (2017)]
- **semi-inclusive dihadron production:** renormalisation of the parton fragmentation functions [M. Fucilla, A. Grabovsky, D. Ivanov, E. Li, L. Szymanowski, S.W., to appear] (see talk of **E. Li**)

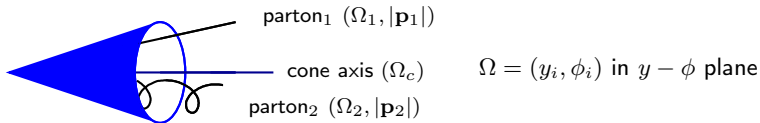
Various diffractive processes in the shockwave approach

Soft and collinear divergence: [dijet case](#)

Cone jet algorithm at NLO (Ellis, Kunszt, Soper)

- Should partons ($|\mathbf{p}_1|, \phi_1, y_1$) and ($|\mathbf{p}_2|, \phi_2, y_2$) combined in a single jet?
 $|\mathbf{p}_i|$ = transverse energy deposit in the calorimeter cell i of parameter $\Omega = (y_i, \phi_i)$ in $y - \phi$ plane
- define transverse energy of the jet: $p_J = |\mathbf{p}_1| + |\mathbf{p}_2|$
- jet axis:

$$\Omega_c \begin{cases} y_J = \frac{|\mathbf{p}_1| y_1 + |\mathbf{p}_2| y_2}{p_J} \\ \phi_J = \frac{|\mathbf{p}_1| \phi_1 + |\mathbf{p}_2| \phi_2}{p_J} \end{cases}$$



If distances $|\Omega_i - \Omega_c|^2 \equiv (y_i - y_c)^2 + (\phi_i - \phi_c)^2 < R^2$ ($i = 1$ and $i = 2$)

\implies partons 1 and 2 are in the same cone Ω_c

Applying this (in the small R^2 limit) cancels our **soft and collinear divergence**

Exclusive dijet diffractive production

Various type of divergences: **dijet case**

Dijet case

- Rapidity divergence

- UV divergence

- Soft divergence $p_g \rightarrow 0$

$$\Phi_{V1} \Phi_0^* + \Phi_0 \Phi_{V1}^*, \Phi_{R1} \Phi_{R1}^*$$

- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$

$$\Phi_{R1} \Phi_{R1}^*$$

- Soft and collinear divergence

The remaining divergences cancel the standard way:
virtual corrections and real corrections cancel each other

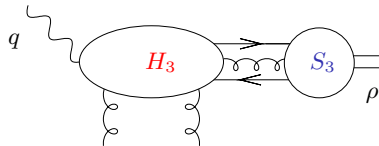
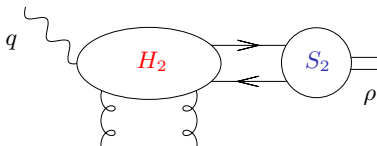
This is done after combining:

- the (LO + NLO) contribution to $q\bar{q}$ production
- the part of the contribution of the $q\bar{q}g$ production where the gluon is either soft or collinear to the quark or to the antiquark, so that they both form a single jet

Various diffractive processes in the shockwave approach

Collinear factorization: **meson case**

The impact factor is the convolution of a **hard part** and the **vacuum-to-meson matrix element** of an operator



$$\int_x (H_2(x))_{ij}^{\alpha\beta} \langle \rho | \bar{\psi}_i^\alpha(x) \psi_j^\beta(0) | 0 \rangle \quad \int_{x_1, x_2} (H_3^\mu(x_1, x_2))_{ij,c}^{\alpha\beta} \langle \rho | \bar{\psi}_i^\alpha(x_1) A_\mu^c(x_2) \psi_j^\beta(0) | 0 \rangle$$

H and S are connected by:

- convolution
- **summation over spinor and color indices**

Once **factorization in the t channel** is done, now **factorize in the s channel** with collinear factorization: **expand the impact factor in powers of the hard scale**

Various diffractive processes in the shockwave approach

Collinear factorization: **meson case**

Collinear factorization at **twist 2**

- Leading twist DA for a **longitudinally polarized** light vector meson

$$\langle \rho | \bar{\psi}(z) \gamma^\mu \psi(0) | 0 \rangle \rightarrow p^\mu f_\rho \int_0^1 dx e^{ix(p \cdot z)} \varphi_1(x)$$

- Leading twist DA for a **transversely polarized** light vector meson

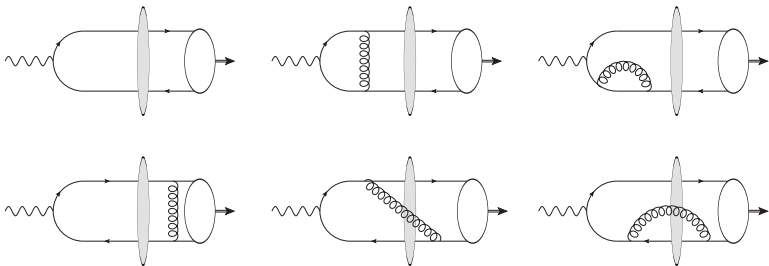
$$\langle \rho | \bar{\psi}(z) \sigma^{\mu\nu} \psi(0) | 0 \rangle \rightarrow i(p^\mu \varepsilon_\rho^\nu - p^\nu \varepsilon_\rho^\mu) f_\rho^T \int_0^1 dx e^{ix(p \cdot z)} \varphi_\perp(x)$$

The twist 2 DA for a transverse meson is **chiral odd**, thus $\gamma^* A \rightarrow \rho_T A$ starts at **twist 3**

Various diffractive processes in the shockwave approach

Collinear factorization: **meson case**

NLO Deep Virtual Meson Production with Pomeron (shockwave) exchange



Leading twist for a longitudinally polarized, C^- meson:

Make the $q\bar{q}$ pair collinear to the meson, and convolute with a Distribution Amplitude (vacuum-to-meson matrix element)

Additional divergence from the collinearity: canceled from the renormalization of the s -channel operator (ERBL evolution equation for the DA)

Probes **gluon GPDs** at low x , as well as **twist 2 DAs**

Conclusion

- There have been very important progresses in the theoretical description of gluonic saturation:
 - Evolution kernels are now known at NLO
 - Many impact factor are now known at NLO
- Description of processes at a complete NLO level remains challenging in view of the complexity of the obtained analytical results:
no full phenomenological NLO description of any process including saturation for the moment
- Understanding the way of including collinear logarithms effects is an important problem (to avoid negative cross-sections!)
- There is a clear hope that these various results should provide precise observables to reveal without ambiguity the saturation of gluons in nucleons and nuclei, and to study the Color Glass Condensate
- Many precision observables could be studied at LHC in UPC and at the future EIC