

# Theory introduction

Samuel Wallon

Université Pierre et Marie Curie  
and  
Laboratoire de Physique Théorique  
CNRS / Université Paris Sud  
Orsay

20th International Workshop on Photon-Photon Collisions  
PHOTON 2013

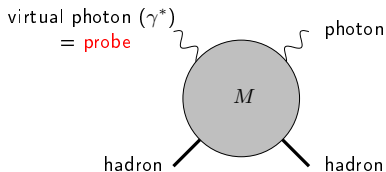
May 22nd 2013

Paris, LPNHE

# Exclusive processes are theoretically challenging

## How to deal with QCD?

example: Compton scattering



- Aim: describe  $M$  by separating:
  - quantities non-calculable perturbatively
    - some tools:
      - Discretization of QCD on a 4-d lattice: numerical simulations
      - AdS/CFT  $\Rightarrow$  AdS/QCD :  $AdS_5 \times S^5 \leftrightarrow$  QCD
        - Polchinski, Strassler '01
        - for some issues related to Deep Inelastic Scattering (DIS):
          - B. Pire, L. Szymanowski, C. Roiesnel, S. W. Phys.Lett.B670 (2008) 84-90
        - for some issues related to Deep Virtual Compton Scattering (DVCS):
          - J.-H. Gao and B.-W. Xiao '10; C. Marquet, C. Roiesnel, S. W. JHEP 1004:051 (2010) 1-26
    - perturbatively calculable quantities
  - We will here focus on theory and phenomenology of exclusive processes for which the dynamics is governed by QCD in the perturbative regime

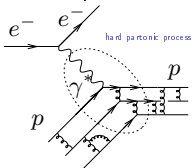
# Exclusive processes are phenomenologically challenging

Key question of QCD:

how to obtain and understand the tri-dimensional structure of hadrons  
in terms of quarks and gluons?

Can this be achieved using **hard** exclusive processes?

- The aim is to reduce the process to interactions involving a small number of *partons* (quarks, gluons), despite confinement
- This is possible if the considered process is driven by short distance phenomena ( $d \ll 1 \text{ fm}$ )  
 $\implies \alpha_s \ll 1$  : **Perturbative methods**
- One should hit strongly enough a hadron  
Example: electromagnetic probe and form factor



$\tau$  electromagnetic interaction  $\sim \tau$  parton life time after interaction  
 $\ll \tau$  characteristic time of strong interaction

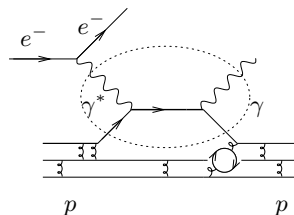
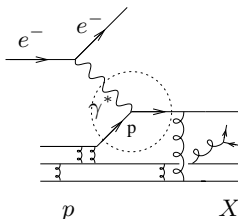
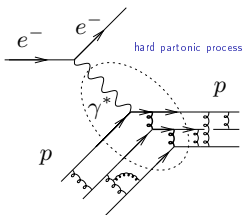
To get such situations in exclusive reactions is very challenging phenomenologically: **the cross sections are very small**

## Introduction

## Hard processes in QCD

## Hard processes in QCD

- This is justified if the process is governed by a **hard scale**:
  - **virtuality of the electromagnetic probe**
    - in elastic scattering  $e^\pm p \rightarrow e^\pm p$
    - in Deep Inelastic Scattering (DIS)  $e^\pm p \rightarrow e^\pm X$
    - in Deep Virtual Compton Scattering (DVCS)  $e^\pm p \rightarrow e^\pm p \gamma$
  - **Total center of mass energy** in  $e^+e^- \rightarrow X$  annihilation
  - **$t$ -channel momentum exchange** in meson photoproduction  $\gamma p \rightarrow M p$
- A precise treatment relies on **factorization theorems**
- The scattering amplitude is described by the **convolution** of the partonic amplitude with the non-perturbative hadronic content



# Introduction

## Counting rules and limitations

### The partonic point of view... and its limitations

- Counting rules:

$$F_n(q^2) \simeq \frac{C}{(Q^2)^{n-1}} \quad n = \text{number of minimal constituents: } \begin{cases} \text{meson: } n = 2 \\ \text{baryon: } n = 3 \end{cases}$$

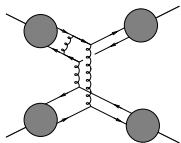
Brodsky, Farrar '73

- Large angle** (i.e.  $s \sim t \sim u$  large) elastic processes  $h_a h_b \rightarrow h_a h_b$   
e.g. :  $\pi\pi \rightarrow \pi\pi$  or  $pp \rightarrow pp$

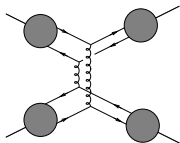
$$\frac{d\sigma}{dt} \sim \left( \frac{\alpha_S(p_\perp^2)}{s} \right)^{n-2} \quad n = \# \text{ of external fermionic lines } (n = 8 \text{ for } \pi\pi \rightarrow \pi\pi)$$

Brodsky, Lepage '81

Other contributions might be significant, even at large angle: e.g.  $\pi\pi \rightarrow \pi\pi$



Brodsky Lepage mechanism:  $\frac{d\sigma_{BL}}{dt} \sim \left( \frac{1}{s} \right)^6$



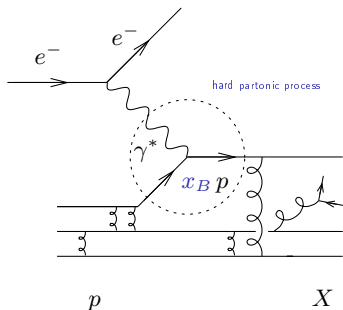
Landshoff '74 mechanism:  $\frac{d\sigma_L}{dt} \sim \left( \frac{1}{s} \right)^5$

absent with at least one  $\gamma^{(*)}$  (point-like coupling)<sub>5 / 46</sub>

## Accessing the perturbative proton content using inclusive processes

no  $1/Q$  suppression

example: DIS



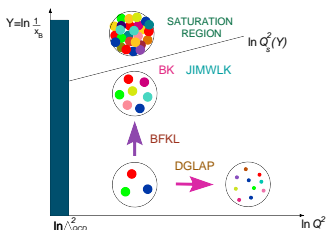
$$s_{\gamma^* p} = (q_\gamma^* + p_p)^2 = 4 E_{\text{c.m.}}^2$$

$$Q^2 \equiv -q_{\gamma^*}^2 > 0$$

$$x_B = \frac{Q^2}{2 p_p \cdot q_\gamma^*} \simeq \frac{Q^2}{s_{\gamma^* p}}$$

- $x_B$  = proton momentum fraction carried by the scattered quark
- $1/Q$  = transverse resolution of the photonic probe  $\ll 1/\Lambda_{\text{QCD}}$

## The various regimes governing the perturbative content of the proton



- “usual” regime:  $x_B$  moderate ( $x_B \gtrsim .01$ ):  
Evolution in  $Q$  governed by the QCD renormalization group  
(Dokshitzer, Gribov, Lipatov, Altarelli, Parisi equation)

$$\sum_n (\alpha_s \ln Q^2)^n \quad \text{LLQ} \quad + \quad \alpha_s \sum_n (\alpha_s \ln Q^2)^n \quad \text{NLLQ} \quad + \dots$$

- perturbative Regge limit:  $s_{\gamma^*p} \rightarrow \infty$  i.e.  $x_B \sim Q^2/s_{\gamma^*p} \rightarrow 0$   
in the perturbative regime (hard scale  $Q^2$ )  
(Balitski Fadin Kuraev Lipatov equation)

$$\sum_n (\alpha_s \ln s)^n \quad \text{LLs} \quad + \quad \alpha_s \sum_n (\alpha_s \ln s)^n \quad \text{NLLs} \quad + \dots$$

# From inclusive to exclusive processes

## Experimental effort

- Inclusive processes are not  $1/Q$  suppressed (e.g. DIS);  
Exclusive processes **are suppressed**
- Going from inclusive to exclusive processes is **difficult**
- High luminosity accelerators and high-performance detection facilities  
HERA (H1, ZEUS), HERMES, JLab@6 GeV (Hall A, CLAS), BaBar, Belle, BEPC-II (BES-III)  
future: LHC, COMPASS-II, JLab@12 GeV, LHeC, EIC, ILC
- What to do, and where?
  - Proton form factor: JLab@6 GeV future: PANDA (timelike proton form factor through  $p\bar{p} \rightarrow e^+e^-$ )
  - $e^+e^-$  in  $\gamma^*\gamma$  single-tagged channel: Transition form factor  $\gamma^*\gamma \rightarrow \pi$ , exotic hybrid meson production BaBar, Belle, BES,...
  - Deep Virtual Compton Scattering (GPD)  
HERA (H1, ZEUS), HERMES, JLab@6 GeV  
future: JLab@12GeV, COMPASS-II, EIC, LHeC
  - Non exotic and exotic hybrid meson electroproduction (GPD and DA), etc...  
NMC (CERN), E665 (Fermilab), HERA (H1, ZEUS), COMPASS, HERMES, CLAS (JLab)
  - TDA (PANDA at GSI)
  - TMDs (BaBar, Belle, COMPASS, ...)
  - Diffractive processes, including ultraperipheral collisions  
LHC (with or without fixed targets), ILC, LHeC



# From inclusive to exclusive processes

## Theoretical efforts

Very important theoretical developments during the last decade

- Key words:

DAs, GPDs, GDAs, TDAs ... TMDs

- Fundamental tools:

- At medium energies:

JLab, HERMES, COMPASS, BaBar, Belle, PANDA, EIC

collinear factorization

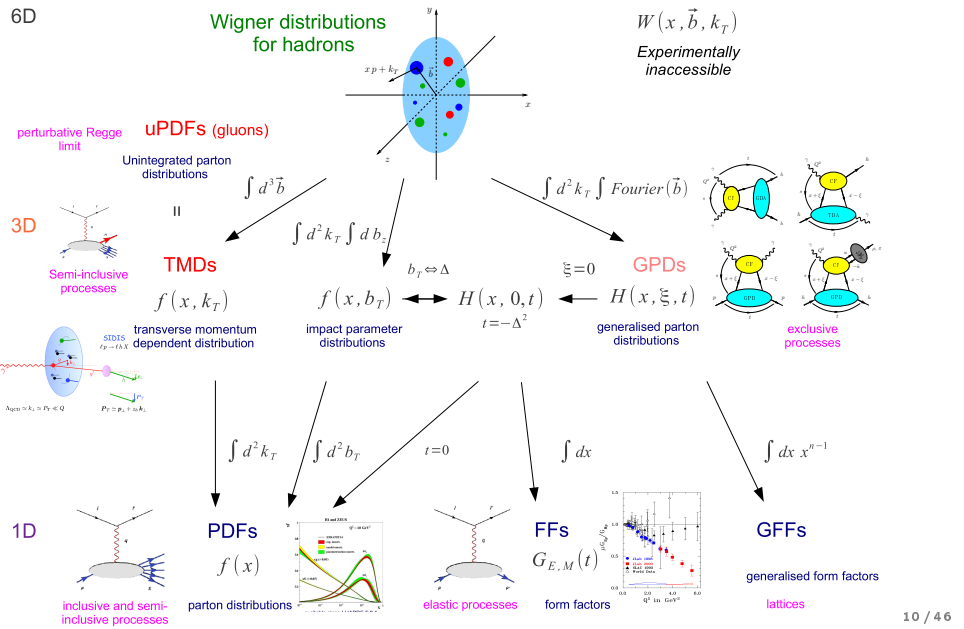
- At asymptotical energies:

HERA, Tevatron, LHC, LHeC, ILC (EIC and COMPASS at the boundary)

$k_T$ -factorization

We will now explain and illustrate these concepts, and discuss issues and possible solutions...

# The ultimate picture



## Extensions from DIS

- **DIS**: inclusive process  $\rightarrow$  forward amplitude ( $t = 0$ ) (optical theorem)

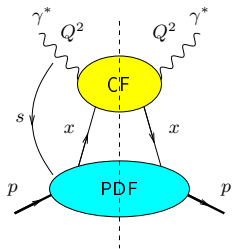
(DIS: Deep Inelastic Scattering)

ex:  $e^\pm p \rightarrow e^\pm X$  at HERA

$x \Rightarrow$  1-dimensional structure

Structure Function

$$= \text{Coefficient Function (hard)} \otimes \text{Parton Distribution Function (soft)}$$



- **DVCS**: exclusive process  $\rightarrow$  non forward amplitude ( $-t \ll s = W^2$ )

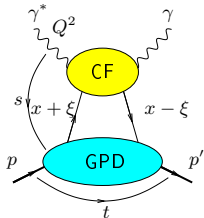
(DVCS: Deep Virtual Compton Scattering)

Fourier transf.:  $t \leftrightarrow$  impact parameter

$(x, t) \Rightarrow$  3-dimensional structure

Amplitude

$$= \text{Coefficient Function (hard)} \otimes \text{Generalized Parton Distribution (soft)}$$



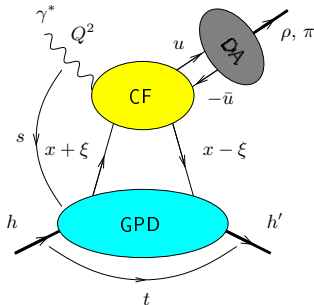
Müller et al. '91 - '94; Radyushkin '96; Ji '97

## Extensions from DVCS

- **Meson production:**  $\gamma$  replaced by  $\rho, \pi, \dots$

Amplitude

$$= \text{GPD (soft)} \otimes \text{CF (hard)} \otimes \text{Distribution Amplitude (soft)}$$

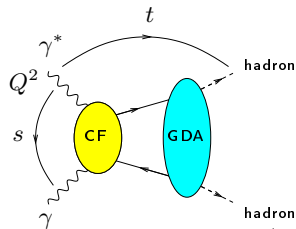


Collins, Frankfurt, Strikman '97; Radyushkin '97

- Crossed process:  $s \ll -t$

Amplitude

$$= \text{Coefficient Function (hard)} \otimes \text{Generalized Distribution Amplitude (soft)}$$



Diehl, Gousset, Pire, Teryaev '98

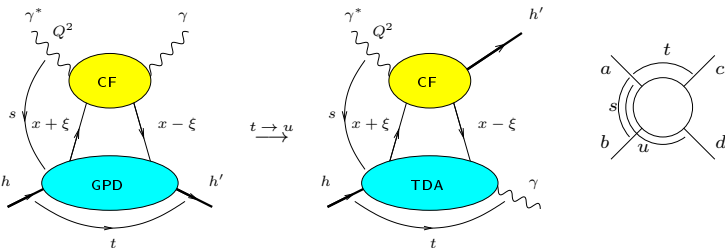
# Extensions from DVCS

- Starting from usual DVCS, one allows: initial hadron  $\neq$  final hadron (in the same octuplet): transition GPDs

Even less diagonal:

baryonic number (initial state)  $\neq$  baryonic number (final state)  $\rightarrow$  TDA

Example:



Pire, Szymanowski '05

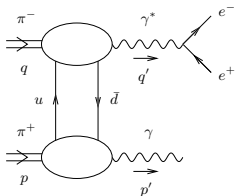
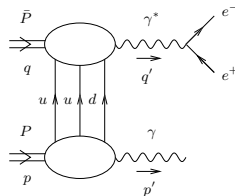
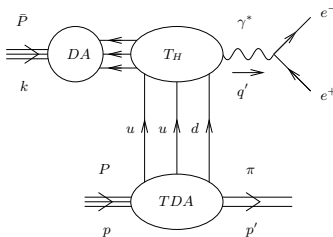
which can be further extended by replacing the outgoing  $\gamma$  by any hadronic state

$$\text{Amplitude} = \text{Transition Distribution Amplitude (soft)} \otimes \text{CF (hard)} \otimes \text{DA (soft)}$$

Lansberg, Pire, Szymanowski '06

## Extensions from DVCS

## TDA at PANDA

TDA  $\pi \rightarrow \gamma$ TDA  $p \rightarrow \gamma$  at PANDA (forward scattering of  $\bar{p}$  on a  $p$  probe)TDA  $p \rightarrow \pi$  at PANDA (forward scattering of  $\bar{p}$  on a  $p$  probe)Spectral model for the  $p \rightarrow \pi$  TDA: [Pire, Semenov, Szymanowski '10](#)

# Collinear factorization

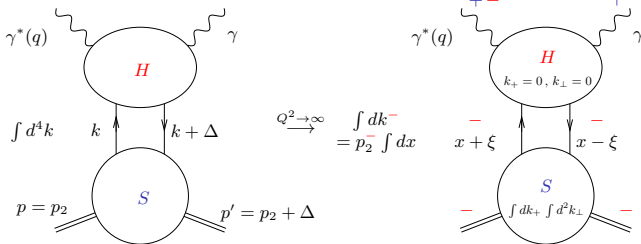
A bit more technical: DVCS and GPDs

## Two steps for factorization

- momentum factorization: light-cone vector dominance for  $Q^2 \rightarrow \infty$

$$p_1, p_2 : \text{the two light-cone directions} \quad \begin{cases} p_1 = \frac{\sqrt{s}}{2}(1, 0_\perp, 1) & p_1^2 = p_2^2 = 0 \\ p_2 = \frac{\sqrt{s}}{2}(1, 0_\perp, -1) & 2p_1 \cdot p_2 = s \sim s_{\gamma^* p} \gtrsim Q^2 \end{cases}$$

$$\text{Sudakov decomposition: } k = \alpha p_1 + \beta p_2 + k_\perp \quad \begin{matrix} a \cdot b = \\ a_+ b_- + a_- b_+ + a_\perp \cdot b_\perp \end{matrix}$$



$$\int d^4 k S(k, k + \Delta) H(q, k, k + \Delta) = \int dk^- \int dk^+ d^2 k_\perp S(k, k + \Delta) H(q, k^-, k^- + \Delta^-)$$

- Quantum numbers factorization (Fierz identity: spinors + color)

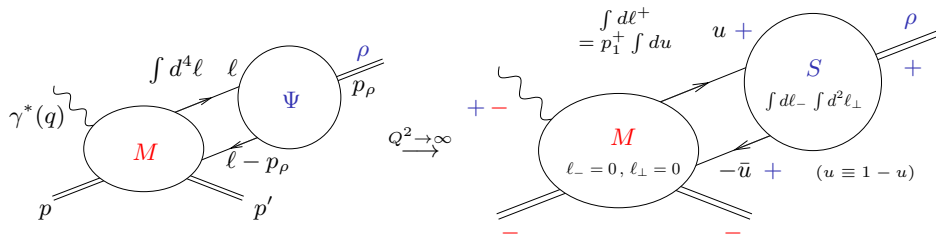
$$\Rightarrow \mathcal{M} = \text{GPD} \otimes \text{Hard part}$$

# Collinear factorization

$\rho$ -meson production: from the wave function to the DA

What is a  $\rho$ -meson in QCD?

