QCD factorization beyond leading twist in exclusive processes: ρ_T-meson production

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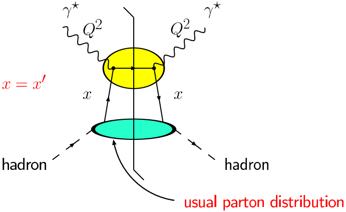
Introduction

Exclusive processes at high energy in QCD: extensions from DIS

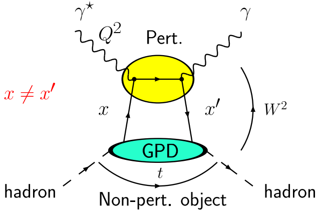
- DIS: inclusive process → forward amplitude ($t = 0$)



Structure Function

= **Coefficient Function** (hard) ⊗ **Parton Distribution Function** (soft)

- DVCS: exclusive process → non forward amplitude ($-t \ll s = W^2$)



Amplitude

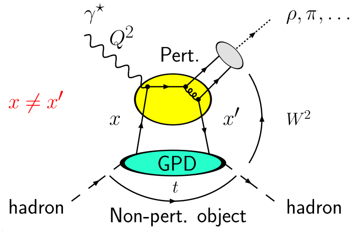
= **Coefficient Function** (hard) ⊗ **Generalized Parton Distribution** (soft)

Introduction
Extensions from GPD

- Meson production: γ replaced by $\rho,\ \pi, \cdots$

Amplitude

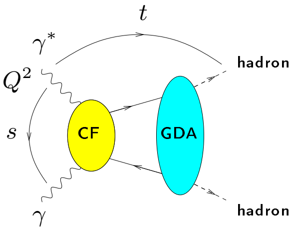
= GPD (soft) ⊗ CF (hard) ⊗ Distribution Amplitude (soft)



- Crossed process: $s \ll -t$

Amplitude

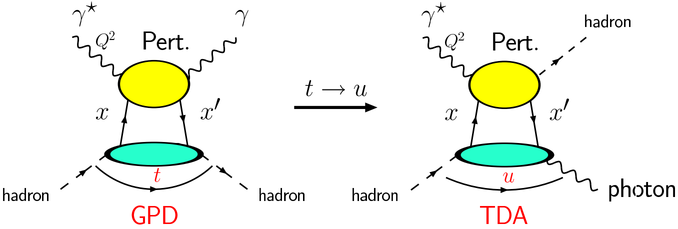
= Coefficient Function (hard) ⊗ Generalized Distribution Amplitude (soft)



Introduction

Extensions from GPD

- starting from usual DVCS, one allows initial hadron $\neq$ final hadron
  example:



which can be further extended by replacing the outoing γ by any hadronic state

Amplitude	=	Transition Distribution Amplitude (soft)	$\otimes$	CF (hard)	$\otimes$	DA (soft)

Introduction: phenomenology of exclusive processes within collinear factorization

- Experimental tests are possible in fixed target experiments
  - $e^{\pm}p$, $\mu^{\pm}p$: HERA (HERMES), JLab, COMPASS...

  as well as in colliders, mainly for medium s
  - $e^{\pm}p$ colliders: HERA (H1, ZEUS)
  - $e^{+}e^{-}$ colliders: LEP, Belle, BaBar, BEPC
- Collinear factorization has been proven only for specific cases:
  e.g.: ρ_T production cannot directly be factorized (appearance of end point singularities)
  ⇒ improvement needed for a consistent approach of exclusive processes

QCD in the perturbative Regge limit

- At the same time, at large s, the interest for phenomenological tests of hard Pomeron and related resummed approaches has become pretty wide:
  - inclusive tests (total cross-section) and semi-inclusive tests (diffraction, forward jets, …)
  - exclusive tests (meson production)
- These tests concern all type of collider experiments:
  - $e^{\pm}p$: HERA: (H1, ZEUS)
  - $p\bar{p}$ and pp: TEVATRON (CDF, D0); LHC (CMS, ATLAS, ALICE)
  - $e^{+}e^{-}$: (LEP, ILC)
- These high energy exclusive processes in the perturbative Regge limit may provide new ideas when dealing with collinear factorization

Introduction

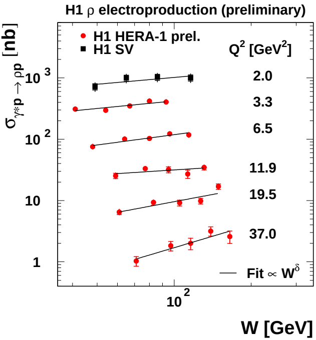
Exclusive ρ-production

Our studies attempt to describe exclusive processes involving the production of ρ-mesons in diffraction-type experiment. We choose $t = t_{min}$ for simplicity.

