# Soft-collinear resummation in deeply virtual Compton scattering

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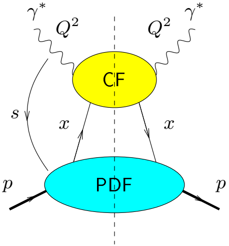
JHEP 1210 (2012) 049 [arXiv:1207.4609 [hep-ph]]
[arXiv:1206.3115 [hep-ph]]

# Extensions from DIS

- DIS: inclusive process $\longrightarrow$ forward amplitude ($t = 0$) *(optical theorem)*

  **(DIS: Deep Inelastic Scattering)**

  ex: $e^{\pm} p \rightarrow e^{\pm} X$ at HERA

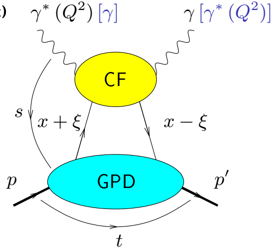



Structure Function

= Coefficient Function (hard) $\otimes$ Parton Distribution Function (soft)

- DVCS (TCS): exclusive process $\longrightarrow$ non forward amplitude ($-t \ll s = W^2$)

  **(DVCS: Deep Vitual Compton Scattering; TCS: Timelike Compton Scattering)**



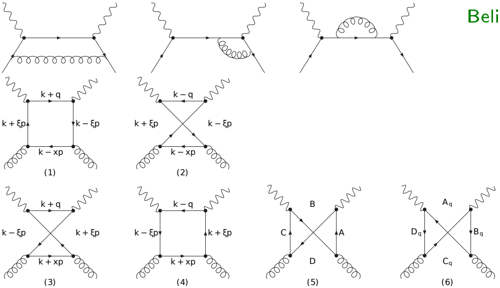

Amplitude

= Coefficient Function (hard) $\otimes$ Generalized Parton Distribution (soft)

Müller et al. '91 - '94; Radyushkin '96; Ji '97

# DVCS and TCS at NLO

## One-loop contributions to the coefficient function

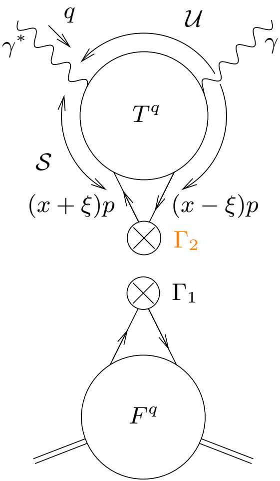



Belitsky, Mueller, Niedermeier, Schafer,
Phys.Lett.B474, 2000
Pire, Szymanowski, Wagner
Phys.Rev.D83, 2011

$$\mathcal{A}^{\mu\nu} = g_T^{\mu\nu} \int_{-1}^{1} dx \left[ \sum_q^{n_F} T^q(x)\, F^q(x) + T^g(x)\, F^g(x) \right]$$

(symmetric part of the factorised amplitude)

# Resummations effects are expected




- The renormalized quark coefficient functions $T^q$ is

$$T^q = C_0^q + C_1^q + C_{coll}^q \log \frac{|Q^2|}{\mu_F^2}$$

$$C_0^q = e_q^2 \left( \frac{1}{x - \xi + i\varepsilon} - (x \to -x) \right)$$

$$C_1^q = \frac{e_q^2 \alpha_S C_F}{4\pi (x - \xi + i\varepsilon)} \left[ \log^2 \left( \frac{\xi - x}{2\xi} - i\varepsilon \right) + \ldots \right] - (x \to -x)$$

- Usual collinear approach: single-scale analysis w.r.t. $Q^2$
- Consider the invariants $\mathcal{S}$ and $\mathcal{U}$:

$$\mathcal{S} = \frac{x-\xi}{2\xi} Q^2 \ll Q^2 \quad \text{when } x \to \xi$$

$$\mathcal{U} = -\frac{x+\xi}{2\xi} Q^2 \ll Q^2 \quad \text{when } x \to -\xi$$

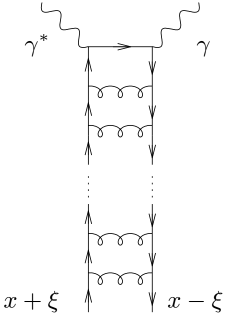
$\Rightarrow$ two scales problem; threshold singularities to be resummed

analogous to the $\log(1-x)$ resummation for DIS coefficient functions

# Resummation for Coefficient functions: our result

Soft-collinear resummation effects for the coefficient function

- The resummation easier when using the axial gauge $p_1 \cdot A = 0$ $(p_\gamma \equiv p_1)$
- The dominant diagrams are ladder-like




resummed formula (for DVCS), for $x \to \xi$ :

$$(T^q)^{\text{res}} = \left( \frac{e_q^2}{x - \xi + i\epsilon} \left\{ \cosh \left[ D \log \left( \frac{\xi - x}{2\xi} - i\epsilon \right) \right] - \frac{D^2}{2} \left[ 9 + 3 \frac{\xi - x}{x + \xi} \log \left( \frac{\xi - x}{2\xi} - i\epsilon \right) \right] \right\} + C_{coll}^q \log \frac{Q^2}{\mu_F^2} \right) - (x \to -x) \quad \text{with} \quad D = \sqrt{\frac{\alpha_s C_F}{2\pi}}$$

T. Altinoluk, B. Pire, L. Szymanowski, S. W.
JHEP 1210 (2012) 049 [arXiv:1206.3115]

## Kinematics, gauge, etc...

- We expand any momentum in the Sudakov basis $p_1$, $p_2$ :

$$k = \alpha\, p_1 + \beta\, p_2 + k_\perp$$

- $p_2$ is the light-cone direction of the two incoming and outgoing partons

$$p_1^2 = p_2^2 = 0\ ,\ \ 2\, p_1 \cdot p_2 = s = \frac{Q^2}{2\xi}$$

- Momenta of the incoming and outgoing photons:

$$q_{\gamma^*} = p_1 - 2\,\xi\, p_2\,, \qquad\qquad p_1 \equiv q_\gamma$$

- The extraction of soft-collinear singularities in the limit $x \to \pm\xi$ is easier in the light-like gauge $p_1 \cdot A = 0$: in this gauge, gluon physical degrees of freedom are manifest and helicity conservation at each vertex implies that collinear singularities only arise in ladder-like diagrams

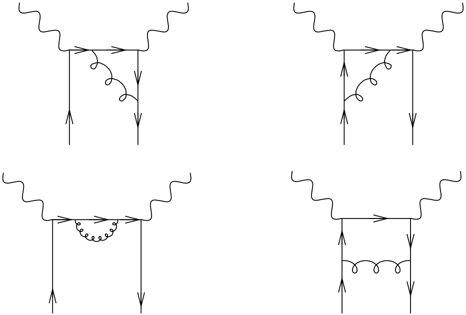
- $K_n$ is the contribution of a $n$-loop ladder to the CF :

$$K_n = -\tfrac{1}{4} e_q^2 \left( -i\, C_F\, \alpha_s \tfrac{1}{(2\pi)^2} \right)^n I_n$$

- The issue related to the $i\epsilon$ prescription is solved by computing the CF in the unphysical region $\xi > 1$. After analytical continuation to the physical region $0 \le \xi \le 1$, the physical prescription is then obtained through the shift $\xi \to \xi - i\epsilon$ .

