

A short review of the theory of hard exclusive processes

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Meeting in honour of J. Cugnon and H.J. Pirner,
"30 years of strong interactions", April 7th 2011

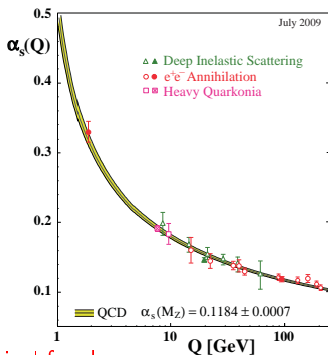
QCD

Quantum chromodynamics (QCD) is THE theory of strong interaction, one of the four elementary interactions of the universe

- it is a relativistic quantum field theory of **Yang-Mills** type (with an $SU(3)$ gauge group)
- the quarks and gluons elementary fields are **confined** in hadrons:
 - mesons ($\pi, \eta, f_0, \rho, \omega \dots$)

$$|q\bar{q}\rangle + |q\bar{q}g\rangle + |qqq\bar{q}\rangle + \dots$$
 - baryons ($p, n, N, \Delta \dots$)

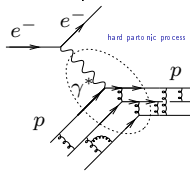
$$|qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle + \dots$$
- in contrast with electrodynamics, strong interaction increases with distance, or equivalently decreases when energy increases: this phenomena is called **asymptotical freedom**



coupling $\alpha_s(Q) \ll 1$ for $Q \gg \Lambda_{\text{QCD}} \simeq 200 \text{ MeV}$

What to do with QCD?

- How however describe and understand the internal structure of hadrons, **starting from their elementary constituents?**
- In the non-perturbative domain, the two available tools are:
 - Chiral perturbation theory: systematic expansion based on the fact that u and d quarks have a very small mass, the π mass being an expansion parameter outside the chiral limit
 - Discretization of QCD on a 4-d lattice: numerical simulations
- Can one extract information reducing the process to interactions involving a small number of *partons* (quarks, gluons), despite confinement?
 - This is possible if the considered process is driven by short distance phenomena ($d \ll 1$ fm)
 $\implies \alpha_s \ll 1$: **Perturbative methods**
 - One should hit strongly enough a hadron
 Example: electromagnetic probe and form factor



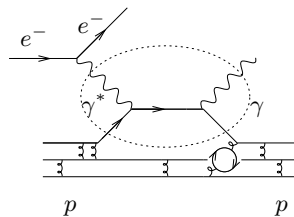
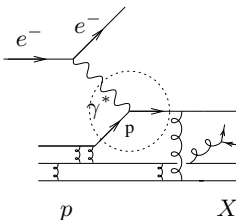
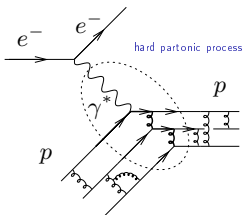
$$\tau_{\text{electromagnetic interaction}} \sim \tau_{\text{parton life time after interaction}} \ll \tau_{\text{characteristic time of strong interaction}}$$

Introduction

Hard processes in QCD

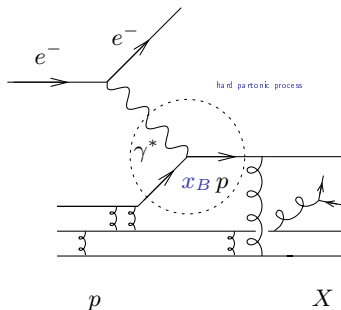
Hard processes in QCD

- This is justified if the process is governed by a **hard scale**:
 - virtuality of the electromagnetic probe
 - in **elastic** scattering $e^\pm p \rightarrow e^\pm p$
 - in **Deep Inelastic Scattering (DIS)** $e^\pm p \rightarrow e^\pm X$
 - in **Deep Virtual Compton Scattering (DVCS)** $e^\pm p \rightarrow e^\pm p \gamma$
 - Total center of mass energy in $e^+e^- \rightarrow X$ **annihilation**
 - t -channel momentum exchange in meson photoproduction $\gamma p \rightarrow Mp$
- A precise treatment relies on **factorization theorems**
- The scattering amplitude is described by the **convolution** of the partonic amplitude with the non-perturbative hadronic content



Accessing to the perturbative proton content

example: DIS



$$s_{\gamma^* p} = (q_{\gamma^*} + p_p)^2 = 4 E_{\text{c.m.}}^2$$

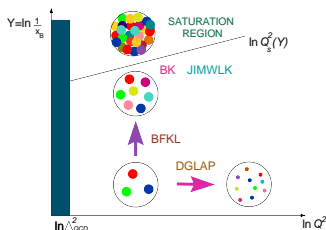
$$Q^2 \equiv -q_{\gamma^*}^2 > 0$$

$$x_B = \frac{Q^2}{2 p_p \cdot q_{\gamma^*}} \simeq \frac{Q^2}{s_{\gamma^* p}}$$

- x_B = proton momentum fraction carried by the scattered quark
- $1/Q$ = transverse resolution of the photonic probe $\ll 1/\Lambda_{\text{QCD}}$

Introduction
DIS

The various regimes governing the perturbative content of the proton



- “usual” regime: x_B moderate ($x_B \gtrsim .01$):
Evolution in Q governed by the QCD renormalization group
(Dokshitzer, Gribov, Lipatov, Altarelli, Parisi equation)

$$\sum_n (\alpha_s \ln Q^2)^n \quad + \quad \alpha_s \sum_n (\alpha_s \ln Q^2)^n + \dots$$

LLQ NLLQ

- perturbative Regge limit: $s_{\gamma^*p} \rightarrow \infty$ i.e. $x_B \sim Q^2/s_{\gamma^*p} \rightarrow 0$
in the perturbative regime (hard scale Q^2)
(Balitski Fadin Kuraev Lipatov equation)

Introduction

Extensions from DIS

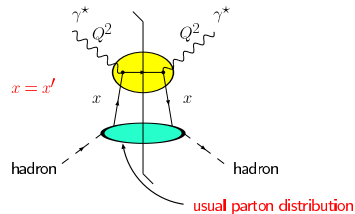
- DIS: inclusive process \rightarrow forward amplitude ($t = 0$) (optical theorem)