It is described by its **wave function**  $\Psi$  which reduces in **hard processes** to its **Distribution Amplitude**



$$\int d^4 l M(q, l, l - p_\rho) \Psi(l, l - p_\rho) = \int dl^+ M(q, l^+, l^+ - p_\rho^+) \int dl^- \int_{|\ell_\perp^2| < \mu_F^2} d^2 l_\perp \Psi(l, l - p_\rho)$$

Hard part

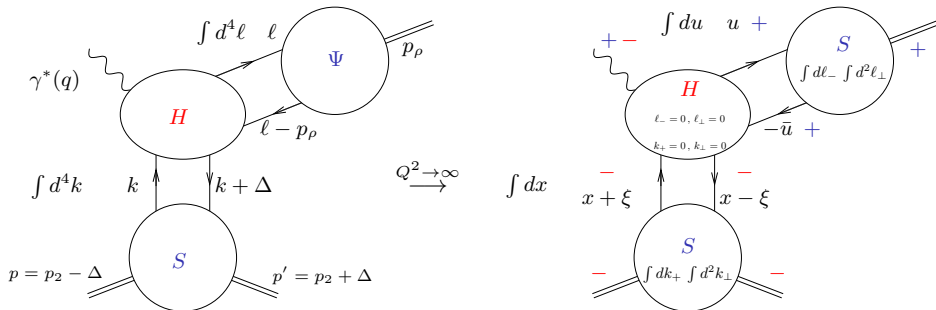
DA  $\Phi(u, \mu_F^2)$

(see Chernyak, Zhitnitsky '77; Brodsky, Lepage '79; Efremov, Radyushkin '80; ... in the case of form-factors studies)



## Collinear factorization

Meson electroproduction: factorization with a GPD and a DA



$$\int d^4 k d^4 \ell \quad S(k, k + \Delta) \quad H(q, k, k + \Delta) \quad \Psi(\ell, \ell - p_\rho)$$

$$= \int dk^- d\ell^+ \int dk^+ \int_{|k_\perp^2| < \mu_{F_2}^2} d^2 k_\perp S(k, k + \Delta) H(q; k^-, k^- + \Delta^-; \ell^+, \ell^+ - p_\rho^+) \int d\ell^- \int_{|\ell_\perp^2| < \mu_{F_1}^2} d^2 \ell_\perp \Psi(\ell, \ell - p_\rho)$$

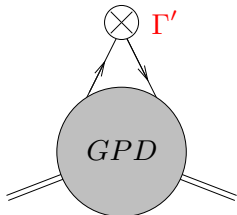
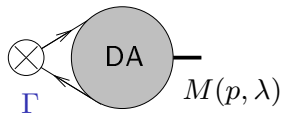
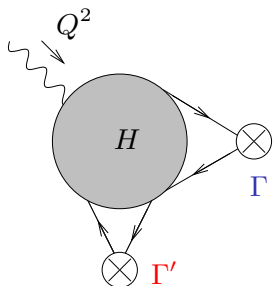
$$\text{GPD } F(x, \xi, t, \mu_{F_2}^2) \quad \text{Hard part } T(x/\xi, u, \mu_{F_1}^2, \mu_{F_2}^2, \mu_R^2) \quad \text{DA } \Phi(u, \mu_{F_1}^2)$$

Collins, Frankfurt, Strikman '97; Radyushkin '97

# Collinear factorization

Meson electroproduction: factorization with a GPD and a DA

## The building blocks



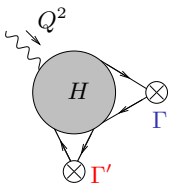
$\Gamma, \Gamma'$  : Dirac matrices compatible  
with quantum numbers:  $C, P, T$ , chirality

Similar structure for gluon exchange

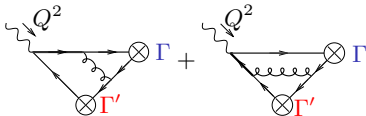
# Collinear factorization

Meson electroproduction: factorization with a GPD and a DA

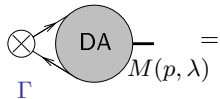
## The building blocks



=



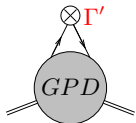
hand-bag diagrams



=

$$\langle M(p, \lambda) | \mathcal{O}(\Psi, \bar{\Psi} A) | 0 \rangle$$

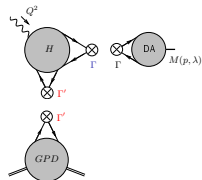
matrix element of a **non-local light-cone**  
operator



=

$$\langle N(p') | \mathcal{O}'(\Psi, \bar{\Psi} A) | N(p) \rangle$$

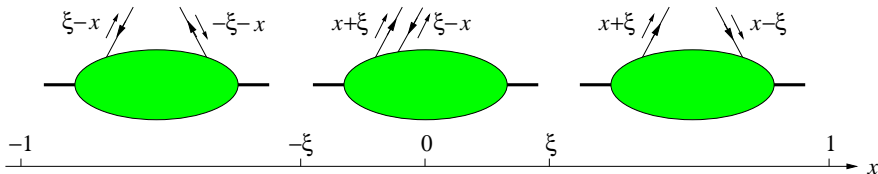
matrix element of a **non-local light-cone**  
operator



## Collinear factorization

## Twist 2 GPDs

## Physical interpretation for GPDs



Emission and reabsorption  
of an antiquark  
~ PDFs for antiquarks  
DGLAP-II region

Emission of a quark and  
emission of an antiquark  
~ meson exchange  
ERBL region

Emission and reabsorption  
of a quark  
~ PDFs for quarks  
DGLAP-I region

## Collinear factorization

## Twist 2 GPDs

## Classification of twist 2 GPDs

- For quarks, one should distinguish the exchanges
  - without helicity flip (chiral-even  $\Gamma'$  matrices): 4 chiral-even GPDs:

$H^q \xrightarrow{\xi=0, t=0}$  PDF  $q$ ,  $E^q$ ,  $\tilde{H}^q \xrightarrow{\xi=0, t=0}$  polarized PDFs  $\Delta q$ ,  $\tilde{E}^q$

$$\begin{aligned}
 F^q &= \frac{1}{2} \int \frac{dz^+}{2\pi} e^{ixP^-z^+} \langle p' | \bar{q}(-\frac{1}{2}z) \gamma^- q(\frac{1}{2}z) | p \rangle \Big|_{z^-=0, z_\perp=0} \\
 &= \frac{1}{2P^-} \left[ H^q(x, \xi, t) \bar{u}(p') \gamma^- u(p) + E^q(x, \xi, t) \bar{u}(p') \frac{i\sigma^{-\alpha} \Delta_\alpha}{2m} u(p) \right],
 \end{aligned}$$

$$\begin{aligned}
 \tilde{F}^q &= \frac{1}{2} \int \frac{dz^+}{2\pi} e^{ixP^-z^+} \langle p' | \bar{q}(-\frac{1}{2}z) \gamma^- \gamma_5 q(\frac{1}{2}z) | p \rangle \Big|_{z^-=0, z_\perp=0} \\
 &= \frac{1}{2P^-} \left[ \tilde{H}^q(x, \xi, t) \bar{u}(p') \gamma^- \gamma_5 u(p) + \tilde{E}^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^-}{2m} u(p) \right].
 \end{aligned}$$

- with helicity flip (chiral-odd  $\Gamma'$  mat.): 4 chiral-odd GPDs:

$H_T^q \xrightarrow{\xi=0, t=0}$  quark transversity PDFs  $\Delta_T q$ ,  $E_T^q$ ,  $\tilde{H}_T^q$ ,  $\tilde{E}_T^q$

$$\begin{aligned}
 &\frac{1}{2} \int \frac{dz^+}{2\pi} e^{ixP^-z^+} \langle p' | \bar{q}(-\frac{1}{2}z) i\sigma^{-i} q(\frac{1}{2}z) | p \rangle \Big|_{z^-=0, z_\perp=0} \\
 &= \frac{1}{2P^-} \bar{u}(p') \left[ H_T^q i\sigma^{-i} + \tilde{H}_T^q \frac{P^- \Delta^i - \Delta^- P^i}{m^2} + E_T^q \frac{\gamma^- \Delta^i - \Delta^- \gamma^i}{2m} + \tilde{E}_T^q \frac{\gamma^- P^i - P^- \gamma^i}{m} \right]
 \end{aligned}$$

# Collinear factorization

## Twist 2 GPDs

### Classification of twist 2 GPDs

- analogously, for gluons:
  - 4 gluonic GPDs without helicity flip:

$$H^g \xrightarrow{\xi=0, t=0} \text{PDF } x g$$

$$E^g$$

$$\tilde{H}^g \xrightarrow{\xi=0, t=0} \text{polarized PDF } x \Delta g$$

$$\tilde{E}^g$$

- 4 gluonic GPDs with helicity flip:

$$H_T^g$$

$$E_T^g$$

$$\tilde{H}_T^g$$

$$\tilde{E}_T^g$$

(no forward limit reducing to gluons PDFs here: a change of 2 units of helicity cannot be compensated by a spin 1/2 target)

## A few applications

Production of an exotic hybrid meson in hard processes

### Quark model and meson spectroscopy

- spectroscopy:  $\vec{J} = \vec{L} + \vec{S}$ ; neglecting any spin-orbital interaction  
 $\Rightarrow S, L =$  additional quantum numbers to classify hadron states

$$\vec{J}^2 = J(J+1), \quad \vec{S}^2 = S(S+1), \quad \vec{L}^2 = L(L+1),$$

with  $J = |L - S|, \dots, L + S$

- In the usual quark-model: meson =  $q\bar{q}$  bound state with

$$C = (-)^{L+S} \quad \text{and} \quad P = (-)^{L+1}.$$

- Thus:

$$S = 0, \quad L = J, \quad J = 0, 1, 2, \dots : \quad J^{PC} = 0^{-+}(\pi, \eta), 1^{+-}(h_1, b_1), 2^{-+}, 3^{+-}, \dots$$

$$S = 1, \quad L = 0, \quad J = 1 : \quad J^{PC} = 1^{--}(\rho, \omega, \phi)$$

$$L = 1, \quad J = 0, 1, 2 : \quad J^{PC} = 0^{++}(f_0, a_0), 1^{++}(f_1, a_1), 2^{++}(f_2, a_2)$$

$$L = 2, \quad J = 1, 2, 3 : \quad J^{PC} = 1^{--}, 2^{--}, 3^{--}$$

...

