Impact factor of $\gamma^* \to \rho_T$ with twist three accuracy

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Outline

Introduction

Exclusive processes at high energy in QCD

Since a decade, there have been much developpements in hard exclusive processes.

- form factors, Distribution Amplitudes $\rightarrow$ Generalized Distribution Amplitudes
- DVCS$\rightarrow$ Generalized Parton Distributions, Transition Distribution Amplitudes

These tests are possible in fixed target experiments

- $e^{\pm}p$: HERA (HERMES), JLab, COMPASS...

as well as in colliders, mainly for medium s

- $e^{\pm}p$ colliders: HERA (H1, ZEUS)
- $e^{+}e^{-}$ colliders: LEP, Belle, BaBar, BEPC

At the same time, at large s, the interest for phenomenological tests of hard Pomeron and related resummed approaches has become pretty wide:

- inclusive tests (total cross-section) and semi-inclusive tests (diffraction, forward jets, ...)
- exclusive tests (meson production)

These tests concern all type of collider experiments:

- $e^{\pm}p$: HERA: (H1, ZEUS)
- $p\bar{p}$ and pp: TEVATRON (CDF, D0); LHC (CMS, ATLAS, ALICE)
- $e^{+}e^{-}$: (LEP, ILC)

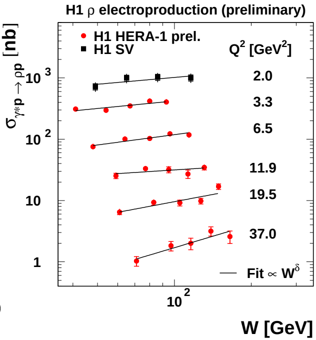
Introduction
Exclusive ρ-production

Our studies attempt to describe exclusive processes involving the production of ρ-mesons in diffraction-type experiment. We choose $t = t_{min}$ for simplicity.

- $\gamma^*(q) + \gamma^*(q') \to \rho_T(p_1) + \rho(p_2)$ process in $e^+e^- \to e^+e^- \rho_T(p_1) + \rho(p_2)$ with double tagged lepton at ILC
- $\gamma^*(q) + P \to \rho_T(p_1) + P$ at HERA

This process was studied by H1 and ZEUS

- the total cross-section strongly decreases with Q^2
- dramatic increase with $W^2 = s_{\gamma^* P}$ (transition from soft to hard regime governed by Q^2)



(from X. Janssen (H1), DIS 2008)

Introduction
Exclusive ρ-production

Polarization effects in $\gamma^* P \to \rho P$ at HERA

- one can experimentally measure all spin density matrix elements

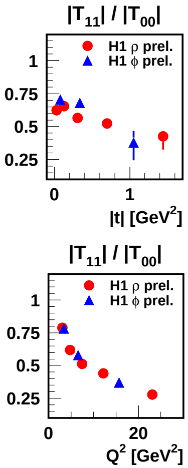
- at $t = t_{min}$ one can experimentally distinguish

$$\begin{cases} \gamma_L^* \to \rho_L : & \text{dominates} \quad (\text{twist 2 dominance}) \\ \gamma_T^* \to \rho_T : & \text{sizable} \quad (\text{twist 3}) \end{cases}$$

- S-channel helicity conservation:

$$\begin{cases} \gamma_L^* \to \rho_L & (\equiv T_{00}) \\ \gamma_{T(+)}^* \to \rho_{T(+)}, \quad \gamma_{T(-)}^* \to \rho_{T(-)} & (\equiv T_{11}) \end{cases}$$

dominate with respect to all other transitions



(from X. Janssen (H1), DIS 2008)

Introduction

Exclusive ρ-production

The processes with vector particle such as rho-meson probe deeper into the fine features of QCD.
It deserves theoretical developpement to describe HERA data in its special kinematical range:

- large $s_{\gamma^* P}$ $\Rightarrow$ small-x effects expected, within k_t-factorization
- large Q^2 $\Rightarrow$ hard scale $\Rightarrow$ perturbative approach and collinear factorization $\Rightarrow$ the ρ can be described through its chiral even Distribution Amplitudes

$$\begin{cases} \rho_L & \text{twist 2} \\ \rho_T & \text{twist 3} \end{cases}$$

The main ingredient is the $\gamma^* \to \rho$ impact factor

- For ρ_T, special care is needed: a pure 2-body description would violate gauge invariance.
- We show that:
  - Including in a consistent way all twist 3 contributions, i.e. 2-body and 3-body correlators, gives a gauge invariant impact factor
  - Our treatment is free of end-point singularities and does not violates the QCD factorization

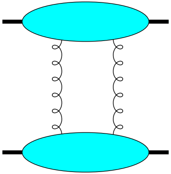
Impact factor for exclusive processes

Theoretical motivations

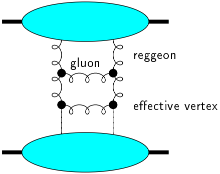
QCD in perturbative Regge limit

- In this limit, the dynamics is dominated by gluons (dominance of spin 1 exchange in t channel)
- BFKL (and extensions: NLL, saturations effects, ...) is expected to dominate with respect to Born order at large relative rapidity.

Born order:



BFKL ladder:

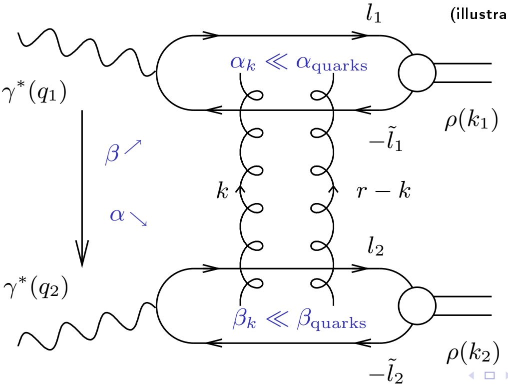


Impact factor for exclusive processes
k_T factorization

$\gamma^* \gamma^* \to \rho \rho$ as an example

- Use Sudakov decomposition $k = \alpha p_1 + \beta p_2 + k_\perp$ $(p_1^2 = p_2^2 = 0,\ 2p_1 \cdot p_2 = s)$
- write $d^4k = \frac{s}{2}\, d\alpha\, d\beta\, d^2k_\perp$
- $t-$channel gluons with non-sense polarizations ($\epsilon_{NS}^{up} = \frac{2}{s}\, p_2,\ \epsilon_{NS}^{down} = \frac{2}{s}\, p_1$) dominates at large s

(illustration for 2-body case)



$\Rightarrow$ set $\alpha_k = 0$ and $\int d\beta$

$\Rightarrow$ set $\beta_k = 0$ and $\int d\alpha$

Impact factor for exclusive processes

k_T factorization

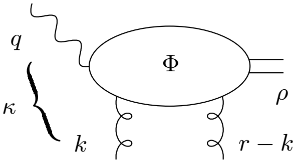
impact representation $\quad \underline{k}$ = Eucl. $\leftrightarrow k_\perp$ = Mink.

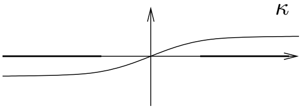
$$\mathcal{M} = is \int \frac{d^2\,\underline{k}}{(2\pi)^2 \underline{k}^2\,(\underline{r}-\underline{k})^2} \Phi^{\gamma^*(q_1)\to\rho(p_1^\rho)}(\underline{k}, \underline{r}-\underline{k})\ \Phi^{\gamma^*(q_2)\to\rho(p_2^\rho)}(-\underline{k}, -\underline{r}+\underline{k})$$