(DIS: Deep Inelastic Scattering)

ex: $e^\pm p \rightarrow e^\pm X$ at HERA

Structure Function

$$= \text{Coefficient Function (hard)} \otimes \text{Parton Distribution Function (soft)}$$

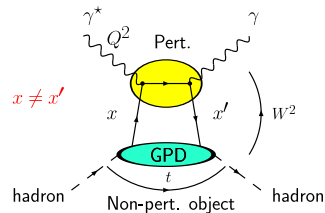


- DVCS: exclusive process \rightarrow non forward amplitude ($-t \ll s = W^2$)

(DVCS: Deep Virtual Compton Scattering)

Amplitude

$$= \text{Coefficient Function (hard)} \otimes \text{Generalized Parton Distribution (soft)}$$



Müller et al. '91 - '94; Radyushkin '96; Ji '97

Introduction

Extensions from GPD

- **Meson production:** γ replaced by ρ, π, \dots

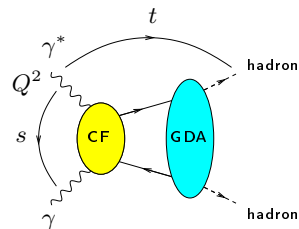
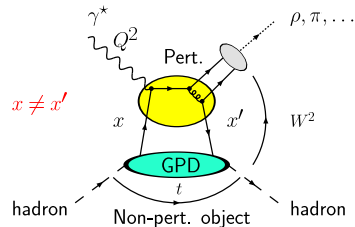
$$\text{Amplitude} = \text{GPD (soft)} \otimes \text{CF (hard)} \otimes \text{Distribution Amplitude (soft)}$$

Collins, Frankfurt, Strikman '97; Radyushkin '97

- **Crossed process:** $s \ll -t$

$$\text{Amplitude} = \text{Coefficient Function (hard)} \otimes \text{Generalized Distribution Amplitude (soft)}$$

Diehl, Gousset, Pire, Teryaev '98



Introduction

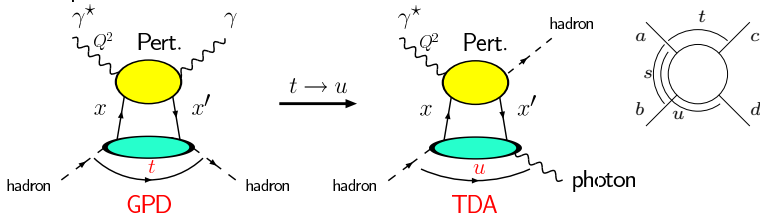
Extensions from GPD

- Starting from usual DVCS, one allows: initial hadron \neq final hadron (in the same octuplet): transition GPDs

Even less diagonal:

baryonic number (initial state) \neq baryonic number (final state) \rightarrow TDA

Example:



Pire, Szymanowski '05

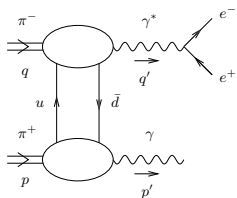
which can be further extended by replacing the outgoing γ by any hadronic state

$$\text{Amplitude} = \text{Transition Distribution Amplitude (soft)} \otimes \text{CF (hard)} \otimes \text{DA (soft)}$$

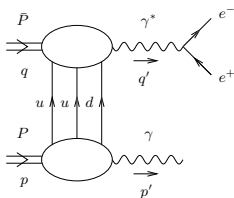
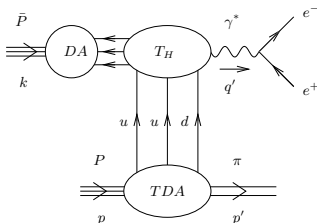
Lansberg, Pire, Szymanowski '06

Introduction

Extensions from GPD

TDA $\pi \rightarrow \gamma$

TDA at PANDA

TDA $p \rightarrow \gamma$ at PANDA (forward scattering of \bar{p} on a p probe)TDA $p \rightarrow \pi$ at PANDA (forward scattering of \bar{p} on a p probe)Spectral model for the $p \rightarrow \pi$ TDA: Pire, Semenov, Szymanowski '10

Collinear factorization

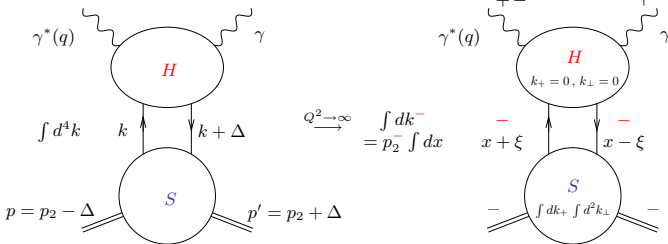
A bit more technical: DVCS and GPDs

Two steps for factorization

- momentum factorization: **light-cone vector dominance for $Q^2 \rightarrow \infty$**

$$p_1, p_2 : \text{the two light-cone directions} \quad \begin{cases} p_1 = \frac{\sqrt{s}}{2}(1, 0_\perp, 1) \\ p_2 = \frac{\sqrt{s}}{2}(1, 0_\perp, -1) \end{cases} \quad (p_1^2 = p_2^2 = 0, 2p_1 \cdot p_2 = s \sim s_{\gamma^* p})$$

$$\text{Sudakov decomposition: } k = \underbrace{\alpha p_1}_+ + \underbrace{\beta p_2}_- + \underbrace{k_\perp}_\perp$$



$$\int d^4k S(k, k + \Delta) H(q, k, k + \Delta) = \int dk^- \int dk^+ d^2k_\perp S(k, k + \Delta) H(q, k^-, k^- + \Delta^-)$$

- Quantum numbers factorization (Fierz identity: spinors + color)

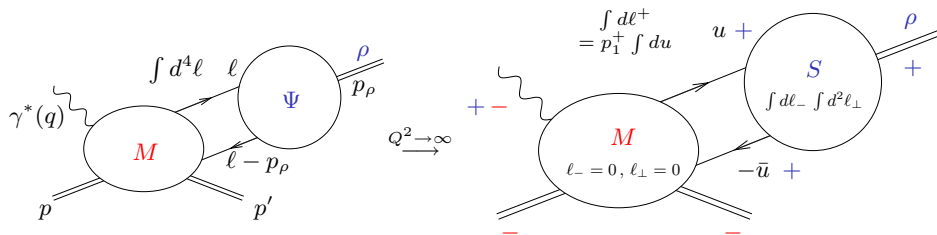
$$\Rightarrow \mathcal{M} = \text{GPD} \otimes \text{Hard part}$$

Collinear factorization

ρ -meson production: from the wave function to the DA

What is a ρ -meson in QCD?