- $\gamma^*(q) + \gamma^*(q') \to \rho_T(p_1) + \rho(p_2)$ process in $e^+ e^- \to e^+ e^- \rho_T(p_1) + \rho(p_2)$ with double tagged lepton at ILC
- $\gamma^*(q) + P \to \rho_T(p_1) + P$ at HERA

This process was studied by H1 and ZEUS

- the total cross-section strongly decreases with Q^2
- dramatic increase with $W^2 = s_{\gamma^* P}$ (transition from soft to hard regime governed by Q^2)



(from X. Janssen (H1), DIS 2008)

Introduction

Exclusive ρ-production

Polarization effects in $\gamma^* \, P \to \rho \, P$ at HERA

- one can experimentally measure all spin density matrix elements

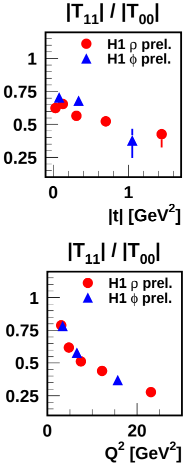
- at $t = t_{min}$ one can experimentally distinguish

$$\begin{cases} \gamma_L^* \to \rho_L : & \text{dominates} & (\text{twist 2 dominance}) \\ \gamma_T^* \to \rho_T : & \text{sizable} & (\text{twist 3}) \end{cases}$$

- S-channel helicity conservation:

$$\begin{cases} \gamma_L^* \to \rho_L & (\equiv T_{00}) \\ \gamma_T^* \to \rho_T \, , & \end{cases}$$

Dominate with respect to all other transitions. Experimentally, $\gamma_T^* \to \rho_T$ is dominated by $\gamma_{T(-)}^* \to \rho_{T(-)}$ and $\gamma_{T(+)}^* \to \rho_{T(+)}$ $(\equiv T_{11})$



(from X. Janssen (H1), DIS 2008)

Introduction

Exclusive ρ-production

The processes with vector particle such as rho-meson probe deeper into the fine features of QCD.
It deserves theoretical developpement to describe HERA data in its special kinematical range:

- large $s_{\gamma^* P}$ ⇒ small-x effects expected, within k_t-factorization
- large Q^2 ⇒ hard scale ⇒ perturbative approach and collinear factorization ⇒ the ρ can be described through its chiral even Distribution Amplitudes

$$\begin{cases} \rho_L & \text{twist 2} \\ \rho_T & \text{twist 3} \end{cases}$$

The main ingredient is the $\gamma^* \to \rho$ impact factor

- For ρ_T, special care is needed: a pure 2-body description would violate gauge invariance.
- We show that:
  - Including in a consistent way all twist 3 contributions, i.e. 2-body and 3-body correlators, gives a gauge invariant impact factor
  - Our treatment is free of end-point singularities and does not violates the QCD factorization

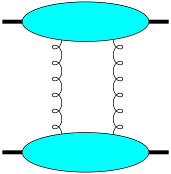
Impact factor for exclusive processes

Theoretical motivations

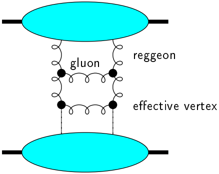
QCD in perturbative Regge limit

- In this limit, the dynamics is dominated by gluons (dominance of spin 1 exchange in t channel)
- BFKL (and extensions: NLL, saturations effects, ...) is expected to dominate with respect to Born order at large relative rapidity.

Born order:



BFKL ladder:

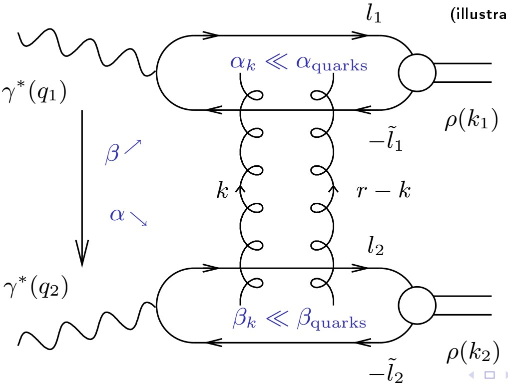


Impact factor for exclusive processes

k_T factorization

$\gamma^* \, \gamma^* \rightarrow \rho \, \rho$ as an example

- Use Sudakov decomposition $k = \alpha \, p_1 + \beta \, p_2 + k_\perp$ $(p_1^2 = p_2^2 = 0, \; 2 p_1 \cdot p_2 = s)$
- write $d^4 k = \frac{s}{2} \, d\alpha \, d\beta \, d^2 k_\perp$
- $t-$channel gluons with non-sense polarizations $(\epsilon_{NS}^{up} = \frac{2}{s} \, p_2, \; \epsilon_{NS}^{down} = \frac{2}{s} \, p_1)$ dominate at large s



(illustration for 2-body case)

$\Rightarrow$ set $\alpha_k = 0$ and $\int d\beta$

$\Rightarrow$ set $\beta_k = 0$ and $\int d\alpha$

Impact factor for exclusive processes

k_T factorization

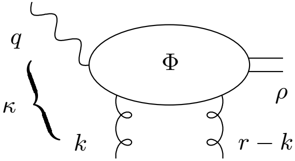
impact representation $\underline{k}$ = Eucl. ↔ $k_\perp$ = Mink.