The $\gamma^*_{L,T}(q) g(k_1) \to \rho_{L,T}\, g(k_2)$ impact factor is normalized as

$$\Phi^{\gamma^*\to\rho}(\underline{k}^2) = e^{\gamma^*\mu}\,\frac{1}{s}\int \frac{d\kappa}{2\pi i}\,\text{Disc}_\kappa\, T_\mu^{\gamma^*\,g\to\rho\,g}(\underline{k}^2),$$

with $\kappa = (q+k)^2 = \beta\, s - Q^2 - \underline{k}^2$



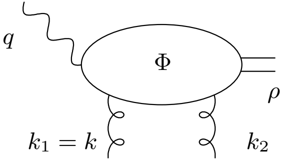


Impact factor for exclusive processes

Gauge invariance within subleading twists

Gauge invariance

- QCD gauge invariance (probes are colorless)
  $\Rightarrow$ impact factor should vanish when $\underline{k} \to 0$ or $\underline{r} - \underline{k} \to 0$
- In the following we will restrict ourselve to the case $t = t_{min}$, i.e. to $\underline{r} = 0$



$$k_1 = \frac{\kappa + Q^2 + \underline{k}^2}{s} p_2 + k_\perp$$

$$k_2 = \frac{\kappa + \underline{k}^2}{s} p_2 + k_\perp ,$$

$$k_1^2 = k_2^2 = -\underline{k}^2$$

This kinematics takes into account skewedness effects along p_2

$\Rightarrow$ restriction to the transitions $\begin{cases} 0 & \to & 0 & \text{(twist 2)} \\ (+ \text{ or } -) & \to & (+ \text{ or } -) & \text{(twist 3)} \end{cases}$

- At twist 3 level (for $\gamma_T^* \to \rho_T$ transition), gauge invariance is a non trivial statement which requires 2 *and* 3 body correlators

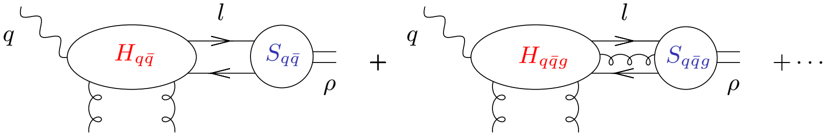
Collinear factorization

Light-Cone Collinear approach

- The impact factor can be written as

$$\Phi = \int d^4 l \cdots \mathrm{tr}[H(l \cdots) \quad S(l \cdots)]$$

hard part   soft part



- At the 2-body level:

$$S_{q\bar{q}}(l) = \int d^4 z \, e^{-il\cdot z} \langle \rho(p) | \psi(0) \, \bar{\psi}(z) | 0 \rangle,$$

- H and S are related by $\int d^4 l$ and by the summation over spinor indices

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

1 - Momentum factorization (1)

- Use Sudakov decomposition in the form ($p = p_1$, $n = 2\, p_2/s$)

$$l_\mu = x\, p_\mu + l^\perp_\mu + (l \cdot p)\, n_\mu, \qquad x = l \cdot n$$

scaling: 1 $\quad 1/Q$ $\quad 1/Q^2$

- decompose $H(k)$ around the p direction:

$$H(l) = H(xp) + \left.\frac{\partial H(l)}{\partial l_\alpha}\right|_{l=xp} (l - x\, p)_\alpha + \ldots \quad \text{with} \quad (l - x\, p)_\alpha \approx l^\perp_\alpha$$

twist 2 — kinematical twist 3

- In Fourier space, the kinematical twist 3 term $k^\perp_\alpha$ turns into a derivative of the soft term

$\Rightarrow$ one will deal with $\int d^4z\; e^{-il\cdot z} \langle \rho(p)|\psi(0)\, i\, \overleftrightarrow{\partial}_{\alpha^\perp} \bar{\psi}(z)|0\rangle$

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

1 - Momentum factorization (2)

- write

$$d^4l \longrightarrow d^4l \; \delta(x - l \cdot n) \; dx$$

- $\int d^4l \, \delta(x - l \cdot n)$ is then absorbed in the soft term:

$$\begin{aligned}(\tilde{S}_{q\bar{q}}, \partial_\perp \tilde{S}_{q\bar{q}}) \quad &\equiv \quad \int d^4l \, \delta(x - l \cdot n) \int d^4z \, e^{-il\cdot z} \langle \rho(p)|\psi(0) \, (1, \, i \; \overleftrightarrow{\partial_\perp} \,)\bar{\psi}(z)|0\rangle \\ &= \quad \int \frac{d\lambda}{2\pi} \, e^{-i\lambda x} \int d^4z \, \delta^{(4)}(z - \lambda n) \, \langle \rho(p)|\psi(0) \, (1, \, i \; \overleftrightarrow{\partial_\perp} \,)\bar{\psi}(z)|0\rangle \\ &= \quad \int \frac{d\lambda}{2\pi} \, e^{-i\lambda x} \langle \rho(p)|\psi(0) \, (1, \, i \; \overleftrightarrow{\partial_\perp} \,)\bar{\psi}(\lambda n)|0\rangle\end{aligned}$$