It is described by its **wave function** Ψ which reduces in **hard processes** to its **Distribution Amplitude**



$$\int d^4 l M(q, l, l - p_\rho) \Psi(l, l - p_\rho) = \int dl^+ M(q, l^+, l^+ - p_\rho^+) \int_{|\ell_\perp^2| < \mu_F^2} dl^- \int d^2 l_\perp \Psi(l, l - p_\rho)$$

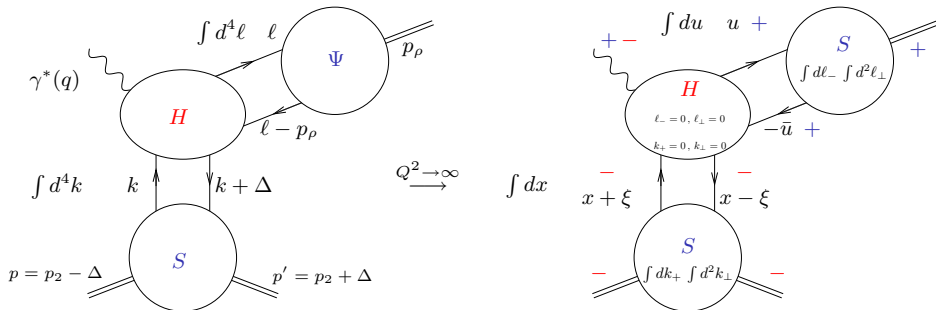
Hard part

DA $\Phi(u, \mu_F^2)$

(see Chernyak, Zhitnitsky '77; Brodsky, Lepage '79; Efremov, Radyushkin '80; ... in the case of form-factors studies)

Collinear factorization

Meson electroproduction: factorization with a GPD and a DA



$$\int d^4k d^4\ell \quad S(k, k + \Delta) \quad H(q, k, k + \Delta) \quad \Psi(\ell, \ell - p_\rho)$$

$$= \int dk^- d\ell^+ \int dk^+ \int_{|k_\perp^2| < \mu_{F_2}^2} d^2k_\perp S(k, k + \Delta) H(q; k^-, k^- + \Delta^-; \ell^+, \ell^+ - p_\rho^+) \int d\ell^- \int_{|\ell_\perp^2| < \mu_{F_1}^2} d^2\ell_\perp \Psi(\ell, \ell - p_\rho)$$

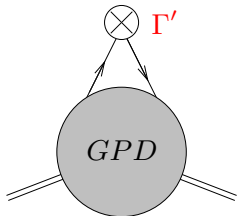
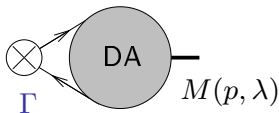
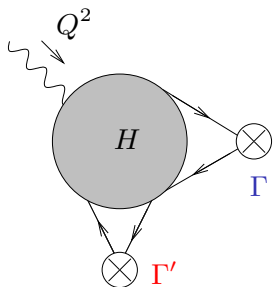
$$\text{GPD } F(x, \xi, t, \mu_{F_2}^2) \quad \text{Hard part } T(x/\xi, u, \mu_{F_1}^2, \mu_{F_2}^2, \mu_R) \quad \text{DA } \Phi(u, \mu_{F_1}^2)$$

Collins, Frankfurt, Strikman '97; Radyushkin '97

Collinear factorization

Meson electroproduction: factorization with a GPD and a DA

The building blocks



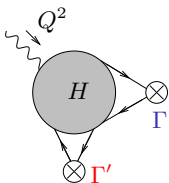
Γ, Γ' : Dirac matrices compatible
with quantum numbers: C, P, T , chirality

Similar structure for gluon exchange

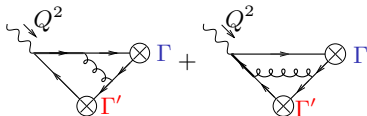
Collinear factorization

Meson electroproduction: factorization with a GPD and a DA

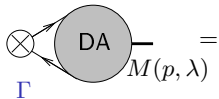
The building blocks



=



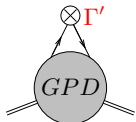
hand-bag diagrams



=

$$\langle M(p, \lambda) | \mathcal{O}(\Psi, \bar{\Psi} A) | 0 \rangle$$

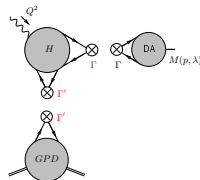
matrix element of a **non-local light-cone operator**



=

$$\langle N(p') | \mathcal{O}'(\Psi, \bar{\Psi} A) | N(p) \rangle$$

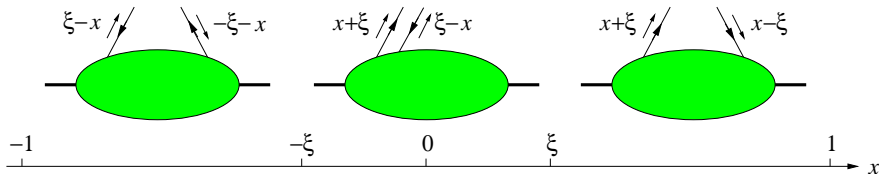
matrix element of a **non-local light-cone operator**



Collinear factorization

Twist 2 GPDs

Physical interpretation for GPDs



Emission and reabsorption
of an antiquark
 \sim PDFs for antiquarks
DGLAP-II region

Emission of a quark and
emission of an antiquark
 \sim meson exchange
ERBL region