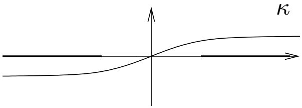
$$\mathcal{M} = is \int \frac{d^2\,\underline{k}}{(2\pi)^2 \underline{k}^2\,(\underline{r}-\underline{k})^2} \Phi^{\gamma^*(q_1)\to\rho(p_1^\rho)}(\underline{k},\underline{r}-\underline{k})\;\Phi^{\gamma^*(q_2)\to\rho(p_2^\rho)}(-\underline{k},-\underline{r}+\underline{k})$$

The $\gamma^*_{L,T}(q)g(k_1) \to \rho_{L,T}\,g(k_2)$ impact factor is normalized as

$$\Phi^{\gamma^*\to\rho}(\underline{k}^2) = e^{\gamma^*\mu}\,\frac{1}{2\,s}\int\frac{d\kappa}{2\pi}\,\text{Disc}_\kappa\,\mathcal{S}_\mu^{\gamma^*\,g\to\rho\,g}(\underline{k}^2),$$

with $\kappa = (q+k)^2 = \beta\,s - Q^2 - \underline{k}^2$



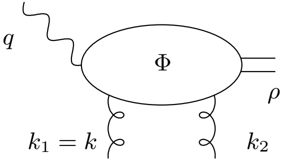


Impact factor for exclusive processes

Gauge invariance within subleading twists

Gauge invariance

- QCD gauge invariance (probes are colorless)
  $\Rightarrow$ impact factor should vanish when $\underline{k} \to 0$ or $\underline{r} - \underline{k} \to 0$
- In the following we will restrict ourselve to the case $t = t_{min}$, i.e. to $\underline{r} = 0$



$$k_1 = \frac{\kappa + Q^2 + \underline{k}^2}{s} p_2 + k_\perp$$

$$k_2 = \frac{\kappa + \underline{k}^2}{s} p_2 + k_\perp ,$$

$$k_1^2 = k_2^2 = -\underline{k}^2$$

This kinematics takes into account skewedness effects along p_2
$t = t_{min} \;\Rightarrow$ restriction to the transitions

$$\left\{ \begin{array}{cccc} 0 & \to & 0 & \text{(twist 2)} \\ \text{(+ or -)} & \to & \text{(+ or -)} & \text{(twist 3)} \end{array} \right.$$

- At twist 3 level (for $\gamma^*_T \to \rho_T$ transition), gauge invariance is a non trivial statement which requires 2 *and* 3 body correlators

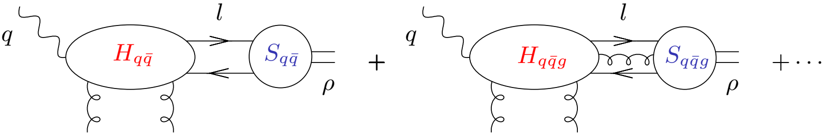
Collinear factorization

Light-Cone Collinear approach

- The impact factor can be written as

$$\Phi = \int d^4 l \cdots \mathrm{tr}[H(l \cdots) \quad S(l \cdots)]$$

hard part soft part



- At the 2-body level:

$$S_{q\bar{q}}(l) = \int d^4 z \, e^{-il\cdot z} \langle \rho(p) | \psi(0) \, \bar{\psi}(z) | 0 \rangle,$$

- H and S are related by $\int d^4 l$ and by the summation over spinor indices

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

1 - Momentum factorization (1)

- Use Sudakov decomposition in the form ($p = p_1$, $n = 2\,p_2/s \Rightarrow p \cdot n = 1$)

$$\begin{array}{lccccccc} l_\mu & = & y\,p_\mu & + & l^\perp_\mu & + & (l\cdot p)\,n_\mu, & y = l\cdot n \\ \text{scaling:} & & 1 & & 1/Q & & 1/Q^2 & \end{array}$$

- decompose $H(k)$ around the p direction:

$$H(l) = H(yp) \quad + \quad \left.\frac{\partial H(l)}{\partial l_\alpha}\right|_{l=yp} (l - y\,p)_\alpha + \ldots \quad \text{with} \quad (l - y\,p)_\alpha \approx l^\perp_\alpha$$

- In Fourier space, the twist 3 term $l^\perp_\alpha$ turns into a derivative of the soft term
  $\Rightarrow$ one will deal with $\int d^4z\; e^{-il\cdot z} \langle \rho(p)|\psi(0)\, i\; \overleftrightarrow{\partial}_{\alpha\perp} \bar\psi(z)|0\rangle$

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

1 - Momentum factorization (2)