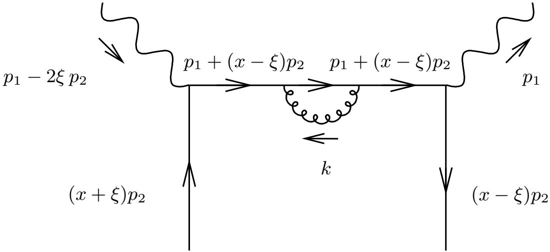
# Full one-loop analysis

- analyzing the one-loop diagrams



- no approximations!
- reduce the number of denominators in order to simplify the calculation.
- aims (we now assume $x \to +\xi$):
  - to understand which diagrams give contribution at order $\dfrac{\alpha_s \log^2(\xi - x)}{(x-\xi)}$
  - identify the part of the phase space that is responsible for this contribution

## Self energy diagram




- numerator for S.E. diagram :

$$(\mathrm{Num})_{\mathrm{S.E.}} = \mathrm{tr}\left\{ \not{p}_2 \gamma_\perp^\sigma \left[\not{p}_1 + (x-\xi)\not{p}_2\right] \gamma^\nu \left[\not{p}_1 + (x-\xi)\not{p}_2 - \not{k}\right] \gamma^\mu \left[\not{p}_1 + (x-\xi)\not{p}_2\right] \gamma_{\perp\sigma} \right\}$$

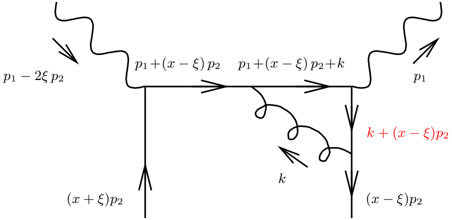
$$\times \left\{ g_{\mu\nu} - \frac{k_\mu p_{1\nu} + k_\nu p_{1\mu}}{k \cdot p_1} \right\} .$$

- a simple algebra shows that $(\mathrm{Num})_{\mathrm{gauge}} = 0 \Rightarrow$ S.E. diagram is the same in Feynman gauge and in light-like gauge.
- In Feynman gauge S.E. diagram gives only single $\log$'s!
  [B. Pire, L. Szymanowski, J. Wagner, Phys.Rev. D83 (2011) 034009]
- S.E. diagram doesn't contribute to $[\log^2(\xi - x)]/(x-\xi)$ terms!

## Right vertex, left vertex and box diagram

- Right Vertex:

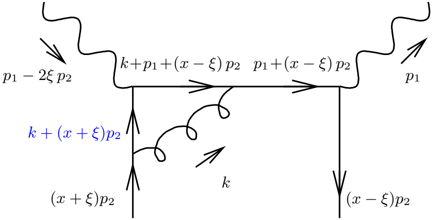
$$(\mathrm{Num})_{\mathrm{R.V.}} = 8s\frac{k_\perp^2}{\beta}(\beta + x - \xi)$$




$$I_{\mathrm{R.V.}} = -\frac{s}{2}\int d\alpha\, d\beta\, d_2\underline{k}\; 8s\frac{\underline{k}^2}{\beta}(\beta + x - \xi)\frac{1}{s(x-\xi)}\frac{1}{\left[k + (x-\xi)p_2\right]^2}\frac{1}{k^2}\frac{1}{\left[k + p_1 + (x-\xi)p_2\right]^2}$$

- Left Vertex:

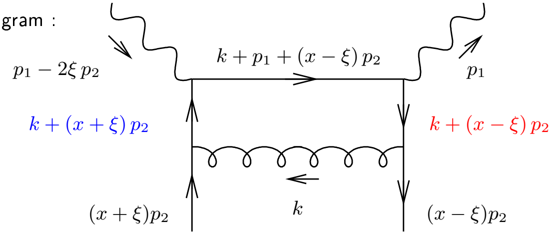
$$(\mathrm{Num})_{\mathrm{L.V.}} = 8s\frac{k_\perp^2}{\beta}(\beta + x + \xi)$$




$$I_{\mathrm{L.V.}} = -\frac{s}{2}\int d\alpha\, d\beta\, d_2\underline{k}\, 8s\frac{\underline{k}^2}{\beta}(\beta + x + \xi)\frac{1}{s(x-\xi)}\frac{1}{\left[k + (x+\xi)p_2\right]^2}\frac{1}{k^2}\frac{1}{\left[k + p_1 + (x-\xi)p_2\right]^2}$$

# Right vertex, left vertex and box diagram

- Box diagram :




The $g_{\mu\nu}$ part of the box diagram reads

$$(\mathrm{Num})_{\mathrm{box}\, g_{\mu\nu}} = -2\,\mathrm{tr}\Big\{ [\not{k} + (x+\xi)\not{p}_2]\not{p}_2[\not{k} + (x-\xi)\not{p}_2]\gamma_\perp^\sigma[\not{k} + \not{p}_1 + (x-\xi)\not{p}_2]\gamma_{\perp\sigma}\Big\}$$

Noting that $p_2$ can be written as (Ward identity)

$$p_2^\mu = \frac{1}{2\xi}\Big([k + (x+\xi)p_2] - [k + (x-\xi)p_2]\Big)^\mu$$

one gets

$$(\mathrm{Num})_{\mathrm{box}\, g_{\mu\nu}} = -\frac{8}{\xi}[k + (x+\xi)p_2]^2\Big\{k_\perp^2 - (\beta + x - \xi)\frac{s}{2}\Big\} + \frac{8}{\xi}[k + (x-\xi)p_2]^2\Big\{k_\perp^2 - (\beta + x + \xi)\frac{s}{2} + \xi\alpha s\Big\}$$