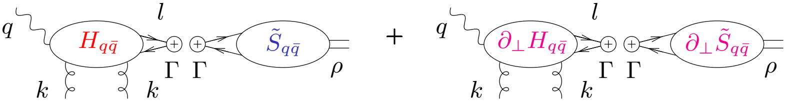
- $\int dx$ performs the longitudinal momentum factorization

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

2 - Spinorial (and color) factorization

- Use Fierz decomposition of the Dirac (and color) matrices $\psi(0)\,\bar\psi(z)$ and $\psi(0)\,i\,\overleftrightarrow{\partial_\perp}\,\bar\psi(z)$:



- Φ has now the simple factorized form:

$$\Phi = \int dx\,\left\{\mathrm{tr}\,[H_{q\bar q}(x\,p)\,\Gamma]\;S^{\Gamma}_{q\bar q}(x) + \mathrm{tr}\,[\partial_\perp H_{q\bar q}(x\,p)\,\Gamma]\;\partial_\perp S^{\Gamma}_{q\bar q}(x)\right\}$$

$\Gamma = \gamma^\mu$ and $\gamma^\mu\,\gamma^5$ matrices

$$S^{\Gamma}_{q\bar q}(x) = \int \frac{d\lambda}{2\pi}\,e^{-i\lambda x}\langle\rho(p)|\bar\psi(\lambda n)\,\Gamma\,\psi(0)|0\rangle$$

$$\partial_\perp S^{\Gamma}_{q\bar q}(x) = \int \frac{d\lambda}{2\pi}\,e^{-i\lambda x}\langle\rho(p)|\bar\psi(\lambda n)\,\Gamma\,i\,\overleftrightarrow{\partial_\perp}\,\psi(0)|0\rangle$$

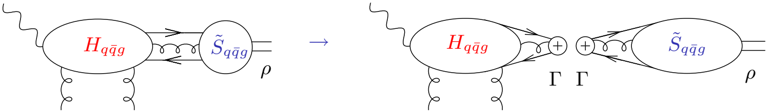
- choose axial gauge condition for gluons, *i.e.* $n\cdot A = 0 \Rightarrow$ no Wilson line

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (3-body case)

Factorization of 3-body contributions

- 3-body contributions start at genuine twist 3
  $\Rightarrow$ no need for Taylor expansion
- Momentum factorization goes in the same way as for 2-body case
- Spinorial (and color) factorization is similar:



Collinear factorization

Parametrization of vacuum–to–rho-meson matrix elements (DAs): 2-body correlators

2-body non-local correlators

ρ_L twist 2

ρ_T kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- vector correlator

$$\langle\rho(p)|\bar{\psi}(z)\gamma_\mu\psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, \left[\varphi_1(x)\,(e\cdot n)p_\mu + \varphi_3(x)\, e^T_\mu\right]$$

- axial correlator

$$\langle\rho(p)|\bar{\psi}(z)\gamma_5\gamma_\mu\psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, i\,\varphi_A(x)\,\varepsilon_{\mu\lambda\beta\delta}\, e^T_\lambda\, p_\beta\, n_\delta$$

- vector correlator with transverse derivative

$$\langle\rho(p)|\bar{\psi}(z)\gamma_\mu\, i\, \overleftrightarrow{\partial^T_\alpha}\, \psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, \varphi^T_1(x)\, p_\mu e^T_\alpha$$

- axial correlator with transverse derivative

$$\langle\rho(p)|\bar{\psi}(z)\gamma_5\gamma_\mu\, i\, \overleftrightarrow{\partial^T_\alpha}\, \psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, i\,\varphi^T_A(x)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e^T_\lambda\, p_\beta\, n_\delta,$$

where x ($\bar{x} = 1 - x$) = momentum fraction along $p \equiv p_1$ of the quark (antiquark) and $\stackrel{\mathcal{F}}{=} \int_0^1 dx \exp\,[ix\, p\cdot z]$, with $z = \lambda n$