- $\Rightarrow$  the exotic mesons with  $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, \dots$  are forbidden

# A few applications

Production of an exotic hybrid meson in hard processes

## Experimental candidates for light hybrid mesons (1)

three candidates:

- $\pi_1(1400)$ 
  - GAMS '88 (SPS, CERN): in  $\pi^- p \rightarrow \eta \pi^0 n$  (through  $\eta \pi^0 \rightarrow 4\gamma$  mode)  
 $M = 1406 \pm 20 \text{ MeV}$     $\Gamma = 180 \pm 30 \text{ MeV}$
  - E852 '97 (BNL):  $\pi^- p \rightarrow \eta \pi^- p$   
 $M = 1370 \pm 16 \text{ MeV}$     $\Gamma = 385 \pm 40 \text{ MeV}$
  - VES '01 (Protvino) in  $\pi^- Be \rightarrow \eta \pi^- Be$ ,  $\pi^- Be \rightarrow \eta' \pi^- Be$ ,  
 $\pi^- Be \rightarrow b_1 \pi^- Be$   
 $M = 1316 \pm 12 \text{ MeV}$     $\Gamma = 287 \pm 25 \text{ MeV}$   
 but resonance hypothesis ambiguous
  - Crystal Barrel (LEAR, CERN) '98 '99 in  $\bar{p} n \rightarrow \pi^- \pi^0 \eta$  and  $\bar{p} p \rightarrow 2\pi^0 \eta$   
 (through  $\pi\eta$  resonance)  
 $M = 1400 \pm 20 \text{ MeV}$     $\Gamma = 310 \pm 50 \text{ MeV}$   
 and  $M = 1360 \pm 25 \text{ MeV}$     $\Gamma = 220 \pm 90 \text{ MeV}$



# A few applications

Production of an exotic hybrid meson in hard processes

## Experimental candidates for light hybrid mesons (2)

- $\pi_1(1600)$ 
  - **E852 (BNL)**: in peripheral  $\pi^- p \rightarrow \pi^+ \pi^- \pi^- p$  (through  $\rho\pi^-$  mode) '98 '02,  $M = 1593 \pm 8$  MeV  $\Gamma = 168 \pm 20$  MeV  $\pi^- p \rightarrow \pi^+ \pi^- \pi^- \pi^0 \pi^0 p$  (in  $b_1(1235)\pi^- \rightarrow (\omega\pi^0)\pi^- \rightarrow (\pi^+ \pi^- \pi^0)\pi^0 \pi^-$  '05 and  $f_1(1285)\pi^-$  '04 modes), in peripheral  $\pi^- p$  through  $\eta'\pi^-$  '01  
 $M = 1597 \pm 10$  MeV  $\Gamma = 340 \pm 40$  MeV  
 but **E852 (BNL)** '06: no exotic signal in  $\pi^- p \rightarrow (3\pi)^- p$  for a larger sample of data!
  - **VES '00 (Protvino)**: in peripheral  $\pi^- p$  through  $\eta'\pi^-$  '93, '00,  $\rho(\pi^+ \pi^-)\pi^-$  '00,  $b_1(1235)\pi^- \rightarrow (\omega\pi^0)\pi^-$  '00
  - **Crystal Barrel (LEAR, CERN)** '03  $\bar{p}p \rightarrow b_1(1235)\pi\pi$
  - **COMPASS '10 (SPS, CERN)**: diffractive dissociation of  $\pi^-$  on  $Pb$  target through Primakov effect  $\pi^- \gamma \rightarrow \pi^- \pi^- \pi^+$  (through  $\rho\pi^-$  mode)  
 $M = 1660 \pm 10$  MeV  $\Gamma = 269 \pm 21$  MeV
- $\pi_1(2000)$ : seen only at **E852 (BNL)** '04 '05 (through  $f_1(1285)\pi^-$  and  $b_1(1235)\pi^-$ )

## A few applications

Production of an exotic hybrid meson in hard processes

### What about hard processes?

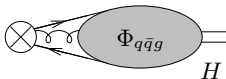
- Is there a hope to see such states in hard processes, with high counting rates, and to exhibit their light-cone wave-function?
- hybrid mesons =  $q\bar{q}g$  states    T. Barnes '77; R. L. Jaffe, K. Johnson, and Z. Ryzak, G. S. Bali
- popular belief:  $H = q\bar{q}g \Rightarrow$  higher Fock-state component  $\Rightarrow$  twist-3  $\Rightarrow$  hard electroproduction of  $H$  versus  $\rho$  suppressed as  $1/Q$
- This is not true!! Electroproduction of hybrid is similar to electroproduction of usual  $\rho$ -meson: it is twist 2 dominated  
I. V. Anikin, B. Pire, O. V. Teryaev, L. Szymanowski, S.W. '04

## A few applications

Production of an exotic hybrid meson in hard processes

### Distribution amplitude of exotic hybrid mesons at twist 2

- One may think that to produce  $|q\bar{q}g\rangle$ , the fields  $\Psi$ ,  $\bar{\Psi}$ ,  $A$  should appear explicitly in the non-local operator  $\mathcal{O}(\Psi, \bar{\Psi} A)$



- If one tries to produce  $H = 1^{-+}$  from a local operator, the dominant operator should be  $\bar{\Psi}\gamma^\mu G_{\mu\nu}\Psi$  of twist = dimension - spin = 5 - 1 = 4
- It means that there should be a  $1/Q^2$  suppression in the production amplitude of  $H$  versus the usual  $\rho$ -production (which is twist 2 dominated)
- But collinear approach describes hard exclusive processes in terms of non-local light-cone operators, among which are the twist 2 operator

$$\bar{\psi}(-z/2)\gamma_\mu[-z/2; z/2]\psi(z/2)$$

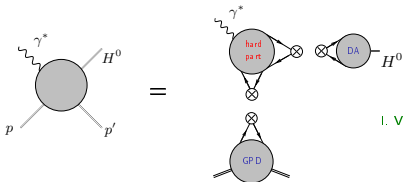
where  $[-z/2; z/2]$  is a Wilson line, necessary to fulfill gauge invariance (i.e. a "color tube" between  $q$  and  $\bar{q}$ ) which thus hides gluonic degrees of freedom: the needed gluon is there, at twist 2. This does not require to introduce explicitly  $A$ !

# A few applications

Production of an exotic hybrid meson in hard processes

## Accessing the partonic structure of exotic hybrid mesons

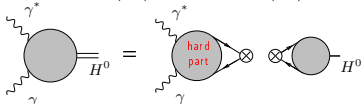
- Electroproduction  $\gamma^* p \rightarrow H^0 p$ : JLab, COMPASS, EIC



$$\text{prediction: } \frac{d\sigma^H}{d\sigma^P} \approx 15\%$$

I. V. Anikin, B. Pire, O. V. Teryaev, L. Szymanowski, S.W.  
 Phys.Rev.D70 (2004) 011501  
 Phys.Rev.D71 (2005) 034021  
 Eur.Phys.J.C42 (2005) 163

- Channels  $\gamma^* \gamma \rightarrow H$  and  $\gamma^* \gamma \rightarrow \pi \eta$ : BaBar, Belle, BES-III



$$\text{prediction: } \frac{|M^{\gamma^* \gamma \rightarrow H}|^2}{|M^{\gamma^* \gamma \rightarrow \pi^0}|^2} \approx 20\%$$

I. V. Anikin, B. Pire, O. V. Teryaev, L. Szymanowski, S.W.  
 Eur.Phys.J.C47 (2006)

[backup]

⇒ the partonic content of exotic hybrid meson is experimentally accessible

# A few applications

## Spin transversity in the nucleon

### What is transversity?

- Transverse spin content of the proton:

$$\begin{array}{lcl} |\uparrow\rangle_{(x)} & \sim & |\rightarrow\rangle + |\leftarrow\rangle \\ |\downarrow\rangle_{(x)} & \sim & |\rightarrow\rangle - |\leftarrow\rangle \\ \text{spin along } x & & \text{helicity state} \end{array}$$

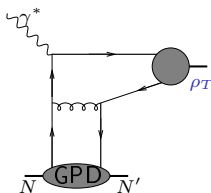
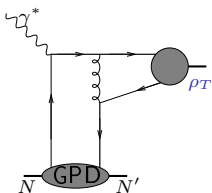
- An observable sensitive to helicity spin flip gives thus access to the transversity  $\Delta_T q(x)$ , which is very badly known (first data have recently been obtained by COMPASS)
- The transversity GPDs are completely unknown
- **Chirality:**  $q_{\pm}(z) \equiv \frac{1}{2}(1 \pm \gamma^5)q(z)$  with  $q(z) = q_+(z) + q_-(z)$   
 Chiral-even: chirality conserving  
 $\bar{q}_{\pm}(z)\gamma^{\mu}q_{\pm}(-z)$  and  $\bar{q}_{\pm}(z)\gamma^{\mu}\gamma^5q_{\pm}(-z)$   
 Chiral-odd: chirality reversing  
 $\bar{q}_{\pm}(z) \cdot 1 \cdot q_{\mp}(-z)$ ,  $\bar{q}_{\pm}(z) \cdot \gamma^5 \cdot q_{\mp}(-z)$  and  $\bar{q}_{\pm}(z)[\gamma^{\mu}, \gamma^{\nu}]q_{\mp}(-z)$
- For a massless (anti)particle, chirality = (-)helicity
- **Transversity is thus a chiral-odd quantity**
- QCD and QED are chiral even  $\Rightarrow \mathcal{A} \sim (\text{Ch.-odd})_1 \otimes (\text{Ch.-odd})_2$