Emission and reabsorption
of a quark
 \sim PDFs for quarks
DGLAP-I region

Collinear factorization

Twist 2 GPDs

Classification of twist 2 GPDs

- For quarks, one should distinguish the exchanges
 - without helicity flip (chiral-even Γ' matrices): 4 chiral-even GPDs:

$H^q \xrightarrow{\xi=0, t=0}$ PDF q , E^q , $\tilde{H}^q \xrightarrow{\xi=0, t=0}$ polarized PDFs Δq , \tilde{E}^q

$$\begin{aligned}
 F^q &= \frac{1}{2} \int \frac{dz^+}{2\pi} e^{ixP^-z^+} \langle p' | \bar{q}(-\frac{1}{2}z) \gamma^- q(\frac{1}{2}z) | p \rangle \Big|_{z^-=0, z_\perp=0} \\
 &= \frac{1}{2P^-} \left[H^q(x, \xi, t) \bar{u}(p') \gamma^- u(p) + E^q(x, \xi, t) \bar{u}(p') \frac{i\sigma^{-\alpha}\Delta_\alpha}{2m} u(p) \right],
 \end{aligned}$$

$$\begin{aligned}
 \tilde{F}^q &= \frac{1}{2} \int \frac{dz^+}{2\pi} e^{ixP^-z^+} \langle p' | \bar{q}(-\frac{1}{2}z) \gamma^- \gamma_5 q(\frac{1}{2}z) | p \rangle \Big|_{z^-=0, z_\perp=0} \\
 &= \frac{1}{2P^-} \left[\tilde{H}^q(x, \xi, t) \bar{u}(p') \gamma^- \gamma_5 u(p) + \tilde{E}^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^-}{2m} u(p) \right].
 \end{aligned}$$

- with helicity flip (chiral-odd Γ' mat.): 4 chiral-odd GPDs:

$H_T^q \xrightarrow{\xi=0, t=0}$ quark transversity PDFs $\Delta_T q$, E_T^q , \tilde{H}_T^q , \tilde{E}_T^q

$$\begin{aligned}
 &\frac{1}{2} \int \frac{dz^+}{2\pi} e^{ixP^-z^+} \langle p' | \bar{q}(-\frac{1}{2}z) i\sigma^{-i} q(\frac{1}{2}z) | p \rangle \Big|_{z^-=0, z_\perp=0} \\
 &= \frac{1}{2P^-} \bar{u}(p') \left[H_T^q i\sigma^{-i} + \tilde{H}_T^q \frac{P^- \Delta^i - \Delta^- P^i}{m^2} + E_T^q \frac{\gamma^- \Delta^i - \Delta^- \gamma^i}{2m} + \tilde{E}_T^q \frac{\gamma^- P^i - P^- \gamma^i}{m} \right]
 \end{aligned}$$

Collinear factorization

Twist 2 GPDs

Classification of twist 2 GPDs

- analogously, for gluons:

- 4 gluonic GPDs without helicity flip:

$$H^g \xrightarrow{\xi=0, t=0} \text{PDF } x g$$

$$E^g$$

$$\tilde{H}^g \xrightarrow{\xi=0, t=0} \text{polarized PDF } x \Delta g$$

$$\tilde{E}^g$$

- 4 gluonic GPDs with helicity flip:

$$H_T^g$$

$$E_T^g$$

$$\tilde{H}_T^g$$

$$\tilde{E}_T^g$$

(no forward limit reducing to gluons PDFs here: a change of 2 units of helicity cannot be compensated by a spin 1/2 target)

A few applications

Electroproduction of an exotic hybrid meson

Quark model and meson spectroscopy

- spectroscopy: $\vec{J} = \vec{L} + \vec{S}$; neglecting any spin-orbital interaction
 $\Rightarrow S, L =$ additional quantum numbers to classify hadron states

$$\vec{J}^2 = J(J+1), \quad \vec{S}^2 = S(S+1), \quad \vec{L}^2 = L(L+1),$$

with $J = |L - S|, \dots, L + S$

- In the usual quark-model: meson = $q\bar{q}$ bound state with

$$C = (-)^{L+S} \quad \text{and} \quad P = (-)^{L+1}.$$

- Thus:

$$S = 0, \quad L = J, \quad J = 0, 1, 2, \dots : \quad J^{PC} = 0^{-+}(\pi, \eta), 1^{+-}(h_1, b_1), 2^{-+}, 3^{+-}, \dots$$

$$S = 1, \quad L = 0, \quad J = 1 : \quad J^{PC} = 1^{--}(\rho, \omega, \phi)$$

$$L = 1, \quad J = 0, 1, 2 : \quad J^{PC} = 0^{++}(f_0, a_0), 1^{++}(f_1, a_1), 2^{++}(f_2, a_2)$$

$$L = 2, \quad J = 1, 2, 3 : \quad J^{PC} = 1^{--}, 2^{--}, 3^{--}$$

...

- \Rightarrow the exotic mesons with $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, \dots$ are forbidden

A few applications

Electroproduction of an exotic hybrid meson

Experimental candidates for light hybrid mesons (1)

three candidates:

- $\pi_1(1400)$
 - GAMS '88 (SPS, CERN): in $\pi^- p \rightarrow \eta \pi^0 n$ (through $\eta \pi^0 \rightarrow 4\gamma$ mode)
 $M = 1406 \pm 20$ MeV $\Gamma = 180 \pm 30$ MeV
 - E852 '97 (BNL): $\pi^- p \rightarrow \eta \pi^- p$
 $M = 1370 \pm 16$ MeV $\Gamma = 385 \pm 40$ MeV
 - VES '01 (Protvino) in $\pi^- Be \rightarrow \eta \pi^- Be$, $\pi^- Be \rightarrow \eta' \pi^- Be$,
 $\pi^- Be \rightarrow b_1 \pi^- Be$
 $M = 1316 \pm 12$ MeV $\Gamma = 287 \pm 25$ MeV
 but resonance hypothesis ambiguous
 - Crystal Barrel (LEAR, CERN) '98 '99 in $\bar{p} n \rightarrow \pi^- \pi^0 \eta$ and $\bar{p} p \rightarrow 2\pi^0 \eta$
 (through $\pi\eta$ resonance)
 $M = 1400 \pm 20$ MeV $\Gamma = 310 \pm 50$ MeV
 and $M = 1360 \pm 25$ MeV $\Gamma = 220 \pm 90$ MeV