Collinear factorization

Parametrization of vacuum–to–rho-meson matrix elements: 3-body correlators

3-body non-local correlators **genuine twist 3**

- vector correlator

$$\langle \rho(p)|\bar{\psi}(z_1)\gamma_\mu g A^T_\alpha(z_2)\psi(0)|0\rangle \stackrel{\mathcal{F}_2}{=} m_\rho\, f_\rho\, B(x_1,x_2)\, p_\mu\, e^T_\alpha,$$

- axial correlator

$$\langle \rho(p)|\bar{\psi}(z_1)\gamma_5\gamma_\mu g A^T_\alpha(z_2)\psi(0)|0\rangle \stackrel{\mathcal{F}_2}{=} m_\rho\, f_\rho\, i\, D(x_1,x_2)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e^T_\lambda\, p_\beta\, n_\delta,$$

where x_1, $\bar{x}_2$, $x_2 - x_1$ = quark, antiquark, gluon momentum fraction

and $\stackrel{\mathcal{F}_2}{=} \int\limits_0^1 dx_1 \int\limits_0^1 dx_2\ \exp\left[i\, x_1\, p\cdot z_1 + i(x_2 - x_1)\, p\cdot z_2\right]$, with $z_{1,2} = \lambda n$

Collinear factorization

Symmetry properties

From C-conjugation on the previous correlators, one gets:

- 2-body correlators:

$$
\begin{aligned}
\varphi_1(y) &= \varphi_1(1-y) \\
\varphi_3(y) &= \varphi_3(1-y) \\
\varphi_A(y) &= -\varphi_A(1-y) \\
\varphi_1^T(y) &= -\varphi_1(1-y) \\
\varphi_A^T(y) &= \varphi_A^T(1-y)
\end{aligned}
$$

- 3-body correlators:

$$
\begin{aligned}
B(x_1, x_2) &= -B(1-x_2, 1-x_1) \\
D(x_1, x_2) &= D(1-x_2, 1-x_1)
\end{aligned}
$$

Collinear factorization
Equations of motion

Equations of motion

twist 2
kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- Dirac equation leads to

$$\langle i(\overrightarrow{\not{D}}(0)\psi(0))_\alpha \bar{\psi}_\beta(z)\rangle = 0 \qquad (i\overrightarrow{D}_\mu = i\overrightarrow{\partial}_\mu + A_\mu)$$

- Apply the Fierz decomposition to 2 and 3-body correlators above

$$-\langle\psi(x)\,\bar{\psi}(z)\rangle = \frac{1}{4}\langle\bar{\psi}(z)\gamma_\mu\psi(x)\rangle\gamma_\mu + \frac{1}{4}\langle\bar{\psi}(z)\gamma_5\gamma_\mu\psi(x)\rangle\gamma_\mu\gamma_5.$$

- $\Rightarrow$ Equation of motion:

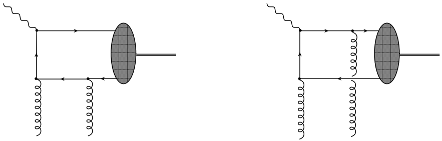
$$\int dx_1[2x_1\,\bar{x_1}\,\varphi_3(x) + (x_1-\bar{x_1})\,\varphi_1^T(x_1) + \varphi_A^T(x_1)]$$
$$+2\int dx_1\,dx_2\;x_1[B(x_1,x_2)+D(x_1,x_2)] = 0$$

- In WW approximation: genuine twist 3 = 0

$$\begin{cases} \varphi_A^T(x) = \frac{1}{2}[(x-\bar{x})\,\varphi_A^{WW}(x) - \varphi_3^{WW}(x)] \\ \varphi_1^T(x) = \frac{1}{2}[(x-\bar{x})\,\varphi_3^{WW}(x) - \varphi_A^{WW}(x)] \end{cases}$$