# A few applications

## Spin transversity in the nucleon

### How to get access to transversity?

- The dominant DA for  $\rho_T$  is of twist 2 and chiral-odd ( $[\gamma^\mu, \gamma^\nu]$  coupling)
- Unfortunately  $\gamma^* N^\uparrow \rightarrow \rho_T N' = 0$ 
  - this is true at any order in perturbation theory (i.e. corrections as powers of  $\alpha_s$ ), since this would require a transfer of 2 units of helicity from the proton: impossible!  
Diehl, Gousset, Pire '99; Collins, Diehl '00
  - diagrammatic argument at Born order:



vanishes:  $\gamma^\alpha [\gamma^\mu, \gamma^\nu] \gamma_\alpha = 0$

# A few applications

## Spin transversity in the nucleon

### Can one circumvent this vanishing?

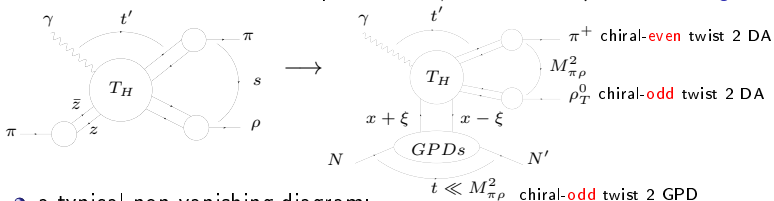
- This vanishing is true only a twist 2
- At twist 3 this process does not vanish
- However processes involving twist 3 DAs may face problems with factorization (end-point singularities: see later)
- The problem of classification of twist 3 chiral-odd GPDs is still open: [Pire, Szymanowski, S.W.](#) in progress, in the spirit of our [Light-Cone Collinear Factorization](#) framework recently developed ([Anikin, Ivanov, Pire, Szymanowski, S. W.](#))

# A few applications

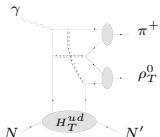
## Spin transversity in the nucleon

$\gamma N \rightarrow \pi^+ \rho_T^0 N'$  gives access to transversity

- Factorization à la Brodsky Lepage of  $\gamma + \pi \rightarrow \pi + \rho$  at large  $s$  and fixed angle (i.e. fixed ratio  $t'/s, u'/s$ )  
 $\implies$  factorization of the amplitude for  $\gamma + N \rightarrow \pi + \rho + N'$  at large  $M_{\pi\rho}^2$



- a typical non-vanishing diagram:



M. El Beiyad, P. Pire, M. Segond, L. Szymanowski, S.W. Phys.Lett. B688:154-167,2010

see also, at large  $s$ , with Pomeron exchange:

R. Ivanov, B. Pire, L. Szymanowski, O. Teryaev '02

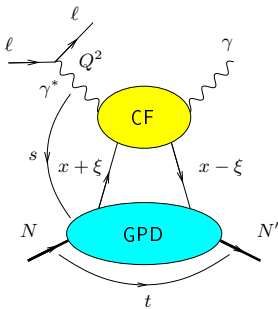
R. Enberg, B. Pire, L. Szymanowski '06

- These processes with 3 body final state can give access to all GPDs:  $M_{\pi\rho}^2$  plays the role of the  $\gamma^*$  virtuality of usual DVCS (here in the time-like domain) JLab, COMPASS

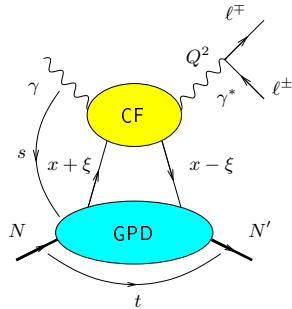


# Threshold effects for DVCS and TCS

## DVCS and TCS



Deeply Virtual Compton Scattering  
 $lN \rightarrow l'N'\gamma$



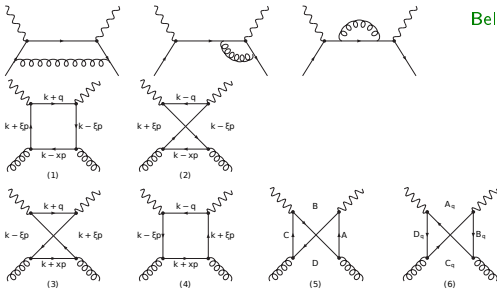
Timelike Compton Scattering  
 $\gamma N \rightarrow l^+l^-N'$

- TCS versus DVCS:
  - **universality of the GPDs**
  - another source for GPDs (special sensitivity on real part)
  - spacelike-timelike crossing and understanding the structure of the NLO corrections
- Where to measure TCS? In **Ultra Peripheral Collisions**  
**LHC, JLab, COMPASS, AFTER**

# Threshold effects for DVCS and TCS

## DVCS and TCS at NLO

### One loop contributions to the coefficient function



Belitsky, Mueller, Niedermeier, Schafer,  
Phys.Lett.B474, 2000  
Pire, Szymanowski, Wagner  
Phys.Rev.D83, 2011

$$\mathcal{A}^{\mu\nu} = g_T^{\mu\nu} \int_{-1}^1 dx \left[ \sum_q^{n_F} T^q(x) F^q(x) + T^g(x) F^g(x) \right]$$

(symmetric part of the factorised amplitude)

# Threshold effects for DVCS and TCS

Resummations effects are expected

- The renormalized quark **coefficient functions**  $T^q$  is

$$T^q = C_0^q + C_1^q + C_{coll}^q \log \frac{|Q^2|}{\mu_F^2}$$

$$C_0^q = e_q^2 \left( \frac{1}{x - \xi + i\epsilon} - (x \rightarrow -x) \right)$$

$$C_1^q = \frac{e_q^2 \alpha_S C_F}{4\pi(x - \xi + i\epsilon)} \left[ \log^2 \left( \frac{\xi - x}{2\xi} - i\epsilon \right) + \dots \right] - (x \rightarrow -x)$$

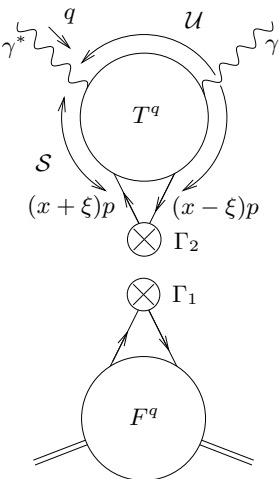
- Usual collinear approach: single-scale analysis w.r.t.  $Q^2$
- Consider the invariants  $S$  and  $U$ :

$$S = \frac{x - \xi}{2\xi} Q^2 \ll Q^2 \quad \text{when } x \rightarrow \xi$$

$$U = -\frac{x + \xi}{2\xi} Q^2 \ll Q^2 \quad \text{when } x \rightarrow -\xi$$

$\Rightarrow$  **two scales problem; threshold singularities to be resummed**

analogous to the  $\log(x - x_{Bj})$  resummation for DIS coefficient functions

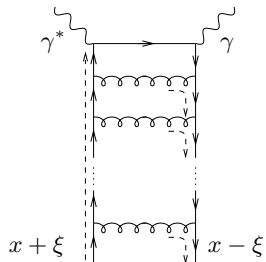


# Threshold effects for DVCS and TCS

## Resummation for Coefficient functions

### Soft-collinear resummation effects for the coefficient function

- The resummation easier when using the axial gauge  $p_1 \cdot A = 0$  ( $p_\gamma \equiv p_1$ )
- The dominant diagram are **ladder-like** [backup]



resummed formula (for DVCS), for  $x \rightarrow \xi$  :

$$(T^q)^{\text{res}} = \left( \frac{e_q^2}{x - \xi + i\epsilon} \left\{ \cosh \left[ D \log \left( \frac{\xi - x}{2\xi} - i\epsilon \right) \right] - \frac{D^2}{2} \left[ 9 + 3 \frac{\xi - x}{x + \xi} \log \left( \frac{\xi - x}{2\xi} - i\epsilon \right) \right] \right\} + C_{\text{coll}}^q \log \frac{Q^2}{\mu_F^2} \right) - (x \rightarrow -x) \quad \text{with} \quad D = \sqrt{\frac{\alpha_s C_F}{2\pi}}$$

T. Altinoluk, B. Pire, L. Szymanowski, S. W.  
JHEP 1210 (2012) 49; [arXiv:1206.3115]

- Our analysis can be used for **the gluon coefficient function** [In progress].
- The measurement of the phenomenological impact of this procedure on the data analysis needs further analysis with the implementation of modeled generalized parton distributions [backup].

# Problems

$\rho$ -electroproduction: Selection rules and factorization status

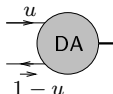
- chirality = helicity for a particule, chirality = -helicity for an antiparticule
- for massless quarks: QED and QCD vertices = chiral even (no chirality flip during the interaction)
  - ⇒ the total helicity of a  $q\bar{q}$  produced by a  $\gamma^*$  should be 0
  - ⇒ helicity of the  $\gamma^* = L_z^{q\bar{q}}$  ( $z$  projection of the  $q\bar{q}$  angular momentum)
- in the pure collinear limit (i.e. twist 2),  $L_z^{q\bar{q}}=0 \Rightarrow \gamma_L^*$
- at  $t = 0$ , no source of orbital momentum from the proton coupling  $\Rightarrow$  helicity of the meson = helicity of the photon
- in the collinear factorization approach,  $t \neq 0$  change nothing from the hard side  $\Rightarrow$  the above selection rule remains true
- thus: 2 transitions possible ( $s$ -channel helicity conservation (SCHC)):
  - $\gamma_L^* \rightarrow \rho_L$  transition: QCD factorization holds at  $t=2$  at any order in perturbation (i.e. LL, NLL, etc...)