A few applications

Electroproduction of an exotic hybrid meson

Experimental candidates for light hybrid mesons (2)

- $\pi_1(1600)$
 - **E852 (BNL)**: in peripheral $\pi^- p \rightarrow \pi^+ \pi^- \pi^- p$ (through $\rho\pi^-$ mode) '98 '02, $M = 1593 \pm 8$ MeV $\Gamma = 168 \pm 20$ MeV $\pi^- p \rightarrow \pi^+ \pi^- \pi^- \pi^0 \pi^0 p$ (in $b_1(1235)\pi^- \rightarrow (\omega\pi^0)\pi^- \rightarrow (\pi^+ \pi^- \pi^0)\pi^0 \pi^-$ '05 and $f_1(1285)\pi^-$ '04 modes), in peripheral $\pi^- p$ through $\eta'\pi^-$ '01
 $M = 1597 \pm 10$ MeV $\Gamma = 340 \pm 40$ MeV
 but **E852 (BNL)** '06: no exotic signal in $\pi^- p \rightarrow (3\pi)^- p$ for a larger sample of data!
 - **VES '00 (Protvino)**: in peripheral $\pi^- p$ through $\eta'\pi^-$ '93, '00, $\rho(\pi^+ \pi^-)\pi^-$ '00, $b_1(1235)\pi^- \rightarrow (\omega\pi^0)\pi^-$ '00
 - **Crystal Barrel (LEAR, CERN)** '03 $\bar{p}p \rightarrow b_1(1235)\pi\pi$
 - **COMPASS '10 (SPS, CERN)**: diffractive dissociation of π^- on Pb target through Primakov effect $\pi^- \gamma \rightarrow \pi^- \pi^- \pi^+$ (through $\rho\pi^-$ mode)
 $M = 1660 \pm 10$ MeV $\Gamma = 269 \pm 21$ MeV
- $\pi_1(2000)$: seen only at **E852 (BNL)** '04 '05 (through $f_1(1285)\pi^-$ and $b_1(1235)\pi^-$)

A few applications

Electroproduction of an exotic hybrid meson

What about hard processes?

- Is there a hope to see such states in **hard processes**, with high counting rates, and to exhibit their light-cone wave-function?
- **hybrid mesons** = $q\bar{q}g$ states T. Barnes '77; R. L. Jaffe, K. Johnson, and Z. Ryzak, G. S. Bali
- popular belief: $H = q\bar{q}g \Rightarrow$ higher Fock-state component \Rightarrow twist-3 \Rightarrow hard electroproduction of H **versus** ρ suppressed as $1/Q$
- **This is not true!!** Electroproduction of hybrid is similar to electroproduction of usual ρ -meson: it is twist 2 dominated

I. V. Anikin, B. Pire, O. V. Teryaev, L. Szymanowski, S.W.

Phys.Rev.D70 (2004) 011501

Phys.Rev.D71 (2005) 034021

Eur.Phys.J.C42 (2005) 163

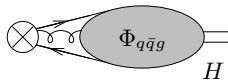
Eur.Phys.J.C47 (2006) 71-79.

A few applications

Electroproduction of an exotic hybrid meson

Distribution amplitude of exotic hybrid mesons at twist 2

- One may think that to produce $|q\bar{q}g\rangle$, the fields Ψ , $\bar{\Psi}$, A should appear explicitly in the non-local operator $\mathcal{O}(\Psi, \bar{\Psi} A)$



- If one tries to produce $H = 1^{-+}$ from a **local operator**, the dominant operator should be $\bar{\Psi}\gamma^\mu G_{\mu\nu}\Psi$ of **twist** = dimension - spin = 5 - 1 = 4
- It means that there should be a $1/Q^2$ suppression in the production amplitude of H versus the usual ρ -production (which is twist 2 dominated)
- But collinear approach describes hard exclusive processes in terms of **non-local light-cone operators**, among which are the **twist 2 operator**

$$\bar{\psi}(-z/2)\gamma_\mu[-z/2; z/2]\psi(z/2)$$

where $[-z/2; z/2]$ is a **Wilson line**, necessary to fulfill gauge invariance (i.e. a "color tube" between q and \bar{q}) **which thus hides gluonic degrees of freedom: the needed gluon is there, at twist 2**. This does not require to introduce explicitly A !

A few applications

Electroproduction of an exotic hybrid meson

Distribution amplitude and quantum numbers: C -parity

- Define the H DA as (for long. pol.)

$$\langle H(p, 0) | \bar{\psi}(-z/2) \gamma_\mu [-z/2; z/2] \psi(z/2) | 0 \rangle \Big|_{\substack{z^2=0 \\ z_+=0 \\ z_\perp=0}} = i f_H M_H e_\mu^{(0)} \int_0^1 dy e^{i(\bar{y}-y)p \cdot z/2} \phi_L^H(y)$$

- Expansion in terms of local operators

$$\langle H(p, \lambda) | \bar{\psi}(-z/2) \gamma_\mu [-z/2; z/2] \psi(z/2) | 0 \rangle = \sum_n \frac{1}{n!} z_{\mu_1} \dots z_{\mu_n} \langle H(p, \lambda) | \bar{\psi}(0) \gamma_\mu \vec{D}_{\mu_1} \dots \vec{D}_{\mu_n} \psi(0) | 0 \rangle,$$

- C -parity: $\begin{cases} H \text{ selects the odd-terms: } C_H = (-) \\ \rho \text{ selects even-terms: } C_\rho = (-) \end{cases}$

$$\langle H(p, \lambda) | \bar{\psi}(-z/2) \gamma_\mu [-z/2; z/2] \psi(z/2) | 0 \rangle = \sum_{n \text{ odd}} \frac{1}{n!} z_{\mu_1} \dots z_{\mu_n} \langle H(p, \lambda) | \bar{\psi}(0) \gamma_\mu \vec{D}_{\mu_1} \dots \vec{D}_{\mu_n} \psi(0) | 0 \rangle$$