- write

$$d^4l \longrightarrow d^4l \; \delta(y - l\cdot n) \; dy$$

- $\int d^4l \, \delta(y - l\cdot n)$ is then absorbed in the soft term:

$$\begin{aligned}
(\tilde{S}_{q\bar{q}}, \partial_\perp \tilde{S}_{q\bar{q}}) &\equiv \int d^4l \, \delta(y - l\cdot n) \int d^4z \, e^{-il\cdot z} \langle \rho(p)|\psi(0) \, (1, \, i \, \overleftrightarrow{\partial_\perp})\bar{\psi}(z)|0\rangle \\
\left(\delta(y - l\cdot n) = \int \tfrac{d\lambda}{2\pi} e^{-i\lambda(y - l\cdot n)} \Rightarrow\right) \quad &= \int \frac{d\lambda}{2\pi} \, e^{-i\lambda y} \int d^4z \, \delta^{(4)}(z - \lambda n) \, \langle \rho(p)|\psi(0) \, (1, \, i \, \overleftrightarrow{\partial_\perp})\bar{\psi}(z)|0\rangle \\
&= \int \frac{d\lambda}{2\pi} \, e^{-i\lambda y} \langle \rho(p)|\psi(0) \, (1, \, i \, \overleftrightarrow{\partial_\perp})\bar{\psi}(\lambda n)|0\rangle
\end{aligned}$$

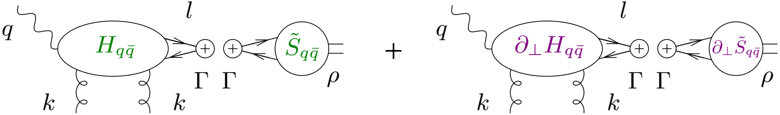
- $\int dy$ performs the longitudinal momentum factorization

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

2 - Spinorial (and color) factorization

- Use Fierz decomposition of the Dirac (and color) matrices $\psi(0)\,\bar{\psi}(z)$ and $\psi(0)\,i\,\overleftrightarrow{\partial_\perp}\,\bar{\psi}(z)$:



- Φ has now the simple factorized form:

$$\Phi = \int dx \left\{ \text{tr}\left[H_{q\bar{q}}(x\,p)\,\Gamma\right]\, S^{\Gamma}_{q\bar{q}}(x) + \text{tr}\left[\partial_\perp H_{q\bar{q}}(x\,p)\,\Gamma\right]\, \partial_\perp S^{\Gamma}_{q\bar{q}}(x) \right\}$$

$\Gamma = \gamma^\mu$ and $\gamma^\mu\,\gamma^5$ matrices

$$S^{\Gamma}_{q\bar{q}}(x) = \int \frac{d\lambda}{2\pi}\, e^{-i\lambda x} \langle \rho(p)|\bar{\psi}(\lambda n)\,\Gamma\,\psi(0)|0\rangle$$

$$\partial_\perp S^{\Gamma}_{q\bar{q}}(x) = \int \frac{d\lambda}{2\pi}\, e^{-i\lambda x} \langle \rho(p)|\bar{\psi}(\lambda n)\,\Gamma\, i\, \overleftrightarrow{\partial_\perp}\,\psi(0)|0\rangle$$

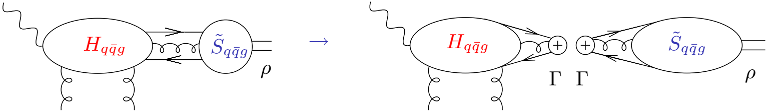
- choose axial gauge condition for gluons, *i.e.* $n \cdot A = 0 \Rightarrow$ no Wilson line

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (3-body case)

Factorization of 3-body contributions

- 3-body contributions start at genuine twist 3
  ⇒ no need for Taylor expansion
- Momentum factorization goes in the same way as for 2-body case
- Spinorial (and color) factorization is similar:



Collinear factorization

Parametrization of vacuum–to–rho-meson matrix elements (DAs): 2-body correlators

2-body non-local correlators

ρ_L twist 2

ρ_T kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- vector correlator

$$\langle\rho(p)|\bar{\psi}(z)\gamma_\mu\psi(0)|0\rangle \overset{\mathcal{F}}{=} m_\rho\, f_\rho\, \left[\varphi_1(y)\,(e^*\cdot n)p_\mu + \varphi_3(y)\, e^{*T}_\mu\right]$$

- axial correlator

$$\langle\rho(p)|\bar{\psi}(z)\gamma_5\gamma_\mu\psi(0)|0\rangle \overset{\mathcal{F}}{=} m_\rho\, f_\rho\, i\,\varphi_A(y)\,\varepsilon_{\mu\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta\, n_\delta$$

- vector correlator with transverse derivative

$$\langle\rho(p)|\bar{\psi}(z)\gamma_\mu\, i\, \overleftrightarrow{\partial^\perp_\alpha}\,\psi(0)|0\rangle \overset{\mathcal{F}}{=} m_\rho\, f_\rho\, \varphi_1^T(y)\, p_\mu e^{*T}_\alpha$$