Collins, Frankfurt, Strikman '97 Radyushkin '97

- $\gamma_T^* \rightarrow \rho_T$  transition: QCD factorization has problems at  $t=3$

Mankiewicz-Piller '00

$$\int_0^1 \frac{du}{u} \text{ or } \int_0^1 \frac{du}{1-u} \text{ diverge (end-point singularity)}$$



# Problems

$\rho$ -electroproduction: Selection rules and factorization status

## Improved collinear approximation: a solution?

- keep a transverse  $\ell_{\perp}$  dependency in the  $q, \bar{q}$  momenta, used to regulate end-point singularities
- soft and collinear gluon exchange between the valence quark are responsible for large double-logarithmic effects which are conjectured to exponentiate
- this is made easier when using the impact parameter space  $b_{\perp}$  conjugated to  $\ell_{\perp} \Rightarrow$  Sudakov factor

$$\exp[-S(u, b, Q)]$$

- $S$  diverges when  $b_{\perp} \sim O(1/\Lambda_{QCD})$  (large transverse separation, i.e. small transverse momenta) or  $u \sim O(\Lambda_{QCD}/Q)$  Botts, Sterman '89  
 $\Rightarrow$  regularization of end-point singularities for  $\pi \rightarrow \pi\gamma^*$  and  $\gamma\gamma^*\pi^0$  form factors, based on the factorization approach Li, Sterman '92
- it has been proposed to combine this perturbative resummation tail effect with an ad-hoc non-perturbative gaussian ansatz for the DAs

$$\exp[-a^2 |k_{\perp}^2|/(u\bar{u})]$$

which gives back the usual asymptotic DA  $6u\bar{u}$  when integrating over  $k_{\perp}$   
 $\Rightarrow$  practical tools for meson electroproduction phenomenology

Goloskokov, Kroll '05

# QCD at large $s$

## Theoretical motivations

A particular regime for QCD:  
The perturbative Regge limit  $s \rightarrow \infty$

Consider the diffusion of two hadrons  $h_1$  and  $h_2$ :

- $\sqrt{s}$  ( $= E_1 + E_2$  in the center-of-mass system)  $\gg$  other scales (masses, transferred momenta, ...) eg  $x_B \rightarrow 0$  in DIS
- other scales comparable (virtualities, etc...)  $\gg \Lambda_{QCD}$

regime  $\alpha_s \ln s \sim 1 \Rightarrow$  dominant sub-series:

$$\begin{aligned}
 \mathcal{A} = & \left( \text{Diagram 1} \right) + \left( \text{Diagram 2} + \text{Diagram 3} + \dots \right) + \left( \text{Diagram 4} + \dots \right) + \dots \\
 & \sim s \qquad \qquad \qquad \sim s (\alpha_s \ln s) \qquad \qquad \qquad \sim s (\alpha_s \ln s)^2
 \end{aligned}$$

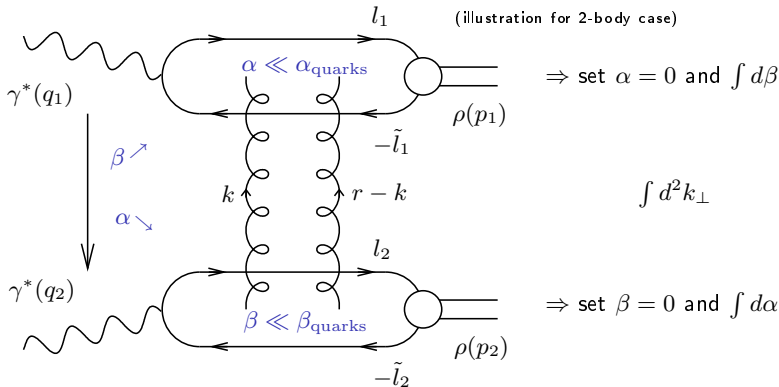
$\Rightarrow \sigma_{tot}^{h_1 h_2 \rightarrow tout} = \frac{1}{s} \text{Im} \mathcal{A} \sim s^{\alpha_{\mathbb{P}}(0)-1}$

with  $\alpha_{\mathbb{P}}(0) - 1 = C \alpha_s$  ( $C > 0$ ) hard Pomeron (Balitsky, Fadin, Kuraev, Lipatov)

- This result violates QCD  $S$  matrix unitarity ( $S S^\dagger = S^\dagger S = 1$  i.e.  $\sum \text{Prob.} = 1$ )
- Until when this result could be applicable, and how to improve it?
- How to test this dynamics experimentally, in particular based on exclusive processes?

QCD at large  $s$  $k_T$  factorization $\gamma^* \gamma^* \rightarrow \rho \rho$  as an example

- Use **Sudakov** decomposition  $k = \alpha p_1 + \beta p_2 + k_\perp$  ( $p_1^2 = p_2^2 = 0$ ,  $2p_1 \cdot p_2 = s$ )
- write  $d^4k = \frac{s}{2} d\alpha d\beta d^2k_\perp$
- $t$ -channel gluons with **non-sense** polarizations ( $\epsilon_{NS}^{up} = \frac{2}{s} p_2$ ,  $\epsilon_{NS}^{down} = \frac{2}{s} p_1$ ) dominate **at large  $s$**



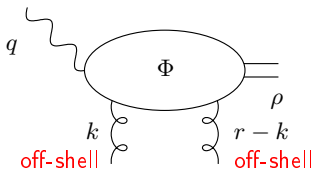


QCD at large  $s$  $k_T$  factorization

Impact representation for exclusive processes  $\underline{k} = \text{Eucl.} \leftrightarrow k_{\perp} = \text{Mink.}$

$$\mathcal{M} = is \int \frac{d^2 \underline{k}}{(2\pi)^2 \underline{k}^2 (\underline{r} - \underline{k})^2} \Phi^{\gamma^*(q_1) \rightarrow \rho(p_1^{\rho})}(\underline{k}, \underline{r} - \underline{k}) \Phi^{\gamma^*(q_2) \rightarrow \rho(p_2^{\rho})}(-\underline{k}, -\underline{r} + \underline{k})$$

$\Phi^{\gamma^*(q_1) \rightarrow \rho(p_1^{\rho})}$ :  $\gamma_{L,T}^*(q)g(k_1) \rightarrow \rho_{L,T}g(k_2)$  impact factor



Gauge invariance of QCD:

- probes are color neutral  
 $\Rightarrow$  their impact factor should vanish when  $\underline{k} \rightarrow 0$  or  $\underline{r} - \underline{k} \rightarrow 0$
- At twist-3 level (for the  $\gamma_T^* \rightarrow \rho_T$  transition), gauge invariance is a non-trivial constraint when combining 2- and 3-body correlators

# QCD at large $s$

Phenomenological applications: Meson production at HERA

## Diffractive meson production at HERA

HERA (DESY, Hamburg): first and single  $e^\pm p$  collider (1992-2007)

- The "easy" case (from factorization point of view):  $J/\Psi$  production ( $u \sim 1/2$ : non-relativistic limit for bound state) combined with  $k_T$ -factorisation  
Ryskin '93; Frankfurt, Koepf, Strikman '98; Ivanov, Kirschner, Schäfer, Szymanowski '00; Motyka, Enberg, Poludniowski '02
- Exclusive vector meson photoproduction at large  $t$  (= hard scale):  
 $\gamma(q) + P \rightarrow \rho_{L,T}(p_1) + P$   
based on  $k_T$ -factorization:  
Forshaw, Ryskin '95; Bartels, Forshaw, Lotter, Wüsthoff '96; Forshaw, Motyka, Enberg, Poludniowski '03
  - H1, ZEUS data seems to favor BFKL
  - but end-point singularities for  $\rho_T$  are regularized with a quark mass:  
 $m = m_\rho/2$
  - the spin density matrix is badly described
- Exclusive electroproduction of vector meson  $\gamma_{L,T}^*(q) + P \rightarrow \rho_{L,T}(p_1) + P$ 
  - phenomenological approach based on improved collinear factorization for the coupling with the meson DA and collinear factorization for GPD coupling Goloskokov, Kroll '05
  - first principle approach based on  $k_T$ -factorisation combined with Light-Cone-Collinear-Factorisation beyond leading twist:  
see talk of A. Besse

# QCD at large $s$

Phenomenological applications: exclusive processes at Tevatron, RHIC, LHC, ILC

Exclusive  $\gamma^{(*)}\gamma^{(*)}$  processes = gold place for testing QCD at large  $s$

Proposals in order to test perturbative QCD in the large  $s$  limit

( $t$ -structure of the hard Pomeron, saturation, Odderon...)