- Special case $n = 1$: $\mathcal{R}_{\mu\nu} = S_{(\mu\nu)} \bar{\psi}(0) \gamma_\mu \vec{D}_\nu \psi(0)$

$S_{(\mu\nu)}$ = symmetrization operator: $S_{(\mu\nu)} T_{\mu\nu} = \frac{1}{2}(T_{\mu\nu} + T_{\nu\mu})$

A few applications

Electroproduction of an exotic hybrid meson

Non perturbative input for the hybrid DA

- We need to fix f_H (the analogue of f_ρ)
- This is a non-perturbative input
- Lattice does not yet give information
- The operator $\mathcal{R}_{\mu\nu}$ is related to quark energy-momentum tensor $\Theta_{\mu\nu}$:

$$\mathcal{R}_{\mu\nu} = -i \Theta_{\mu\nu}$$

- Rely on QCD sum rules: resonance for $M \approx 1.4$ GeV
I. I. Balitsky, D. Diakonov, and A. V. Yung

$$f_H \approx 50 \text{ MeV}$$

$$f_\rho = 216 \text{ MeV}$$

- Note: f_H evolves according to the γ_{QQ} anomalous dimension

$$f_H(Q^2) = f_H \left(\frac{\alpha_S(Q^2)}{\alpha_S(M_H^2)} \right)^{K_1} \quad K_1 = \frac{2\gamma_{QQ}(1)}{\beta_0},$$

A few applications

Electroproduction of an exotic hybrid meson

Counting rates for H versus ρ electroproduction: order of magnitude

- Ratio:

$$\frac{d\sigma^H(Q^2, x_B, t)}{d\sigma^\rho(Q^2, x_B, t)} = \left| \frac{f_H (e_u \mathcal{H}_{uu}^- - e_d \mathcal{H}_{dd}^-) \mathcal{V}^{(H,-)}}{f_\rho (e_u \mathcal{H}_{uu}^+ - e_d \mathcal{H}_{dd}^+) \mathcal{V}^{(\rho,+)}} \right|^2$$

- Rough estimate:

- neglect \bar{q} i.e. $x \in [0, 1]$

$\Rightarrow Im\mathcal{A}_H$ and $Im\mathcal{A}_\rho$ are equal up to the factor \mathcal{V}^M

- Neglect the effect of $Re\mathcal{A}$

$$\frac{d\sigma^H(Q^2, x_B, t)}{d\sigma^\rho(Q^2, x_B, t)} \approx \left(\frac{5f_H}{3f_\rho} \right)^2 \approx 0.15$$

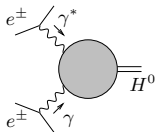
- More precise study based on *Double Distributions* to model GPDs + effects of varying μ_R : order of magnitude unchanged
- The range around 1400 MeV is dominated by the $a_2(1329)(2^{++})$ resonance
 - possible interference between H and a_2
 - identification through the $\pi\eta$ GDA, main decay mode for the $\pi_1(1400)$ candidate, through angular asymmetry in θ_π in the $\pi\eta$ cms

A few applications

Electroproduction of an exotic hybrid meson

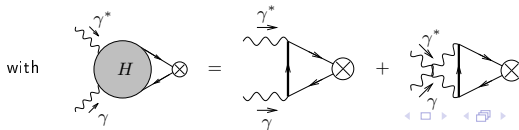
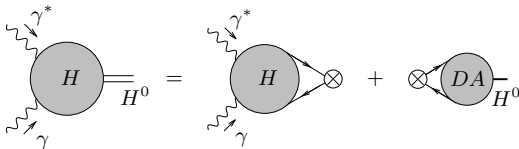
Hybrid meson production in e^+e^- colliders

- Hybrid can be copiously produced in $\gamma^*\gamma$, i.e. at e^+e^- colliders **with one tagged out-going electron**



BaBar, Belle, Super-B

- This can be described in a hard factorization framework:



A few applications

Electroproduction of an exotic hybrid meson

Counting rates for H^0 versus π^0

- Factorization gives:

$$\mathcal{A}^{\gamma\gamma^* \rightarrow H^0}(\gamma\gamma^* \rightarrow H_L) = (\epsilon_\gamma \cdot \epsilon_\gamma^*) \frac{(e_u^2 - e_d^2) f_H}{2\sqrt{2}} \int_0^1 dz \Phi^H(z) \left(\frac{1}{\bar{z}} - \frac{1}{z} \right)$$

- Ratio H^0 versus π^0 :

$$\frac{d\sigma^H}{d\sigma^{\pi^0}} = \left| \frac{f_H \int_0^1 dz \Phi^H(z) \left(\frac{1}{z} - \frac{1}{\bar{z}} \right)}{f_\pi \int_0^1 dz \Phi^\pi(z) \left(\frac{1}{z} + \frac{1}{\bar{z}} \right)} \right|^2$$

- This gives, with *asymptotical* DAs (i.e. limit $Q^2 \rightarrow \infty$):

$$\frac{d\sigma^H}{d\sigma^{\pi^0}} \approx 38\%$$

still larger than 20% at $Q^2 \approx 1 \text{ GeV}^2$ (including kinematical twist-3 effects à la [Wandzura-Wilczek](#) for the H^0 DA) and similarly

$$\frac{d\sigma^H}{d\sigma^\eta} \approx 46\%$$

A few applications

Spin transversity in the nucleon

What is transversity?