# Right vertex, left vertex and box diagram

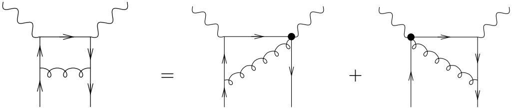
The gauge part of the numerator for the box diagram reads

$$
\begin{aligned}
(\mathrm{Num})_{\mathrm{box}\,gauge} \quad = \quad & -\frac{2}{\beta s}\mathrm{tr}\Big\{\big[\not{k}+(x+\xi)\not{p}_2\big]\not{p}_1\not{p}_2\not{k}\big[\not{k}+(x-\xi)\not{p}_2\big]\gamma_\perp^\sigma\big[\not{k}+\not{p}_1+(x-\xi)\not{p}_2\big]\gamma_{\perp\sigma}\Big\} \\
& -\frac{2}{\beta s}\mathrm{tr}\Big\{\big[\not{k}+(x+\xi)\not{p}_2\big]\not{k}\not{p}_2\not{p}_1\big[\not{k}+(x-\xi)\not{p}_2\big]\gamma_\perp^\sigma\big[\not{k}+\not{p}_1+(x-\xi)\not{p}_2\big]\gamma_{\perp\sigma}\Big\}
\end{aligned}
$$

Using the fact that $p_2^2 = 0$, then one can write $k \to k + (x \pm \xi)p_2$ inside the trace and gets

$$
\begin{aligned}
(\mathrm{Num})_{\mathrm{box}} \quad = \quad & 8\big[k+(x-\xi)p_2\big]^2\left\{\frac{1}{\xi}\left[k_\perp^2-(\beta+x+\xi)\frac{s}{2}+\xi\alpha s\right]+\frac{s}{\beta}(1+\alpha)(\beta+x+\xi)\right\} \\
- \quad & 8\big[k+(x+\xi)p_2\big]^2\left\{\frac{1}{\xi}\left[k_\perp^2-(\beta+x-\xi)\frac{s}{2}\right]-\frac{s}{\beta}(1+\alpha)(\beta+x-\xi)\right\}
\end{aligned}
$$

$\Longrightarrow$ box diagram = right + left vertices:

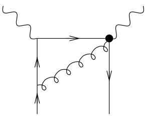


# Right vertex, left vertex and box diagram

Combining right vertex, left vertex and box diagram

$$I_{\text{box}+\text{L.V.}+\text{R.V.}} = I_{\text{E.L.V.}} + I_{\text{E.R.V.}}$$

Effective Left Vertex: $I_{\text{E.L.V.}} =$



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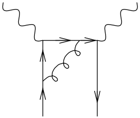
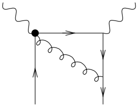
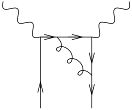


$$I_{\text{E.L.V.}} = -\frac{s}{2}\int d\alpha\, d\beta\, d_2\underline{k}\; 8\left\{\frac{1}{\xi}\left[\underline{k}^2 + (\beta + x + \xi)\frac{s}{2} - \xi\alpha s\right] - \frac{s}{\beta}(1+\alpha)(\beta + x + \xi) + \frac{\underline{k}^2}{\beta}\frac{(\beta + x + \xi)}{(x-\xi)}\right\}$$
$$\times \frac{1}{k^2}\frac{1}{\left[k + (x+\xi)p_2\right]^2}\frac{1}{\left[k + p_1 + (x-\xi)p_2\right]^2}$$

Effective Right Vertex: $I_{\text{E.R.V.}} =$



$+$



$$I_{\text{E.R.V.}} = -\frac{s}{2}\int d\alpha\, d\beta\, d_2\underline{k}\; (-8)\left\{\frac{1}{\xi}\left[\underline{k}^2 + (\beta + x - \xi)\frac{s}{2}\right] + \frac{s}{\beta}(1+\alpha)(\beta + x - \xi) - \frac{\underline{k}^2}{\beta}\frac{(\beta + x - \xi)}{(x-\xi)}\right\}$$
$$\times \frac{1}{k^2}\frac{1}{\left[k + p_1 + (x-\xi)p_2\right]^2}\frac{1}{\left[k + (x-\xi)p_2\right]^2}$$

## Loop integration

$I_{\mathrm{E.L.V.}}$

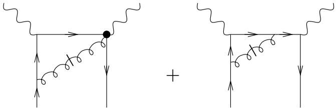
- Write $d^4k = \frac{s}{2}\, d\alpha\, d\beta\, d^2\, k_\perp \qquad (k_\perp^2 = -\underline{k}^2)$
- We use Cauchy integration to integrate over $\alpha$
- There are two contributions :
  - cutting the gluonic line $\to \alpha_g = \frac{\underline{k}^2}{s\beta}$
  - cutting the fermionic line $\to \alpha_f = \frac{\underline{k}^2}{s(\beta+x+\xi)}$
- distribution of the poles in $\alpha$ sets the integration region of $\beta$:

$$I_{\mathrm{E.L.V.}} = -2\pi i \left[ \int_0^{\xi-x} d\beta \int_0^{\infty} d_N\underline{k}\ Res_{\alpha_g} + \int_{-\xi-x}^{\xi-x} d\beta \int_0^{\infty} d_N\underline{k}\ Res_{\alpha_f} \right]$$

- integration over $\underline{k}$ is performed by using dimensional regularization: $N = 2 - \epsilon_{UV} = 2 + \epsilon_{IR}$
- the ultraviolet divergence in $\underline{k}$ integral is taken into account by renormalization
- the IR divergent part is absorbed by the DGLAP-ERBL evolution kernel
- We are only interested in the finite part, which is reminiscent of the IR soft and collinear divergencies

# Loop integration

$I_{\text{E.L.V.}}$: the gluonic pole contribution

$I_{\text{E.L.V., g}} =$ 

The integration over $\underline{k}$ gives

$$I_{\text{E.L.V., g}} = 4\frac{2\pi i}{x-\xi}\int_0^{\xi-x} d\beta \left[\frac{\beta}{\xi(x+\xi)} - \frac{1}{(x+\xi)} + \frac{(\beta+x+\xi)}{(x+\xi)(x-\xi)} - \frac{(\beta+x+\xi)}{2\xi(\beta+x-\xi)}\right]\Gamma(\epsilon_{UV}) \times \left[\frac{s\beta(\beta+x-\xi)}{x-\xi}\right]^{\epsilon_{IR}}$$