- axial correlator with transverse derivative

$$\langle\rho(p)|\bar{\psi}(z)\gamma_5\gamma_\mu\, i\, \overleftrightarrow{\partial^\perp_\alpha}\,\psi(0)|0\rangle \overset{\mathcal{F}}{=} m_\rho\, f_\rho\, i\,\varphi_A^T(y)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta\, n_\delta,$$

where y ($\bar{y}\equiv 1-y$) = momentum fraction along $p\equiv p_1$ of the quark (antiquark) and
$\overset{\mathcal{F}}{=}\int_0^1\, dy\,\exp\,[i\,y\,p\cdot z]$, with $z=\lambda n$

$\Rightarrow$ 5 2-body DAs

Collinear factorization

Parametrization of vacuum–to–rho-meson matrix elements: 3-body correlators

3-body non-local correlators **genuine twist 3**

- vector correlator

$$\langle \rho(p)|\bar{\psi}(z_1)\gamma_\mu g A^T_\alpha(z_2)\psi(0)|0\rangle \overset{\mathcal{F}_2}{=} m_\rho\, f_3^V\, B(y_1,y_2)\, p_\mu\, e^{*T}_\alpha ,$$

- axial correlator

$$\langle \rho(p)|\bar{\psi}(z_1)\gamma_5\gamma_\mu g A^T_\alpha(z_2)\psi(0)|0\rangle \overset{\mathcal{F}_2}{=} m_\rho\, f_3^A\, i\, D(y_1,y_2)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta\, n_\delta ,$$

where y_1, $\bar{y}_2$, $y_2 - y_1$ = quark, antiquark, gluon momentum fraction

and $\overset{\mathcal{F}_2}{=} \int_0^1 dy_1 \int_0^1 dy_2 \; \exp\left[i\, y_1\, p\cdot z_1 + i(y_2-y_1)\, p\cdot z_2\right]$, with $z_{1,2} = \lambda n$

$\Rightarrow$ 2 3-body DAs

Collinear factorization

Symmetry properties

From C-conjugation on the previous correlators, one gets:

- 2-body correlators:

$$\begin{aligned}\varphi_1(y) &= \varphi_1(1-y)\\ \varphi_3(y) &= \varphi_3(1-y)\\ \varphi_A(y) &= -\varphi_A(1-y)\\ \varphi_1^T(y) &= -\varphi_1^T(1-y)\\ \varphi_A^T(y) &= \varphi_A^T(1-y)\end{aligned}$$

- 3-body correlators:

$$\begin{aligned}B(y_1,y_2) &= -B(1-y_2,1-y_1)\\ D(y_1,y_2) &= D(1-y_2,1-y_1)\end{aligned}$$

Collinear factorization
Equations of motion

Equations of motion

twist 2
kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- Dirac equation leads to

$$\langle i(\overrightarrow{\not{D}}(0)\psi(0))_\alpha\, \bar{\psi}_\beta(z)\rangle = 0 \qquad (i\overrightarrow{D}_\mu = i\overrightarrow{\partial}_\mu + A_\mu)$$

- Apply the Fierz decomposition to the above 2 and 3-body correlators

$$-\langle\psi(x)\,\bar{\psi}(z)\rangle = \frac{1}{4}\langle\bar{\psi}(z)\gamma_\mu\psi(x)\rangle\gamma_\mu + \frac{1}{4}\langle\bar{\psi}(z)\gamma_5\gamma_\mu\psi(x)\rangle\gamma_\mu\gamma_5 .$$

- $\Rightarrow$ 2 Equations of motion:

$$\begin{aligned}&\bar{y}_1\,\varphi_3(y_1) + \bar{y}_1\,\varphi_A(y_1) + \varphi_1^T(y_1) + \varphi_A^T(y_1)\\ &+ \int dy_2\left[\zeta_3^V\, B(y_1,\,y_2) + \zeta_3^A\, D(y_1,\,y_2)\right] = 0 \qquad \text{and} \quad (\bar{y}_1 \leftrightarrow y_1)\end{aligned}$$

- In WW approximation: genuine twist 3 = 0 i.e. $B = D = 0$

$$\begin{cases}\varphi_A^T(y) = \frac{1}{2}[(y-\bar{y})\,\varphi_A^{WW}(y) - \varphi_3^{WW}(y)]\\[2ex] \varphi_1^T(y) = \frac{1}{2}[(y-\bar{y})\,\varphi_3^{WW}(y) - \varphi_A^{WW}(y)]\end{cases}$$

Collinear factorization

$n-$independence

A minimal set of DAs

- The non-perturbative correlators cannot be obtained from perturbative QCD (!)
- one should reduce them to a minimal set before using any model
- this can be achieved by using an additional condition:
  independency of the full amplitude with respect to the light-cone direction n

  $\Rightarrow$ we prove that 3 independent Distribution Amplitudes are needed:

  $\phi_1(y)$ $\leftarrow$ 2 body twist 2 correlator

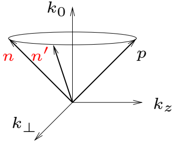
  $B(y_1,\, y_2)$ $\leftarrow$ 3 body genuine twist 3 vector correlator