- Tranverse spin content of the proton:

$$\begin{array}{l} |\uparrow\rangle_{(x)} \quad \sim \quad |\rightarrow\rangle + |\leftarrow\rangle \\ |\downarrow\rangle_{(x)} \quad \sim \quad |\rightarrow\rangle - |\leftarrow\rangle \\ \text{spin along } x \quad \quad \quad \text{helicity state} \end{array}$$

- An observable sensitive to helicity spin flip gives thus access to the transversity $\Delta_T q(x)$, which is very badly known (first data have recently been obtained by COMPASS)

- The transversity GPDs are completely unknown

- Chirality:** $q_{\pm}(z) \equiv \frac{1}{2}(1 \pm \gamma^5)q(z)$ avec $q(z) = q_+(z) + q_-(z)$

Chiral-even: chirality conserving

$$\bar{q}_{\pm}(z)\gamma^{\mu}q_{\pm}(-z) \text{ et } \bar{q}_{\pm}(z)\gamma^{\mu}\gamma^5q_{\pm}(-z)$$

Chiral-odd: chirality reversing

$$\bar{q}_{\pm}(z) \cdot 1 \cdot q_{\mp}(-z), \quad \bar{q}_{\pm}(z) \cdot \gamma^5 \cdot q_{\mp}(-z) \text{ et } \bar{q}_{\pm}(z)[\gamma^{\mu}, \gamma^{\nu}]q_{\mp}(-z)$$

- For a massless (anti)particle, chirality = (-)helicity

- Transversity is thus a chiral-odd quantity**

- QCD and QED are chiral even $\Rightarrow \mathcal{A} \sim (\text{Ch.-odd})_1 \otimes (\text{Ch.-even})_2$

A few applications

Spin transversity in the nucleon

Can one circumvent this vanishing?

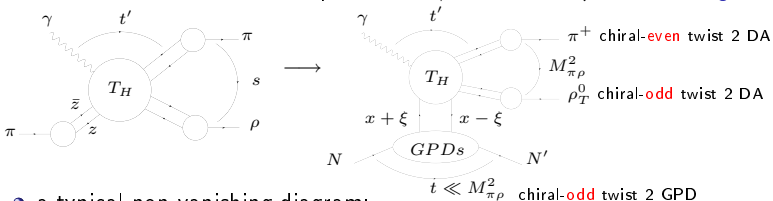
- This vanishing is true only a twist 2
- At twist 3 this process does not vanish
- However processes involving twist 3 DAs may face problems with factorization (end-point singularities: see later)
- The problem of classification of twist 3 chiral-odd GPDs is still open:
Pire, Szymanowski, S.W. in progress, in the spirit of our
Light-Cone Collinear Factorization framework recently developed
(Anikin, Ivanov, Pire, Szymanowski, S. W.)

A few applications

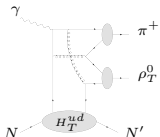
Spin transversity in the nucleon

$\gamma N \rightarrow \pi^+ \rho_T^0 N'$ gives access to transversity

- Factorization à la Brodsky Lepage of $\gamma + \pi \rightarrow \pi + \rho$ at large s and fixed angle (i.e. fixed ratio $t'/s, u'/s$)
 \implies factorization of the amplitude for $\gamma + N \rightarrow \pi + \rho + N'$ at large $M_{\pi\rho}^2$



- a typical non-vanishing diagram:



M. El Beiyad, P. Pire, M. Segond, L. Szymanowski, S.W. Phys.Lett. B688:154-167,2010

see also, at large s , with Pomeron exchange:

R. Ivanov, B. Pire, L. Szymanowski, O. Teryaev '02

R. Enberg, B. Pire, L. Szymanowski '06

- These processes with 3 body final state can give access to all GPDs: $M_{\pi\rho}^2$ plays the role of the γ^* virtuality of usual DVCS (here in the time-like domain) JLab, COMPASS

Problems

ρ -electroproduction: Selection rules and factorization status

Improved collinear approximation: a solution?

- keep a transverse ℓ_{\perp} dependency in the q, \bar{q} momenta, used to regulate end-point singularities
- soft and collinear gluon exchange between the valence quark are responsible for large double-logarithmic effects which are conjectured to exponentiate
- this is made easier when using the impact parameter space b_{\perp} conjugated to $\ell_{\perp} \Rightarrow$ Sudakov factor

$$\exp[-S(u, b, Q)]$$

- S diverges when $b_{\perp} \sim O(1/\Lambda_{QCD})$ (large transverse separation, i.e. small transverse momenta) or $u \sim O(\Lambda_{QCD}/Q)$ Botts, Serman '89
 \Rightarrow regularization of end-point singularities for $\pi \rightarrow \pi\gamma^*$ and $\gamma\gamma^*\pi^0$ form factors, based on the factorization approach Li, Serman '92
- it has been proposed to combine this perturbative resummation tail effect with an ad-hoc non-perturbative gaussian ansatz for the DAs

$$\exp[-a^2 |k_{\perp}^2|/(u\bar{u})]$$

which gives back the usual asymptotic DA $6u\bar{u}$ when integrating over k_{\perp}
 \Rightarrow practical tools for meson electroproduction phenomenology

Goloskokov, Kroll '05

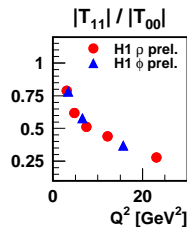
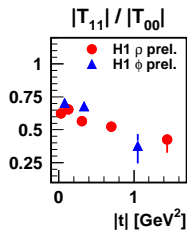
QCD at large s

Meson production at HERA

Polarization effects in $\gamma^* P \rightarrow \rho P$ at HERA

- Very precise experimental data on the spin density matrix (i.e. correlations between γ^* and ρ polarizations)
- for $t = t_{min}$ one can experimentally distinguish

$$\left\{ \begin{array}{l} \gamma_L^* \rightarrow \rho_L : \text{dominates ("twist 2": amplitude } |\mathcal{A}| \sim \frac{1}{Q}) \\ \gamma_T^* \rightarrow \rho_T : \text{visible ("twist 3": amplitude } |\mathcal{A}| \sim \frac{1}{Q^2}) \end{array} \right.$$
- How to calculate the $\gamma_T^* \rightarrow \rho_T$ transition from first principles?