- We are only interested in terms that contribute to $\frac{\log^2(\xi-x)}{(x-\xi)}$ terms
- These corresponds to most singular terms, at the limits of $\beta$ integration.
- For $I_{\text{E.L.V.}}$
  - $\frac{1}{\beta}$ terms that are singular at 0
  - $\frac{1}{\beta+x-\xi}$ terms that are singular at $\xi - x$
- There are no $\frac{1}{\beta}$ terms in $I_{\text{E.L.V., g}}$

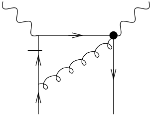
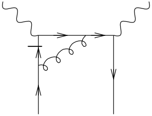
For $\frac{1}{\beta+x-\xi}$ type of singularity, the contribution is

$$I_{\text{E.L.V., g}} = -4\frac{2\pi i}{x-\xi}\frac{1}{2!}\log^2(\xi-x)$$

Actually, this contributions originates from the box diagram term

# Loop integration

$I_{\mathrm{E.L.V.}}$: the fermionic pole contribution

$I_{\mathrm{E.L.V.,\,f}} =$  $+$ 

The integration over $\underline{k}$ gives

$$I_{\mathrm{E.L.V.,\,f}} \;=\; 4\frac{2\pi i}{(x+\xi)2\xi}\int_{-\xi-x}^{\xi-x} d\beta \Big\{(\beta+x+\xi)\Big[\frac{1}{\xi}+\frac{1}{x-\xi}+\frac{(x+\xi)}{(x-\xi)}\frac{1}{(\beta+x-\xi)}\Big]-1\Big\}\Gamma(\epsilon_{UV}) \times\left[\frac{s(\beta+x+\xi)(\beta+x-\xi)}{2\xi}\right]^{\epsilon_{IR}}$$

- There are no $\frac{1}{\beta+x+\xi}$ type of terms!

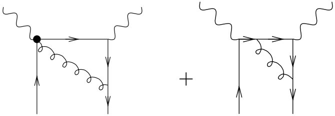
For $\frac{1}{\beta+x-\xi}$ type of singularity, the contribution is

$$I_{\mathrm{E.L.V.,\,f}} = 4\frac{2\pi i}{x-\xi}\frac{1}{2!}\log^2(2\xi)$$

this term is less singular than the term we are looking for

# Loop integration

$I_{\text{E.R.V.}}$

$I_{\text{E.R.V.}} =$ 

- gluonic contribution $\rightarrow \alpha_g = \frac{\underline{k}^2}{s\beta}$  fermionic contribution $\rightarrow \alpha_f = \frac{\underline{k}^2}{s(\beta+x-\xi)}$

$$I_{\text{E.R.V.}} = -2\pi i\left[\int_0^{\xi-x} d\beta \int_0^{\infty} d_N\underline{k}\ Res_{\alpha_g} + \int_{-\xi-x}^{\xi-x} d\beta \int_0^{\infty} d_N\underline{k}\ Res_{\alpha_f}\right]$$

with

$$Res_{\alpha_g} = 4\frac{1}{(x-\xi)^2}\left[\frac{\beta}{\xi}+1-\frac{(\beta+x-\xi)}{x-\xi}-\frac{(x-\xi)}{2\xi}\right]\frac{1}{\underline{k}^2+\frac{\beta(\beta+x+\xi)s}{x-\xi}}+4\frac{1}{(x-\xi)}\left[\frac{1}{2\xi}+\frac{1}{\beta}\right]\frac{1}{\underline{k}^2}$$

$$Res_{\alpha_f} = -4\frac{1}{(x-\xi)}\left\{\left[\frac{1}{\xi(\beta+x-\xi)}+\frac{1}{\beta(\beta+x-\xi)}-\frac{1}{\beta(x-\xi)}\right]+s\left(\frac{1}{2\xi}+\frac{1}{\beta}\right)\frac{1}{\underline{k}^2}\right\}$$

$\Rightarrow$ fermionic contribution vanishes

$\Rightarrow$ no $1/\beta$ or $1/(\beta+x-\xi)$ type of singularity in gluonic contribution

no contribution from $I_{E.R.V.}$

# Full one-loop analysis: summary

The only contribution to $[\log^2(\xi - x)]/x - \xi$ terms comes from the box diagram in the case of cutting the gluonic line around $\beta + x - \xi \approx 0$ in the phase space

The precision of our calculation does not permit us to fix the multiplicative coefficient $a$ of $(\xi - x)$ under logarithm, i.e. our result can be equivalently written as

$$I^{\text{dominant}}_{\text{one loop}} \approx -4\frac{2\pi i}{x-\xi}\frac{1}{2!}\log^2[a(\xi - x)]$$

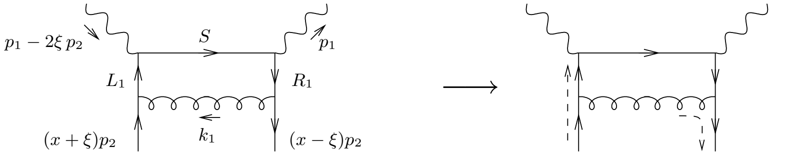
- The coefficient $a$ is fixed to $\frac{1}{2\xi}$ by comparing the $\log^2(\xi - x)$ terms in the exact NLO result.
- The shift $\xi \to \xi - i\epsilon$ correctly takes into account the imaginary part.

our final formula reads:

$$I^{\text{dominant}}_{\text{one loop}} \approx -4\frac{2\pi i}{x-\xi+i\epsilon}\frac{1}{2!}\log^2\left[\frac{\xi - x}{2\xi} - i\epsilon\right]$$

# One-loop in semi-eikonal approximation

Aim : *obtain the same result by using eikonal techniques on the left fermionic line of the box diagram*




dominant momentum flow along $p_2$ direction

The corresponding integral $\rightarrow I_1 = \frac{s}{2} \int d\alpha_1\, d\beta_1\, d_2\underline{k}_1\; (\mathrm{Num})_1 \frac{1}{L_1^2} \frac{1}{S^2} \frac{1}{R_1^2} \frac{1}{k_1^2}$

with $(\mathrm{Num})_1 = \mathrm{tr}\{\; \not{p}_2 \gamma_\mu [\not{k}_1 + (x-\xi)\; \not{p}_2] \theta [\not{k}_1 + (x+\xi)\; \not{p}_2] \gamma_\nu \} d^{\mu\nu}$

and $L_1^2 = [k_1 + (x+\xi)p_2]^2\,, S^2 = [k_1 + p_1 + (x-\xi)p_2]^2\,, R_1^2 = [k_1 + (x-\xi)p_2]^2$

- use eikonal coupling on the left quark line and treat the gluon as soft with respect to this quark $\Rightarrow$ in the quark numerator $L_1$:

$$[k_1 + (x+\xi)p_2] \rightarrow (x+\xi)p_2$$

- gluon is soft w.r.t. $s$-channel fermionic line $\Rightarrow \alpha_1 \ll 1$.