  $D(y_1,\, y_2)$ $\leftarrow$ 3 body genuine twist 3 axial correlator

Collinear factorization

$n-$independence

$n-$independence in practice

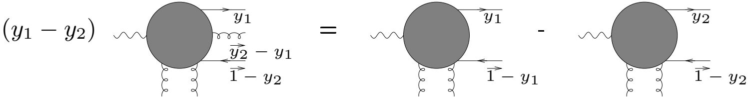


- ρ_T polarization: $e^{*T}_\mu = e^*_\mu - p_\mu \, e^* \cdot n$ keeping $n \cdot p = 1$
- for the full factorized amplitude:

$$\mathcal{A} = H \otimes S \qquad \frac{d\mathcal{A}}{dn^\mu} = 0\,, \qquad \text{where} \quad \frac{d}{dn^\mu} = \frac{\partial}{\partial n^\mu} + e^*_\mu \frac{\partial}{\partial (e^* \cdot n)}$$

- rewrite hard terms in one single form, of 2-body type: use Ward identities
  Example: hard 3-body ⟶ hard 2-body

$$\mathrm{tr}\left[H_{3\rho}(y_1, y_2)\, p^\rho \, \not{p}\right] \; B(y_1,\, y_2) \; = \; \frac{1}{y_1 - y_2}\left(\mathrm{tr}\left[H_2(y_1)\,\not{p}\right] - \mathrm{tr}\left[H_2(y_2)\,\not{p}\right]\right) B(y_1,\, y_2)\,,$$



- thus, symbolically,

$$\frac{dS}{dn^\mu} = 0$$

Collinear factorization

$n-$independence

Constraints from $n-$independence

twist 2
kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- vector correlators

$$\frac{d}{dy_1}\varphi_1^T(y_1) = -\varphi_1(y_1) + \varphi_3(y_1)$$
$$-\zeta_3^V \int_0^1 \frac{dy_2}{y_2 - y_1}\left(B(y_1, y_2) + B(y_2, y_1)\right)$$

- axial correlators

$$\frac{d}{dy_1}\varphi_A^T(y_1) = \varphi_A(y_1) - \zeta_3^A \int_0^1 \frac{dy_2}{y_2 - y_1}\left(D(y_1, y_2) + D(y_2, y_1)\right)$$

Collinear factorization

A set of independent non-perturbative correlators

Solution

twist 2
kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- the set of 4 equations (2 EOM + 2 n-independence relations) can be solved analytically
- 7 $\longrightarrow$ 3 independent DAs

Computation and results
Computation of the hard part

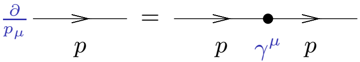
2-body diagrams

- without derivative

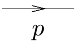
twist 2 $(\gamma_L^* \to \rho_L)$

twist 3 $(\gamma_T^* \to \rho_T)$

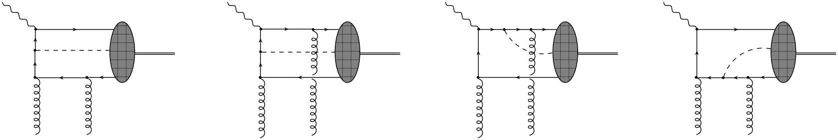
- practical trick for computing $\partial_\perp H$: use the Ward identity



where



$= \frac{1}{m - \not{p} - i\epsilon}$

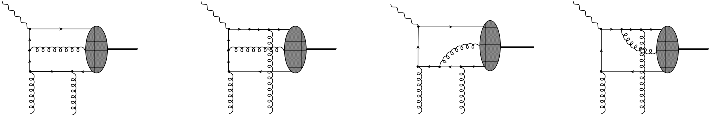


Computation and results

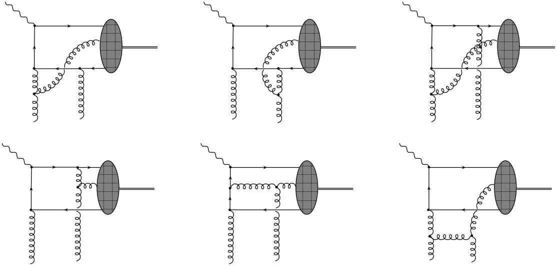
Computation of the hard part

3-body diagrams

- “abelian” type



- “non-abelian” type



Computation and results

Recall: $\gamma_L^* \to \rho_L$ impact factor

$\gamma_L^* \to \rho_L$ impact factor

$$\Phi^{\gamma_L^* \to \rho_L}(\underline{k}^2) = \frac{2\, e\, g^2\, f_\rho}{Q} \frac{\delta^{ab}}{2\, N_c} \int dy\, \varphi_1(y) \frac{\underline{k}^2}{y\, \bar{y}\, Q^2 + \underline{k}^2}$$

pure twist 2 scaling (from ρ-factorization point of view)

Computation and results

Results: $\gamma_T^* \to \rho_T$ impact factor

$\gamma_T^* \to \rho_T$ impact factor:

Spin Non-Flip/Flip separation appears

$$\Phi^{\gamma_T^* \to \rho_T}(\underline{k}^2) = \Phi_{n.f.}^{\gamma_T^* \to \rho_T}(\underline{k}^2)\, T_{n.f.} + \Phi_{f.}^{\gamma_T^* \to \rho_T}(\underline{k}^2)\, T_{f.}$$

where

$$T_{n.f.} = -(e_\gamma \cdot e^*) \qquad \text{and} \qquad T_{f.} = \frac{(e_\gamma \cdot k)(e^* k)}{\underline{k}^2} + \frac{(e_\gamma \cdot e^*)}{2}$$

non-flip transitions $\left\{ \begin{array}{l} + \to + \\ - \to - \end{array} \right.$ flip transitions $\left\{ \begin{array}{l} + \to - \\ - \to + \end{array} \right.$

Computation and results

Results: $\gamma^*_T \to \rho_T$ impact factor

$$\Phi^{\gamma^*_T \to \rho_T}_{n.f.}(\underline{k}^2)$$

pure twist 3 scaling (from ρ-factorization point of view)

$$= -\frac{e\,g^2 m_\rho f_\rho}{2\sqrt{2}\,Q^2}\frac{\delta^{ab}}{2\,N_c}\left\{-2\int dy_1 \frac{\left(\underline{k}^2+2\,Q^2\,y_1\,(1-y_1)\right)\underline{k}^2}{y_1\,(1-y_1)\,(\underline{k}^2+Q^2\,y_1\,(1-y_1))^2}\left[(2y_1-1)\,\varphi_1^T(y_1)+\varphi_A^T(y_1)\right]\right.$$

$$+2\int dy_1\,dy_2\left[\zeta_3^V\,B(y_1,y_2)-\zeta_3^A\,D(y_1,y_2)\right]\frac{y_1\,(1-y_1)\,\underline{k}^2}{\underline{k}^2+Q^2\,y_1\,(1-y_1)}\left[\frac{(2-N_c/C_F)Q^2}{\underline{k}^2\,(y_1-y_2+1)+Q^2\,y_1\,(1-y_2)}\right.$$

$$\left.-\frac{N_c}{C_F}\frac{Q^2}{y_2\underline{k}^2+Q^2\,y_1\,(y_2-y_1)}\right]-2\int dy_1\,dy_2\left[\zeta_3^V\,B(y_1,y_2)+\zeta_3^A\,D(y_1,y_2)\right]\left[\frac{2+N_c/C_F}{1-y_1}\right.$$

$$+\frac{y_1\,Q^2}{\underline{k}^2+Q^2y_1\,(1-y_1)}\left(\frac{(2-N_c/C_F)\,y_1\,\underline{k}^2}{\underline{k}^2\,(y_1-y_2+1)+Q^2y_1\,(1-y_2)}-2\right)$$

$$\left.\left.+\frac{N_c}{C_F}\frac{(y_1-y_2)\,(1-y_2)}{1-y_1}\frac{Q^2}{\underline{k}^2\,(1-y_1)+Q^2\,(y_2-y_1)\,(1-y_2)}\right]\right\}$$

and

$$\Phi^{\gamma^*_T\to\rho_T}_{f.}(\underline{k}^2) = -\frac{e\,g^2 m_\rho f_\rho}{2\sqrt{2}\,Q^2}\frac{\delta^{ab}}{2\,N_c}\left\{4\int dy_1\frac{\underline{k}^2\,Q^2}{(\underline{k}^2+Q^2\,y_1\,(1-y_1))^2}\left[\varphi_A^T(y_1)-(2y_1-1)\,\varphi_1^T(y_1)\right]\right.$$

$$-4\int dy_1\,dy_2\frac{y_1\,\underline{k}^2}{\underline{k}^2+Q^2\,y_1\,(1-y_1)}\left[\zeta_3^A\,D(y_1,y_2)\,(-y_1+y_2-1)+\zeta_3^V\,B(y_1,y_2)\,(y_1+y_2-1)\right]$$

$$\left.\times\left[\frac{(2-N_c/C_F)Q^2}{\underline{k}^2\,(y_1-y_2+1)+Q^2\,y_1\,(1-y_2)}-\frac{N_c}{C_F}\frac{Q^2}{y_2\,\underline{k}^2+Q^2y_1\,(y_2-y_1)}\right]\right\}$$