Computation and results
2-body Diagrams

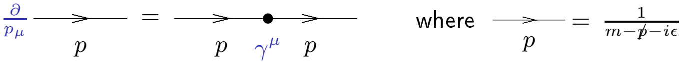
- without derivative

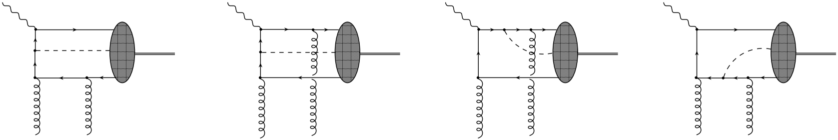


twist 2 $(\gamma^*_L \to \rho_L)$

twist 3 $(\gamma^*_T \to \rho_T)$

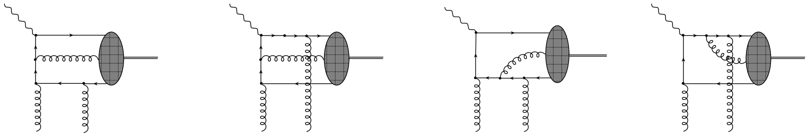
- practical trick for computing $\partial_\perp H$: use the Ward identity



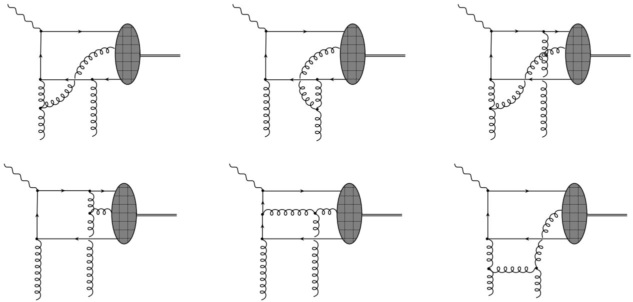


Computation and results
3-body Diagrams

- "abelian" type



- "non-abelian" type



Computation and results

Recall: $\gamma_L^* \to \rho_L$ impact factor

$\gamma_L^* \to \rho_L$ impact factor

$$\Phi^{\gamma_L^* \to \rho_L}(\underline{k}^2) = -i \frac{4 C_F \, e_q \, f_\rho}{Q} \int dx \, \varphi_1(x) \frac{\underline{k}^2}{x \, \bar{x} \, Q^2 + \underline{k}^2}$$

pure twist 2 scaling

Computation and results

Results: $\gamma_T^* \to \rho_T$ impact factor

$\gamma_T^* \to \rho_T$ impact factor:

Spin Non-Flip/Flip separation appears

$$\Phi^{\gamma_T^* \to \rho_T}(\mathbf{k}_T^2) = \Phi_{n.f.}^{\gamma_T^* \to \rho_T}(\mathbf{k}_T^2)\, T_{n.f.} + \Phi_{f.}^{\gamma_T^* \to \rho_T}(\mathbf{k}_T^2)\, T_{f.}$$

where

$$T_{n.f.} = -(e_\gamma e^*) \quad \text{and} \quad T_{f.} = \frac{(e_\gamma k)(e^* k)}{\vec{k}^2} + \frac{(e_\gamma e^*)}{2}$$

non-flip transitions $\left\{ \begin{array}{l} + \to + \\ - \to - \end{array} \right.$ flip transitions $\left\{ \begin{array}{l} + \to - \\ - \to + \end{array} \right.$

Computation and results

Results: $\gamma_T^* \to \rho_T$ impact factor

pure twist 3 scaling

$$\Phi_{n.f.}^{\gamma_T^*\to\rho_T}(\underline{k}^2)$$

$$= \frac{i\,m_\rho f_\rho}{Q^2}\Bigg\{-2\,C_F\int dx_1\,\frac{\left(\underline{k}^2+2\,Q^2\,x_1\,(1-x_1)\right)\underline{k}^2}{x_1\,(1-x_1)\,(\underline{k}^2+Q^2\,x_1\,(1-x_1))^2}\left[(2x_1-1)\,\varphi_1^T(x_1)+\varphi_A^T(x_1)\right]$$