$\theta = \gamma_\perp^\sigma [\not{k}_1 + \not{p}_1 + (x-\xi)\not{p}_2] \gamma_{\sigma\perp} \rightarrow -2\not{p}_1$

# One-loop in semi-eikonal approximation

- The dominant contribution comes from the gluon pole.

on mass shell: $$d^{\mu\nu} = -\sum_\lambda \epsilon^\mu_{(\lambda)}\epsilon^\nu_{(\lambda)}$$

- The numerator becomes

$$(\mathrm{Num})_1 = -2(x+\xi)\sum_\lambda \mathrm{tr}\{\ \not{p}_2\gamma_\mu[\not{k}_1 + (x-\xi)\ \not{p}_2]\ \not{p}_1\ \not{p}_2\ \not{\epsilon}_{(\lambda)}\}(-\epsilon^\mu_{(\lambda)})$$

- Sudakov decomposition of $\epsilon^\mu_{(\lambda)}$ in $p_1$ gauge $\rightarrow$ $\epsilon^\mu_{(\lambda)} = \epsilon^\mu_{\perp(\lambda)} - 2\frac{\epsilon_{\perp(\lambda)}\cdot k_{\perp 1}}{\beta_1 s}p_1^\mu$
- Summing over the polarizations $\rightarrow$ $\sum_\lambda \epsilon_{\perp(\lambda)}\cdot k_{\perp 1}\,\epsilon^\mu_{(\lambda)} = \left(-k^\mu_{\perp 1} + 2\frac{k^2_{\perp 1}}{\beta_1 s}p_1^\mu\right)$

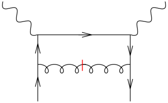
Then $$(\mathrm{Num})_1 = \frac{2(x+\xi)}{\beta_1}\left[\frac{2(x-\xi)}{\beta_1} + 1\right]4\,s\,\underline{k}_1^2$$

left eikonal coupling    right eikonal coupling    non eikonal correction

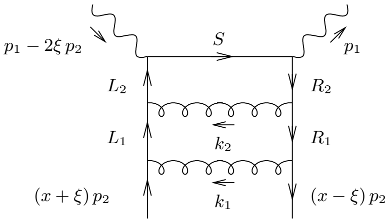
- After Cauchy integration over $\alpha_1$ and considering only the $1/(\beta+x-\xi)$ type of singularities one gets

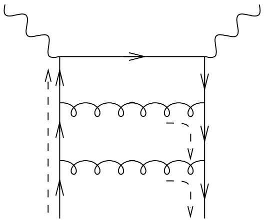
$$I_1 = -4\frac{2\pi i}{x-\xi}\int_0^{\xi-x}d\beta_1\int_0^\infty d_N\underline{k}_1\frac{1}{(\beta_1+x-\xi)}\frac{1}{\underline{k}_1^2-(\beta_1+x-\xi)s}$$

- The integration over $\underline{k}$ and $\beta$ leads to

$$I^{\mathrm{dominant}}_{\mathrm{one\,loop}} =$$  $$= -4\frac{2\pi i}{x-\xi+i\epsilon}\frac{1}{2!}\log^2\left[\frac{\xi-x}{2\xi}-i\epsilon\right]$$

# Two-loop in semi-eikonal approximation



dominant momentum flow along $p_2$ direction

- The $\log^2$ terms we are resumming arise from soft-collinear singularities :
  - Dominance of on-shell gluons contributions
  - Strong ordering in $|\underline{k}_i|$ and $\beta_i$

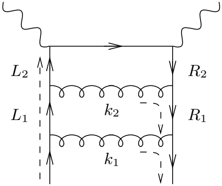
$$|\underline{k}_2| \gg |\underline{k}_1| \quad \text{and} \quad x \sim \xi \gg |\beta_1| \sim |x-\xi| \gg |x-\xi+\beta_1| \sim |\beta_2| \quad \text{and} \quad 1 \gg |\alpha_2| \gg |\alpha_1|$$

- Other diagrams which are not ladder-like or do not respect this strong ordering are suppressed [backup]

# Two-loop in semi-eikonal approximation

The integral for the 2-loop case is

$$I_2 = \left(\frac{s}{2}\right)^2 \int d\alpha_1\, d\beta_1\, d_2\underline{k}_1 \int d\alpha_2\, d\beta_2\, d_2\underline{k}_2 (\mathrm{Num})_2 \frac{1}{L_1^2}\frac{1}{R_1^2}\frac{1}{S^2}\frac{1}{L_2^2}\frac{1}{R_2^2}\frac{1}{\underline{k}_1^2}\frac{1}{\underline{k}_2^2}$$




Using eikonal coupling on the left fermionic line, the numerator is given as

$$(\mathrm{Num})_2 = -4\, s\, \underbrace{\frac{-2\underline{k}_1^2\,(x+\xi)}{\beta_1}\left[1+\frac{2(x-\xi)}{\beta_1}\right]}_{\text{gluon 1}} \underbrace{\frac{-2\underline{k}_2^2\,(x+\xi)}{\beta_2}\left[1+\frac{2(\beta_1+x-\xi)}{\beta_2}\right]}_{\text{gluon 2}}$$

and the propagators

$$\begin{aligned} L_1^2 &= \alpha_1(x+\xi)s \quad , \quad R_1^2 = -\underline{k}_1^2 + \alpha_1(\beta_1+x-\xi)s \quad , \quad S^2 = -\underline{k}_2^2 + (\beta_1+\beta_2+x-\xi)s \\ L_2^2 &= \alpha_2(x+\xi)s \quad , \quad R_2^2 = -\underline{k}_2^2 + \alpha_2(\beta_1+\beta_2+x-\xi)s\,, \end{aligned}$$

After integrating over $\alpha_1$ and $\alpha_2$ and using the properties of dimensional regularization