Computation and results

Results: $\gamma_T^* \to \rho_T$ impact factor

WW limit

- WW limit: keep only twist 2 + kinematical twist 3 terms (i.e $B = D = 0$)
- The only remaining contributions come from the two-body correlators
- non-flip transition

$$\Phi_{n.f.}^{\gamma_T^* \to \rho_T}(\underline{k}^2) = \frac{-e\, m_\rho f_\rho}{2\sqrt{2}\, Q^2} \frac{\delta^{ab}}{2\, N_c} \int_0^1 dy \left\{ \frac{(y-\bar{y})\varphi_1^{T\,WW}(y) + 2\, y\, \bar{y}\, \varphi_3^{WW}(y) + \varphi_A^{T\,WW}(y)}{y\, \bar{y}} - \frac{2\, \underline{k}^2 \left(\underline{k}^2 + 2\, Q^2\, y\, \bar{y}\right) \left((y-\bar{y})\, \varphi_1^{T\,WW}(y) + \varphi_A^{T\,WW}(y)\right)}{y\, \bar{y}\, \left(\underline{k}^2 + Q^2\, y\, (1-y)\right)^2} \right\}$$

which simplifies, using equation of motion:

$$\int dy \left[(y-\bar{y})\varphi_1^{T\,WW}(y) + 2\, y\, \bar{y}\, \varphi_3^{WW}(y) + \varphi_A^{T\,WW}(y)\right] = 0$$

$$\Phi_{n.f.}^{\gamma_T^* \to \rho_T}(\underline{k}^2) = \frac{e\, m_\rho f_\rho}{\sqrt{2}\, Q^2} \frac{\delta^{ab}}{2\, N_c} \int_0^1 dy \frac{2\, \underline{k}^2 \left(\underline{k}^2 + 2\, Q^2\, y\, \bar{y}\right)}{y\, \bar{y}\, \left(\underline{k}^2 + Q^2\, y\, \bar{y}\right)^2} \left[(2\, y - 1)\, \varphi_1^{T\,WW}(y) + \varphi_A^{T\,WW}(y)\right].$$

- flip transition:

$$\Phi_{n.f.}^{\gamma_T^* \to \rho_T}(\underline{k}^2) = -\frac{e\, m_\rho f_\rho}{\sqrt{2}\, Q^2} \frac{\delta^{ab}}{2\, N_c} \int_0^1 \frac{2\, \underline{k}^2\, Q^2}{\left(\underline{k}^2 + Q^2\, y\, \bar{y}\right)^2} \left[(1-2\, y)\varphi_1^{T\,WW}(y) + \varphi_A^{T\,WW}(y)\right].$$

Computation and results

Discussion: gauge invariance

- The obtained results are gauge invariant:

$$\Phi^{\gamma_T^* \to \rho_T} \to 0 \quad \text{when} \quad \underline{k} \to 0$$

  - this is straightforward in the WW limit
  - at the full twist 3 order:
    - the C_F part of the abelian 3-body contribution cancels the 2-body contribution after using the equation of motion
    - the N_c part of the abelian 3-body contribution cancels the 3-body non-abelian contribution
    - thus $\gamma_T^* \to \rho_T$ impact factor is gauge-invariant only provided the 3-body contributions have been taken into account

Computation and results

Discussion: consistence with factorization

- Our results are free of end-point singularities, in both WW approximation and full twist-3 order calculation:
    - the flip contribution obviously does not have any end-point singularity because of the $\underline{k}^2$ which regulates them
    - the potential end-point singularity for the non-flip contribution is spurious since $\varphi_A^T(y)$, $\varphi_1^T(y)$ vanishes at $y = 0, 1$ as well as $B(y_1, y_2)$ and $D(y_1, y_2)$.

Conclusions

1

- We have performed a full up to twist 3 computation of the $\gamma^* \to \rho$ impact factor, in the $t = t_{min}$ limit.
- Our result respects gauge invariance. This is achieved only after including 2 and 3 body correlators.
- It is free of end-point singularities (this should be contrasted with standard collinear treatment, at moderate s, where k_T-factorization is NOT applicable: see Mankiewicz-Piller).
- Phenomenological applications will be done in the near future.
- In this talk we relied on the Light-Cone Collinear approach (Ellis + Furmanski + Petronzio; Efremov + Teryaev; Anikin + Teryaev), which is non-covariant, but very efficient for practical computations.
- This Light-Cone Collinear approach is systematic, and can be extended to any process, including higher twist effects (but does not preclude potential end-point singularities)

Conclusions
2

- Comparison with a fully covariant approach by Ball+Braun et al:
  The dictionnary between the two approaches within a full twist 3 treatment is now established:

$$\begin{aligned}
B(y_1,\,y_2) &= -\frac{V(y_1,\,1-y_2,\,y_2-y_1)}{y_2-y_1}\,,\\
D(y_1,\,y_2) &= \frac{A(y_1,\,1-y_2,\,y_2-y_1)}{y_2-y_1}\\
\varphi_1(y) &= f_\rho\, m_\rho\, \phi_{||}(y)\\
\varphi_3(y) &= f_\rho\, m_\rho\, g^{(v)}(y)\,,\\
\varphi_A(y) &= -\frac{1}{4} f_\rho\, m_\rho \frac{\partial g^{(a)}(y)}{\partial y}
\end{aligned}$$

- We also performed calculations of the same impact factor within the covariant approach by Ball+Braun et al: calculations proceed in quite different way : eg. no $\varphi^T_{1,A}$−DAs but Wilson line effects are important !!
  We got a full agreement with our approach