$$+2\int dx_1\,dx_2\,[B\,(x_1,x_2)-D\,(x_1,x_2)]\,\frac{x_1\,(1-x_1)\,\underline{k}^2}{\underline{k}^2+Q^2\,x_1\,(1-x_1)}\left[\frac{(2\,C_F-N_c)Q^2}{\underline{k}^2\,(x_1-x_2+1)+Q^2\,x_1\,(1-x_2)}\right.$$

$$\left.-\frac{N_c\,Q^2}{x_2\underline{k}^2+Q^2\,x_1\,(x_2-x_1)}\right]-2\int dx_1\,dx_2\,[B\,(x_1,x_2)+D\,(x_1,x_2)]\left[\frac{2\,C_F+N_c}{1-x_1}\right.$$

$$+\frac{x_1\,Q^2}{\underline{k}^2+Q^2x_1\,(1-x_1)}\left(\frac{(2\,C_F-N_c)\;x_1\,\underline{k}^2}{\underline{k}^2\,(x_1-x_2+1)+Q^2x_1\,(1-x_2)}-2C_F\right)$$

$$\left.\left.+N_c\,\frac{(x_1-x_2)\,(1-x_2)}{1-x_1}\,\frac{Q^2}{\underline{k}^2\,(1-x_1)+Q^2\,(x_2-x_1)\,(1-x_2)}\right]\right\}$$

and

$$\Phi_{f.}^{\gamma_T^*\to\rho_T}(\underline{k}^2)=\frac{i\,m_\rho f_\rho}{Q^2}\Bigg\{4\,C_F\int dx_1\,\frac{\underline{k}^2\,Q^2}{(\underline{k}^2+Q^2\,x_1\,(1-x_1))^2}\left[\varphi_A^T(x_1)-(2x_1-1)\,\varphi_1^T(x_1)\right]$$

$$-4\int dx_1\,dx_2\,\frac{x_1\,\underline{k}^2}{\underline{k}^2+Q^2\,x_1\,(1-x_1)}\,[D\,(x_1,x_2)\,(-x_1+x_2-1)+B\,(x_1,x_2)\,(x_1+x_2-1)]$$

$$\times\left[\frac{(2\,C_F-N_c)Q^2}{\underline{k}^2\,(x_1-x_2+1)+Q^2\,x_1\,(1-x_2)}-\frac{N_c\,Q^2}{x_2\,\underline{k}^2+Q^2x_1\,(x_2-x_1)}\right]\Bigg\}$$

Computation and results
Discussion

- The obtained results are gauge invariant:

$$\Phi^{\gamma_T^* \to \rho_T} \to 0 \quad \text{when} \quad \underline{k} \to 0$$

  - the C_F part of the abelian 3-body contribution cancels the 2-body contribution after using the equation of motion
  - the N_c part of the abelian 3-body contribution cancels the 3-body non-abelian contribution
  - thus $\gamma_T^* \to \rho_T$ impact factor is gauge-invariant only provided the 3-body contributions have been taken into account

- Our results are free of end-point singularities, in both WW approximation and full twist-3 order calculation:
  - the flip contribution obviously does not have any end-point singularity because of the $\underline{k}^2$ which regulates them
  - the potential end-point singularity for the non-flip contribution is spurious since $\varphi_A^T(x_1)$, $\varphi_1^T(x_1)$ vanishes at $x_1 = 0, 1$ as well as $B(x_1, x_2)$ and $D(x_1, x_2)$.

Conclusions

- We have performed a full up to twist 3 computation of the $\gamma^* \to \rho$ impact factor, in the $t = t_{min}$ limit
- Our result respects gauge invariance
- It is free of end-point singularities (this should be contrasted with standard collinear treatment, at moderate s, where no k_T-factorization is applicable: see Mankiewicz-Piller).
- In this talk we relied on the Light-Cone Collinear approach (Anikin, Teryaev), which is non-covariant, but very efficient for practical computations. We also performed calculations of the same impact factor using the fully covariant approach in the coordinate space (Braun, Ball). We got identical results and developped the corresponding dictionnary between the two approaches.
- Phenomenological applications will be done in the near future