- $\gamma^{(*)}(q) + \gamma^{(*)}(q') \rightarrow J/\Psi J/\Psi$  Kwiecinski, Motyka '98
- $\gamma_{L,T}^*(q) + \gamma_{L,T}^*(q') \rightarrow \rho_L(p_1) + \rho_L(p_2)$  process in  $e^+e^- \rightarrow e^+e^- \rho_L(p_1) + \rho_L(p_2)$  with double tagged lepton at ILC

Pire, Szymanowski, S. W. '04; Pire, Szymanowski, Enberg, S. W. '06; Ivanov, Papa '06; Second, Szymanowski, S. W. '07

conclusion: feasible at ILC (high energy and high luminosity); BFKL NLL enhancement with respect to Born and DGLAP contributions

- What about the Odderon?  $C$ -parity of Odderon = -1  
consider  $\gamma + \gamma \rightarrow \pi^+\pi^-\pi^+\pi^-$ :  $\pi^+\pi^-$  pair has no fixed  $C$ -parity  
 $\Rightarrow$  Odderon and Pomeron can interfere  
 $\Rightarrow$  Odderon appears linearly in the charge asymmetry

Pire, Schwennsen, Szymanowski, S. W. '07

= example of possibilities offered by ultraperipheral exclusive processes at LHC [backup]

( $p$ ,  $\bar{p}$  or  $A$  as effective sources of photon)

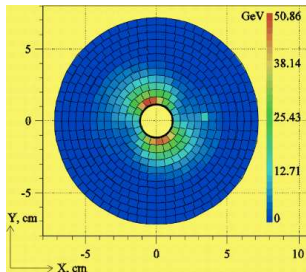
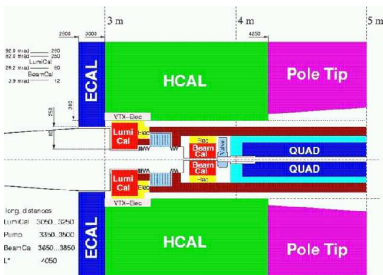
but the distinction with pure QCD processes (with gluons instead of a photon) is tricky...

# QCD at large $s$

Phenomenological applications: exclusive test of Pomeron

An example of realistic exclusive test of Pomeron:  $\gamma^{(*)}\gamma^{(*)} \rightarrow \rho\rho$   
 as a subprocess of  $e^-e^+ \rightarrow e^-e^+ \rho_L^0 \rho_L^0$

- ILC should provide  $\left\{ \begin{array}{l} \text{very large } \sqrt{s} \text{ (} = 500 \text{ GeV)} \\ \text{very large luminosity (} \simeq 125 \text{ fb}^{-1}/\text{year)} \end{array} \right.$
- detectors are planned to cover the **very forward** region, close from the beampipe (directions of out-going  $e^+$  and  $e^-$  at large  $s$ )



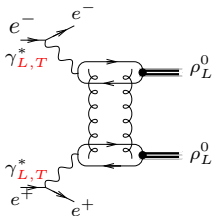
good efficiency of tagging for outgoing  $e^\pm$  for  $E_e > 100$  GeV and  $\theta > 4$  mrad (illustration for LDC concept)

- could be equivalently done at LHC based on the AFP project

# QCD at large s

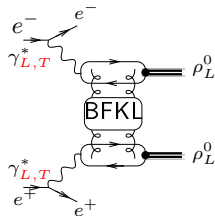
Phenomenological applications: exclusive test of Pomeron

QCD effects in the Regge limit on  $\gamma^{(*)}\gamma^{(*)} \rightarrow \rho\rho$



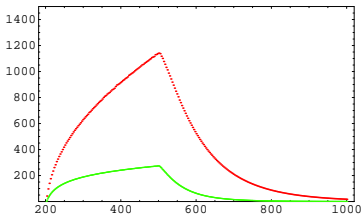
$\simeq 4 \cdot 10^3$  events/year

→



$\simeq 2 \cdot 10^4$  events/year

$\frac{d\sigma^{tmin}}{dt} (fb/GeV^2)$



$\sqrt{s_{e+e-}} [GeV]$

proof of feasibility:

B. Pire, L. Szymanowski and S. W.  
Eur.Phys.J.C44 (2005) 545

proof of visible BFKL enhancement:

R. Enberg, B. Pire, L. Szymanowski and S. W.  
Eur.Phys.J.C45 (2006) 759

comprehensive study of  $\gamma^*$  polarization effects  
and event rates:

M. Segond, L. Szymanowski and S. W.  
Eur. Phys. J. C 52 (2007) 93

NLO BFKL study:

Ivanov, Papa '06 '07; Caporale, Papa, Vera '08

# Conclusion

- Since a decade, there have been much progress in the understanding of **hard** exclusive processes
  - at medium energies, there is now a conceptual framework starting from first principle, allowing to describe a huge number of processes
  - at high energy, the impact representation is a powerful tool for describing exclusive processes in diffractive experiments; they are and will be essential for studying QCD in the hard Regge limit (Pomeron, Odderon, saturation...)
- Still, some problems remain:
  - proofs of factorization have been obtained only for very few processes (ex.:  $\gamma^* p \rightarrow \gamma p$ ,  $\gamma_L^* p \rightarrow \rho_L p$ )
  - for some other processes factorization is highly plausible, but not fully demonstrated at any order (ex.: processes involving GDAs and TDAs)
  - some processes explicitly show sign of breaking of factorization (ex.:  $\gamma_T^* p \rightarrow \rho_T p$  which has end-point singularities at Leading Order)
  - models and results from the lattice or from AdS/QCD for the non-perturbative correlators entering GPDs, DAs, GDAs, TDAs are needed, even at a qualitative level!
  - QCD evolution, NLO corrections, choice of renormalization/factorization scale, power corrections, threshold resummations will be very relevant to interpret and describe the forthcoming data
- Constructing a consistent framework including GPDs (skewness) and TMDs/uPDFs ( $k_T$ -dependency) with realistic experimental observables is an (almost) open problem (GTMDs)
- Links between theoretical and experimental communities are very fruitful!

# A few applications

Production of an exotic hybrid meson in hard processes

## Distribution amplitude and quantum numbers: $C$ -parity

- Define the  $H$  DA as (for long. pol.)

$$\langle H(p, 0) | \bar{\psi}(-z/2) \gamma_\mu [-z/2; z/2] \psi(z/2) | 0 \rangle \Big|_{\substack{z^2=0 \\ z_+=0 \\ z_\perp=0}} = i f_H M_H e_\mu^{(0)} \int_0^1 dy e^{i(\bar{y}-y)p \cdot z/2} \phi_L^H(y)$$

- Expansion in terms of local operators

$$\langle H(p, \lambda) | \bar{\psi}(-z/2) \gamma_\mu [-z/2; z/2] \psi(z/2) | 0 \rangle = \sum_n \frac{1}{n!} z_{\mu_1} \dots z_{\mu_n} \langle H(p, \lambda) | \bar{\psi}(0) \gamma_\mu \overleftrightarrow{D}_{\mu_1} \dots \overleftrightarrow{D}_{\mu_n} \psi(0) | 0 \rangle,$$

- $C$ -parity:  $\begin{cases} H \text{ selects the odd-terms: } C_H = (-) \\ \rho \text{ selects even-terms: } C_\rho = (-) \end{cases}$

$$\langle H(p, \lambda) | \bar{\psi}(-z/2) \gamma_\mu [-z/2; z/2] \psi(z/2) | 0 \rangle = \sum_{n \text{ odd}} \frac{1}{n!} z_{\mu_1} \dots z_{\mu_n} \langle H(p, \lambda) | \bar{\psi}(0) \gamma_\mu \overleftrightarrow{D}_{\mu_1} \dots \overleftrightarrow{D}_{\mu_n} \psi(0) | 0 \rangle$$

- Special case  $n = 1$ :  $\mathcal{R}_{\mu\nu} = S_{(\mu\nu)} \bar{\psi}(0) \gamma_\mu \overleftrightarrow{D}_\nu \psi(0)$

$S_{(\mu\nu)}$  = symmetrization operator:  $S_{(\mu\nu)} T_{\mu\nu} = \frac{1}{2}(T_{\mu\nu} + T_{\nu\mu})$

# A few applications

Electroproduction of an exotic hybrid meson

## Non perturbative input for the hybrid DA

- We need to fix  $f_H$  (the analogue of  $f_\rho$ )
- This is a non-perturbative input
- Lattice does not yet give information
- The operator  $\mathcal{R}_{\mu\nu}$  is related to quark energy-momentum tensor  $\Theta_{\mu\nu}$  :

$$\mathcal{R}_{\mu\nu} = -i \Theta_{\mu\nu}$$

- Rely on QCD sum rules: resonance for  $M \approx 1.4$  GeV  
I. I. Balitsky, D. Diakonov, and A. V. Yung

$$f_H \approx 50 \text{ MeV}$$

$$f_\rho = 216 \text{ MeV}$$

- Note:  $f_H$  evolves according to the  $\gamma_{QQ}$  anomalous dimension

$$f_H(Q^2) = f_H \left( \frac{\alpha_S(Q^2)}{\alpha_S(M_H^2)} \right)^{K_1} \quad K_1 = \frac{2\gamma_{QQ}(1)}{\beta_0},$$



# A few applications

Electroproduction of an exotic hybrid meson

Counting rates for  $H$  versus  $\rho$  electroproduction: order of magnitude

- Ratio:

$$\frac{d\sigma^H(Q^2, x_B, t)}{d\sigma^\rho(Q^2, x_B, t)} = \left| \frac{f_H (e_u \mathcal{H}_{uu}^- - e_d \mathcal{H}_{dd}^-) \mathcal{V}^{(H,-)}}{f_\rho (e_u \mathcal{H}_{uu}^+ - e_d \mathcal{H}_{dd}^+) \mathcal{V}^{(\rho,+)}} \right|^2$$

- Rough estimate:

- neglect  $\bar{q}$  i.e.  $x \in [0, 1]$

$\Rightarrow Im\mathcal{A}_H$  and  $Im\mathcal{A}_\rho$  are equal up to the factor  $\mathcal{V}^M$

- Neglect the effect of  $Re\mathcal{A}$

$$\frac{d\sigma^H(Q^2, x_B, t)}{d\sigma^\rho(Q^2, x_B, t)} \approx \left( \frac{5f_H}{3f_\rho} \right)^2 \approx 0.15$$