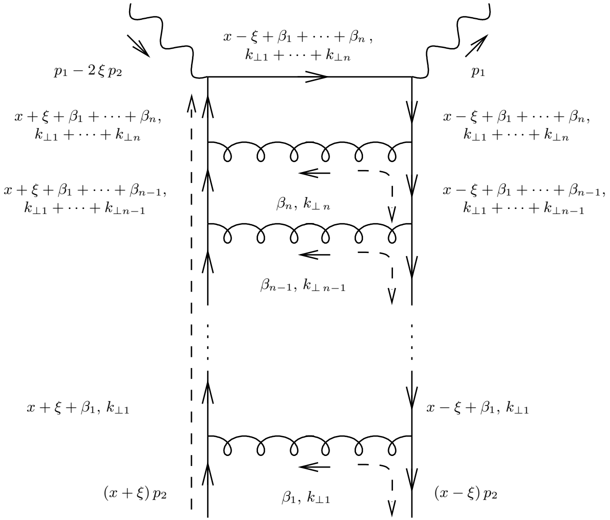
$$\begin{aligned} I_2 &= -4\frac{(2\pi i)^2}{x-\xi}\int_0^{\xi-x} d\beta_1 \int_0^{\xi-x-\beta_1} d\beta_2 \frac{1}{\beta_1+x-\xi}\,\frac{1}{\beta_1+\beta_2+x-\xi} \\ &\qquad \times \int_0^\infty d_N\underline{k}_2 \int_{\underline{k}_2^2}^\infty d_N\underline{k}_1 \frac{1}{\underline{k}_1^2}\,\frac{1}{\underline{k}_2^2-(\beta_1+\beta_2+x-\xi)s} \end{aligned}$$

Integrating over $\beta_i$ and $\underline{k}_i$ and using the matching condition, the final result is

$$I_2^{\text{fin.}} = -4\frac{(2\pi i)^2}{x-\xi+i\epsilon}\,\frac{1}{4!}\,\log^4\left[\frac{\xi-x}{2\xi} - i\epsilon\right]$$

# Computation of the $n$-loop ladder-like diagram

Generalisation of the 1- and 2-loop diagrams




- All gluons are assumed to be on mass shell.
- Strong ordering in $\underline{k}_i$, $\alpha_i$ and $\beta_i$.
- The dominant momentum flows along $p_2$ are indicated

# Computation of the $n$-loop ladder-like diagram

- Strong ordering is given as :

$$|\underline{k}_n| \gg |\underline{k}_{n-1}| \gg \cdots \gg |\underline{k}_1| \quad , \quad 1 \gg |\alpha_n| \gg |\alpha_{n-1}| \gg \cdots \gg |\alpha_1|$$

$$x \sim \xi \gg |\beta_1| \sim |x-\xi| \gg |x-\xi+\beta_1| \sim |\beta_2| \gg \cdots \gg |x-\xi+\beta_1+\beta_2-\cdots+\beta_{n-1}| \sim |\beta_n|$$

- eikonal coupling on the left
- coupling on the right goes beyond eikonal
- Integral for $n$-loop:

$$I_n = \left(\frac{s}{2}\right)^n \int d\alpha_1\, d\beta_1\, d_2\underline{k}_1 \cdots \int d\alpha_n\, d\beta_n\, d_2\underline{k}_n\, (\mathrm{Num})_n \frac{1}{L_1^2} \cdots \frac{1}{L_n^2} \frac{1}{S^2} \frac{1}{R_1^2} \cdots \frac{1}{R_n^2} \frac{1}{\underline{k}_1^2} \cdots \frac{1}{\underline{k}_n^2}$$

- Numerator:

$$(\mathrm{Num})_2 = -4\,s \underbrace{\frac{-2\underline{k}_1^2\,(x+\xi)}{\beta_1}\left[1+\frac{2(x-\xi)}{\beta_1}\right]}_{\text{gluon 1}} \underbrace{\frac{-2\underline{k}_2^2\,(x+\xi)}{\beta_2}\left[1+\frac{2(\beta_1+x-\xi)}{\beta_2}\right]}_{\text{gluon 2}} \cdots \underbrace{\frac{-2\underline{k}_n^2\,(x+\xi)}{\beta_n}\left[1+\frac{2(\beta_{n-1}+\cdots+\beta_1+x-\xi)}{\beta_n}\right]}_{\text{gluon n}}$$

- Propagators:

$$L_1^2 = \alpha_1(x+\xi)s\,, \qquad R_1^2 = -\underline{k}_1^2 + \alpha_1(\beta_1+x-\xi)s\,,$$

$$L_2^2 = \alpha_2(x+\xi)s\,, \qquad R_2^2 = -\underline{k}_2^2 + \alpha_2(\beta_1+\beta_2+x-\xi)s\,,$$

$$\vdots$$

$$L_n^2 = \alpha_n(x+\xi)s\,, \qquad R_n^2 = -\underline{k}_n^2 + \alpha_n(\beta_1+\cdots+\beta_n+x-\xi)s\,,$$

## Computation of the $n$-loop ladder-like diagram

Final step

$$I_n = -4\frac{(2\pi i)^n}{x-\xi}\int_0^{\xi-x} d\beta_1 \cdots \int_0^{\xi-x-\beta_1-\cdots-\beta_{n-1}} d\beta_n \frac{1}{\beta_1+x-\xi}\cdots\frac{1}{\beta_1+\cdots+\beta_n+x-\xi}$$
$$\times \int_0^\infty d_N\underline{k}_n \cdots \int_{\underline{k}_2^2}^\infty d_N\underline{k}_1 \frac{1}{\underline{k}_1^2}\cdots\frac{1}{\underline{k}_{n-1}^2}\frac{1}{\underline{k}_n^2-(\beta_1+\cdots+\beta_n+x-\xi)s}$$

integration over $\underline{k}_i$ and $\beta_i$ leads to our final result :

$$I_n^{\text{fin.}} = -4\frac{(2\pi i)^n}{x-\xi+i\epsilon}\frac{1}{(2n)!}\log^{2n}\left[\frac{\xi-x}{2\xi}-i\epsilon\right]$$

Resummation :

remember that $K_n = -\frac{1}{4}e_q^2\left(-i\,C_F\,\alpha_s\frac{1}{(2\pi)^2}\right)^n I_n$

$$\left(\sum_{n=0}^\infty K_n\right) - (x\to -x) = \frac{e_q^2}{x-\xi+i\epsilon}\cosh\left[D\log\left(\frac{\xi-x}{2\xi}-i\epsilon\right)\right] - (x\to -x)$$

where $D=\sqrt{\frac{\alpha_s C_F}{2\pi}}$

# Resummed formula

Inclusion of our resummed formula into the NLO coefficient function

The inclusion procedure is not unique and it is natural to propose two choices:

- modifying only the Born term and the $\log^2$ part of the $C_1^q$ and keeping the rest of the terms untouched :

$$(T^q)^{\text{res1}} = \left( \frac{e_q^2}{x-\xi+i\epsilon} \left\{ \cosh\left[ D \log\left( \frac{\xi-x}{2\xi} - i\epsilon \right) \right] - \frac{D^2}{2}\left[ 9 + 3\frac{\xi-x}{x+\xi} \log\left( \frac{\xi-x}{2\xi} - i\epsilon \right) \right] \right\} + C_{coll}^q \log \frac{Q^2}{\mu_F^2} \right) - (x \to -x)$$

- the resummation effects are accounted for in a multiplicative way for $C_0^q$ and $C_1^q$ :

$$(T^q)^{\text{res2}} = \left( \frac{e_q^2}{x-\xi+i\epsilon} \cosh\left[ D \log\left( \frac{\xi-x}{2\xi} - i\epsilon \right) \right] \left[ 1 - \frac{D^2}{2}\left\{ 9 + 3\frac{\xi-x}{x+\xi} \log\left( \frac{\xi-x}{2\xi} - i\epsilon \right) \right\} \right] + C_{coll}^q \log \frac{Q^2}{\mu_F^2} \right) - (x \to -x)$$

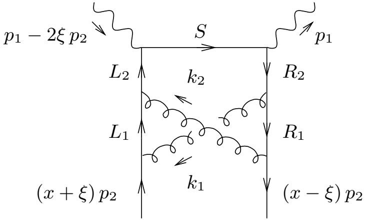
These resummed formulas differ through logarithmic contributions which are beyond the precision of our study.

# Conclusions

- The resummation of soft-collinear gluon radiation effects allowed us to get a close all-order formula that modifies significantly the coefficient function in the specific region $x$ near $\pm\xi$.
- Our analysis can be used for the gluon coefficient function [In progress].
- The measurement of the phenomenological impact of this procedure on the data analysis needs further analysis with the implementation of modeled generalized parton distributions [backup].
- Our analysis could and should be applied to other processes:
  TCS [done], exclusive meson production, form factors... [In progress].
- A formulation of resummation in our exclusive case in terms of (conformal) moments is not yet available. This would generalize analogous resummation of inclusive DIS cross-section which were performed in terms of Mellin moments.
- Our one-loop treatment involves a non-symmetric treatment for gluon emission. This whole result can presumably be obtained based on the Low theorem (known for the Bremsstrahlung in QED) [F. Low 1958 PRD]:
  the classical radiation should be fully extracted from the elastic amplitude (in our case the Born order hand-bag diagram) [In progress]

# Suppressed 2-loop diagrams

## Cross diagram




- The dominant contribution is provided by a strong ordering
  - of transverse momenta
  - of collinear momenta

$$|\underline{k}_2| \gg |\underline{k}_1| \quad \text{and} \quad x \sim \xi \gg |\beta_1| \gg |\beta_2|$$

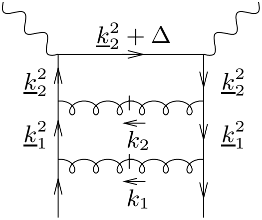
- Within this ordering:

$$I = 4s(2\pi i)^2 \int_0^{\xi - x} d\beta_1 \int_0^{\xi - x - \beta_1} d\beta_2 \int_0^{\infty} d_2\underline{k}_2 \int_0^{\underline{k}_2{}^2} d_2\underline{k}_1 \frac{1}{x-\xi} \frac{1}{\underline{k}_2{}^2 (x-\xi)} \frac{1}{\underline{k}_2{}^2} \frac{1}{\underline{k}_2{}^2 - (\beta_1+\beta_2+x-\xi)s}$$

  - no $\underline{k}_1$ dependence! $\Rightarrow$ one less power of $\log(\xi - x)$
  - this cross diagram does not generate maximal collinear singularity!

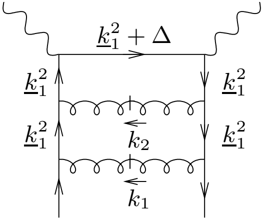
# Suppressed 2-loop diagrams

Ladder diagram with reverse ordering




$|\underline{k}_1| \ll |\underline{k}_2|$

dominates




$|\underline{k}_2| \ll |\underline{k}_1|$

suppressed

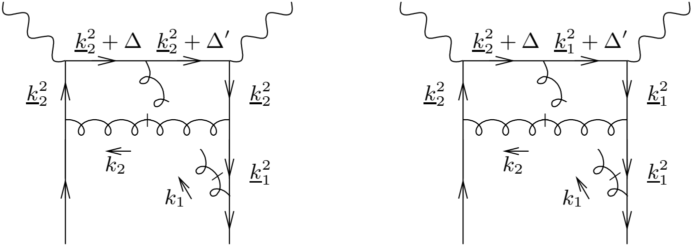
- Left : natural ordering gives $\log^4(\xi - x)$. Maximal number of $\underline{k}_i$ for each $i$
- Right : reverse ordering gives less powers of $\log^4(\xi - x)$. No $\underline{k}_2$!

$\Rightarrow$ Second rule:

*(ii) Each loop should involve a maximal number of collinear singularities, which manifest themselves as maximal powers of $1/\underline{k}_i^2$ for each $i$, after the $\alpha_i$ integration.*

## Suppressed 2-loop diagrams

Diagram with gluon coupled to the $s-$channel quark



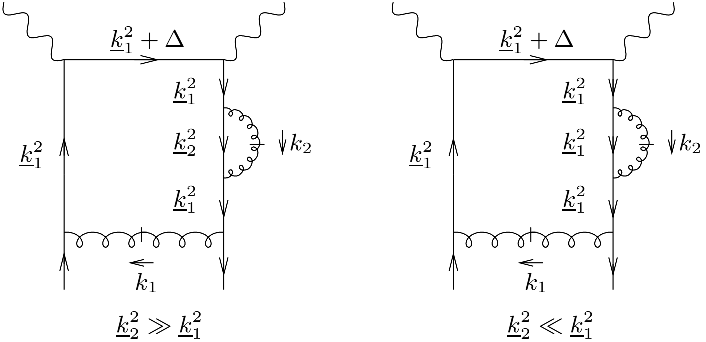

- Left: $\underline{k}_2^2 \gg \underline{k}_1^2$ : the number of collinear singularities originating from $k_1$ is not maximal $\Rightarrow$ violates rule (ii)!
- Right: $\underline{k}_1 \gg \underline{k}_2$ : the virtuality of the upper left fermionic propagator is $\underline{k}_2^2 + \Delta$ where $\Delta = -(x - \xi + \beta_2)s$. This lowers the level of singularity, again leading to a suppressed contribution.