QCD at large s

Meson production at HERA

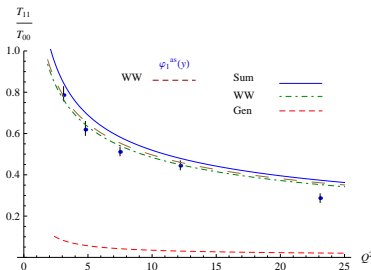
Exclusive vector meson production: Comparison of our model with H1 data

- Model for the proton impact factor:

$$\Phi_{N \rightarrow N}(\underline{k}, \underline{\Delta}; M^2) = A \delta_{ab} \left[\frac{1}{M^2 + \left(\frac{\underline{\Delta}}{2}\right)^2} - \frac{1}{M^2 + \left(\underline{k} - \frac{\underline{\Delta}}{2}\right)^2} \right].$$

$\Phi_{N \rightarrow N} \rightarrow 0$ if $\underline{k} \rightarrow 0$ or $\underline{\Delta} - \underline{k} \rightarrow 0$

- Very satisfying results: (note that the **sign** is also a prediction)



A. Besse, I. V. Anikin, D. Y. Ivanov, B. Pire, L. Szymanowski and S. W, to be submitted

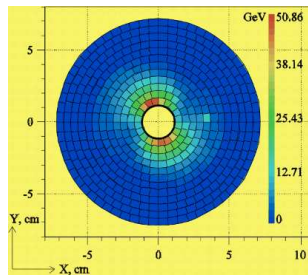
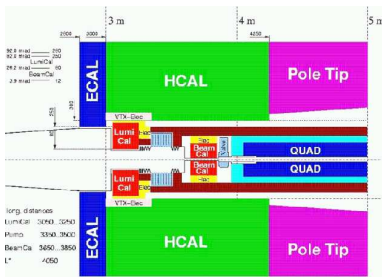
The specific case of QCD at large s

Phenomenological applications: exclusive test of Pomeron

An example of realistic exclusive test of Pomeron: $\gamma^{(*)}\gamma^{(*)} \rightarrow \rho\rho$
 as a subprocess of $e^-e^+ \rightarrow e^-e^+ \rho_L^0 \rho_L^0$

It make sense to focus on tests of QCD in the perturbative Regge limit at future ILC for rare **exclusive** processes:

- ILC should provide **very large \sqrt{s}** (= 500 GeV) and **luminosity** ($\simeq 125 \text{ fb}^{-1}/\text{year}$)
- detectors are planned to cover the **very forward** region, close from the beampipe (directions of out-going e^+ and e^- at large s)

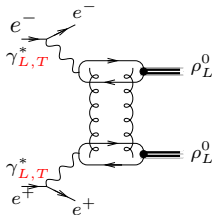


good efficiency of tagging for outgoing e^\pm for $E_e > 100 \text{ GeV}$ and $\theta > 4 \text{ mrad}$
 (illustration for LDC concept)

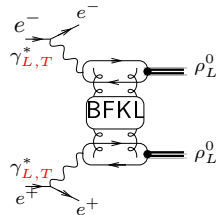
The specific case of QCD at large s

Phenomenological applications: exclusive test of Pomeron

QCD effects in the Regge limit on $\gamma^{(*)}\gamma^{(*)} \rightarrow \rho\rho$

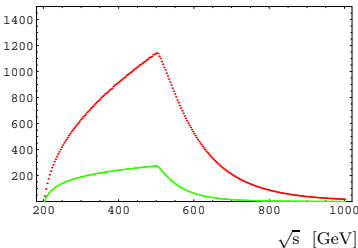


$\simeq 4 \cdot 10^3$ events/year



$\simeq 2 \cdot 10^4$ events/year

$\frac{d\sigma^{tmin}}{dt} (fb/GeV^2)$



proof of feasibility:

B. Pire, L. Szymanowski and S. W.
Eur.Phys.J.C44 (2005) 545

proof of visible BFKL enhancement:

R. Enberg, B. Pire, L. Szymanowski and S. W.
Eur.Phys.J.C45 (2006) 759

comprehensive study of γ^* polarization effects
and event rates:

M. Segond, L. Szymanowski and S. W.
Eur. Phys. J. C 52 (2007) 93

Beyond leading twist

Light-Cone Collinear Factorization

Minimal set of DAs

- Number of non-perturbative quantities Φ : a priori 7 at twist 3
- Non-perturbative correlators cannot be obtained perturbatively!
- One should reduce their number to a minimal set before any use of a model or any measure on the QCD lattice
- independence w.r.t the choice of the vector n defining

- the light-cone direction z : $z = \lambda n$
- the ρ_T polarization vector: $e_T \cdot n = 0$
- the axial gauge: $n \cdot A = 0$

$$\mathcal{A} = H \otimes S \quad \frac{d\mathcal{A}}{dn_{\perp}^{\mu}} = 0 \Rightarrow S \text{ are related}$$

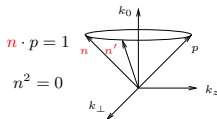
- We have proven that 3 independent Distribution Amplitude are necessary:

$$\begin{cases} \text{Equations of motion} & 2 \text{ equations} \\ \text{Arbitrariness in the choice of } n & 2 \text{ equations} \end{cases}$$

$\phi_1(y)$ ← 2-body twist 2 correlator

$B(y_1, y_2)$ ← 3-body genuine twist 3 vector correlator

$D(y_1, y_2)$ ← 3-body genuine twist 3 axial correlator



Conclusion

- Since a decade, there have been much progress in the understanding of **hard** exclusive processes
 - at medium energies, there is now a conceptual framework starting from first principle, allowing to describe a huge number of processes
 - at high energy, the impact representation is a powerful tool for describing exclusive processes in diffractive experiments; they are and will be essential for studying QCD in the hard Regge limit (Pomeron, Odderon, saturation...)
- till, some problems remain:
 - proofs of factorization have been obtained only for very few processes (ex.: $\gamma^* p \rightarrow \gamma p$, $\gamma_L^* p \rightarrow \rho_L p$)
 - for some other processes factorization is highly plausible, but not fully demonstrated at any order (ex.: processes involving GDAs and TDAs)
 - some processes explicitly show sign of breaking of factorization (ex.: $\gamma_T^* p \rightarrow \rho_T p$ which has end-point singularities at Leading Order)
 - models and results from the lattice for the non-perturbative correlators entering GPDs, DAs, GDAs, TDAs are needed, even at a qualitative level!
 - the effect of QCD evolution, the NLO corrections (see talk of L. Szymanowski), choice of renormalization/factorization scale, power corrections will be very relevant to interpret and describe the forthcoming data
- Links between theoretical and experimental communities are very fruitful HERA, HERMES, Tevatron, LHC, JLab, Compass, BaBar, BELLE, Super-B, ILC
This is very hot and pleasant domain. Everybody is welcome!