$\Rightarrow$ Third rule :

*(iii) Any coupling of a gluon to the $s-$channel fermionic line leads to a suppressed contribution.*

# Suppressed 2-loop diagrams

Fermion self-energy diagrams




key point : $s$-channel fermion virtuality $= \underline{k}_1^2 + \Delta$, where $\Delta = -(x - \xi + \beta_1)s$ .

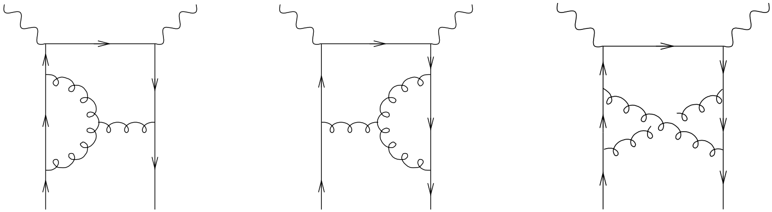
$\Delta$ does not involve $\beta_2$ $\Rightarrow$ reduces the power of $\log(\xi - x)$ after $\beta_2$ integration

$\Rightarrow$ Fourth rule :

*(iv) The diagram should be sufficiently non-local in order that the $s-$channel fermionic line involves the whole $p_2$ flux.*

## Suppressed 2-loop diagrams

Other suppressed diagrams (rule (ii))

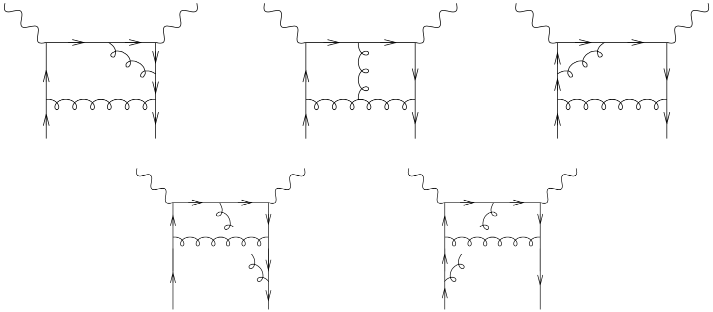


violate the rule:

*(ii) Each loop should involve a maximal number of collinear singularities, which manifest themselves as maximal powers of* $1/\underline{k}_i^2$ *for each* $i$*, after the* $\alpha_i$ *integration.*

## Suppressed 2-loop diagrams

Other suppressed diagrams (rule (iii))

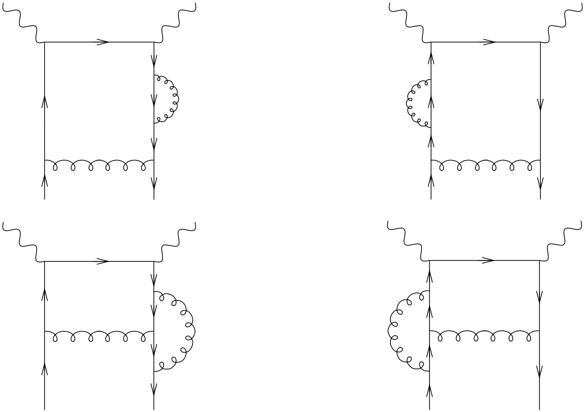


violate the rule:

*(iii) Any coupling of a gluon to the $s-$channel fermionic line leads to a suppressed contribution.*

# Suppressed 2-loop diagrams

Other suppressed diagrams (rule (iv))



violate the rule:

*(iv) The diagram should be sufficiently non-local in order that the $s-$channel fermionic line involves the whole $p_2$ flux.*

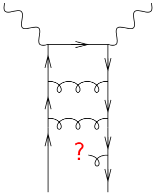
# Beyond the 2-loop level

Dominance of the ladder-like diagrams

The two-loop analysis showed that only ladder-like diagrams give contribution to $\alpha_s^2 \frac{\log^4(\xi - x)}{x - \xi}$ terms.

- Beyond the 2-loop level : recursive argument.
  - at 3-loop level the only missing building block is the four-gluon vertex
  - four-gluon vertex = contraction of two 3-gluon (subleading) diagrams with one less propagator.

    $\Rightarrow$ this kind of diagrams are also subleading
- Dress a 2-loop (or n loop) ladder diagram from the right fermionic line :



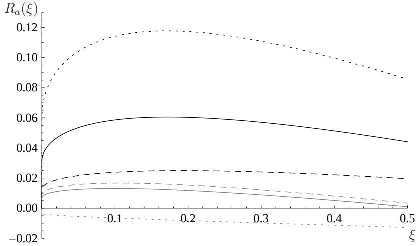

- only abelian-like diagrams are allowed
- can not end on the right fermionic line $\rightarrow$ (local) violates rule (iv)
- can not end on the $s$-channel fermionic line $\rightarrow$ violates rule (iii)
- crossing of any gluon line is not permitted $\rightarrow$ violates rule (ii)

$\Rightarrow$ Only ladder-like diagrams are allowed

# Phenomenological implications

- We use a Double Distribution based model
  S. V. Goloskokov and P. Kroll, Eur. Phys. J. C **50**, 829 (2007)
- Blind integral in the whole $x-$range: amplitude = NLO result $\pm\ 1\%$
- To respect the domain of applicability of our resummation procedure:
  - restrict the use of our formula to $\xi - a\gamma < |x| < \xi + a\gamma$
  - width $a\gamma$ defined through $|D \log(\gamma/(2\xi))| = 1$
  - theoretical uncertainty evaluated by varying $a$
  - a more precise treatment is beyond the leading logarithmic approximation

$$R_a(\xi) = \frac{[\int_{\xi-a\gamma}^{\xi+a\gamma} + \int_{-\xi-a\gamma}^{-\xi+a\gamma}]\, dx (C^{\text{res}} - C_0 - C_1) H(x,\xi,0)}{|\int_{-1}^{1} dx\, (C_0 + C_1) H(x,\xi,0)|}.$$




$Re[R_a(\xi)]$ : black upper curves

$Im[R_a(\xi)]$ : grey lower curves

$a = 1$ (solid)

$a = 1/2$ (dotted)

$a = 2$ (dashed)