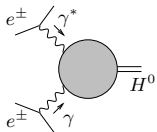
- More precise study based on *Double Distributions* to model GPDs + effects of varying  $\mu_R$ : order of magnitude unchanged
- The range around 1400 MeV is dominated by the  $a_2(1329)(2^{++})$  resonance
  - possible interference between  $H$  and  $a_2$
  - identification through the  $\pi\eta$  GDA, main decay mode for the  $\pi_1(1400)$  candidate, through angular asymmetry in  $\theta_\pi$  in the  $\pi\eta$  cms

# A few applications

Electroproduction of an exotic hybrid meson

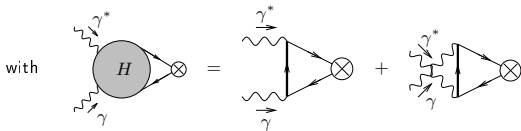
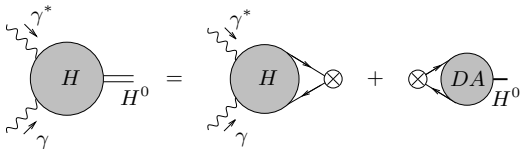
## Hybrid meson production in $e^+e^-$ colliders

- Hybrid can be copiously produced in  $\gamma^*\gamma$ , i.e. at  $e^+e^-$  colliders **with one tagged out-going electron**



BaBar, Belle

- This can be described in a hard factorization framework:



# A few applications

Electroproduction of an exotic hybrid meson

## Counting rates for $H^0$ versus $\pi^0$

- Factorization gives:

$$\mathcal{A}^{\gamma\gamma^* \rightarrow H^0}(\gamma\gamma^* \rightarrow H_L) = (\epsilon_\gamma \cdot \epsilon_\gamma^*) \frac{(e_u^2 - e_d^2) f_H}{2\sqrt{2}} \int_0^1 dz \Phi^H(z) \left( \frac{1}{z} - \frac{1}{\bar{z}} \right)$$

- Ratio  $H^0$  versus  $\pi^0$ :

$$\frac{d\sigma^H}{d\sigma^{\pi^0}} = \left| \frac{f_H \int_0^1 dz \Phi^H(z) \left( \frac{1}{z} - \frac{1}{\bar{z}} \right)}{f_\pi \int_0^1 dz \Phi^\pi(z) \left( \frac{1}{z} + \frac{1}{\bar{z}} \right)} \right|^2$$

- This gives, with *asymptotical* DAs (i.e. limit  $Q^2 \rightarrow \infty$ ):

$$\frac{d\sigma^H}{d\sigma^{\pi^0}} \approx 38\%$$

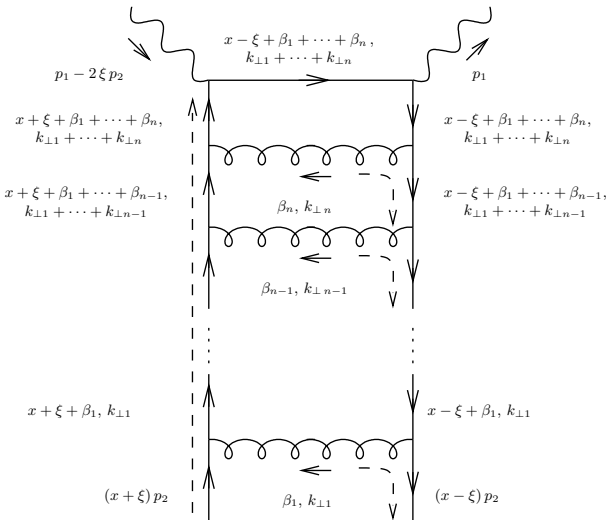
still larger than 20% at  $Q^2 \approx 1 \text{ GeV}^2$  (including kinematical twist-3 effects à la [Wandzura-Wilczek](#) for the  $H^0$  DA) and similarly

$$\frac{d\sigma^H}{d\sigma^\eta} \approx 46\%$$

# Threshold effects for DVCS and TCS

## Resummation for Coefficient functions (1)

### Computation of the $n$ -loop ladder-like diagram



- All gluons are assumed to be on mass shell.
- Strong ordering in  $\underline{k}_i$ ,  $\alpha_i$  and  $\beta_i$ .
- The dominant momentum flows along  $p_2$  are indicated

## Threshold effects for DVCS and TCS

## Resummation for Coefficient functions

Computation of the  $n$ -loop ladder-like diagram (2)

- Strong ordering is given as :

$$|\underline{k}_n| \gg |\underline{k}_{n-1}| \gg \dots \gg |\underline{k}_1| \quad , \quad 1 \gg |\alpha_n| \gg |\alpha_{n-1}| \gg \dots \gg |\alpha_1|$$

$$x \sim \xi \gg |\beta_1| \sim |x - \xi| \gg |x - \xi + \beta_1| \sim |\beta_2| \gg \dots \gg |x - \xi + \beta_1 + \beta_2 - \dots + \beta_{n-1}| \sim |\beta_n|$$

- eikonal coupling on the left
- coupling on the right goes beyond eikonal
- Integral for  $n$ -loop:

$$I_n = \left(\frac{s}{2}\right)^n \int d\alpha_1 d\beta_1 d_2\underline{k}_1 \dots \int d\alpha_n d\beta_n d_2\underline{k}_n (\text{Num})_n \frac{1}{L_1^2} \dots \frac{1}{L_n^2} \frac{1}{S^2} \frac{1}{R_1^2} \dots \frac{1}{R_n^2} \frac{1}{k_1^2} \dots \frac{1}{k_n^2}$$

- Numerator:

$$(\text{Num})_2 = -4s \underbrace{\frac{-2\underline{k}_1^2(x+\xi)}{\beta_1} \left[1 + \frac{2(x-\xi)}{\beta_1}\right]}_{\text{gluon 1}} \underbrace{\frac{-2\underline{k}_2^2(x+\xi)}{\beta_2} \left[1 + \frac{2(\beta_1+x-\xi)}{\beta_2}\right]}_{\text{gluon 2}} \dots \underbrace{\frac{-2\underline{k}_n^2(x+\xi)}{\beta_n} \left[1 + \frac{2(\beta_{n-1}+\dots+\beta_1+x-\xi)}{\beta_n}\right]}_{\text{gluon n}}$$

- Propagators:

$$L_1^2 = \alpha_1(x+\xi)s, \quad R_1^2 = -\underline{k}_1^2 + \alpha_1(\beta_1+x-\xi)s,$$

$$L_2^2 = \alpha_2(x+\xi)s, \quad R_2^2 = -\underline{k}_2^2 + \alpha_2(\beta_1+\beta_2+x-\xi)s,$$

$$\vdots$$

$$L_n^2 = \alpha_n(x+\xi)s, \quad R_n^2 = -\underline{k}_n^2 + \alpha_n(\beta_1+\dots+\beta_n+x-\xi)s,$$

## Threshold effects for DVCS and TCS

## Resummation for Coefficient functions

Computation of the  $n$ -loop ladder-like diagram (3)

$$I_n = -4 \frac{(2\pi i)^n}{x - \xi} \int_0^{\xi-x} d\beta_1 \cdots \int_0^{\xi-x-\beta_1-\cdots-\beta_{n-1}} d\beta_n \frac{1}{\beta_1 + x - \xi} \cdots \frac{1}{\beta_1 + \cdots + \beta_n + x - \xi} \\ \times \int_0^\infty d_N \underline{k}_n \cdots \int_{\underline{k}_2}^\infty d_N \underline{k}_1 \frac{1}{\underline{k}_1^2} \cdots \frac{1}{\underline{k}_{n-1}^2} \frac{1}{\underline{k}_n^2 - (\beta_1 + \cdots + \beta_n + x - \xi)s}$$

integration over  $\underline{k}_i$  and  $\beta_i$  leads to our final result :

$$I_n^{\text{fin.}} = -4 \frac{(2\pi i)^n}{x - \xi + i\epsilon} \frac{1}{(2n)!} \log^{2n} \left[ \frac{\xi - x}{2\xi} - i\epsilon \right]$$

Resummation :

remember that  $K_n = -\frac{1}{4} e_q^2 \left( -i C_F \alpha_s \frac{1}{(2\pi)^2} \right)^n I_n$

$$\left( \sum_{n=0}^{\infty} K_n \right) - (x \rightarrow -x) = \frac{e_q^2}{x - \xi + i\epsilon} \cosh \left[ D \log \left( \frac{\xi - x}{2\xi} - i\epsilon \right) \right] - (x \rightarrow -x)$$

where  $D = \sqrt{\frac{\alpha_s C_F}{2\pi}}$

# Threshold effects for DVCS and TCS

## Resummed formula

### Inclusion of our resummed formula into the NLO coefficient function

The inclusion procedure is not unique and it is natural to propose two choices:

- modifying only the Born term and the  $\log^2$  part of the  $C_1^q$  and keeping the rest of the terms untouched :

$$(T^q)^{\text{res1}} = \left( \frac{e_q^2}{x-\xi+i\epsilon} \left\{ \cosh \left[ D \log \left( \frac{\xi-x}{2\xi} - i\epsilon \right) \right] - \frac{D^2}{2} \left[ 9 + 3 \frac{\xi-x}{x+\xi} \log \left( \frac{\xi-x}{2\xi} - i\epsilon \right) \right] \right\} \right. \\ \left. + C_{coll}^q \log \frac{Q^2}{\mu_F^2} \right) - (x \rightarrow -x)$$

- the resummation effects are accounted for in a multiplicative way for  $C_0^q$  and  $C_1^q$  :

$$(T^q)^{\text{res2}} = \left( \frac{e_q^2}{x-\xi+i\epsilon} \cosh \left[ D \log \left( \frac{\xi-x}{2\xi} - i\epsilon \right) \right] \left[ 1 - \frac{D^2}{2} \left\{ 9 + 3 \frac{\xi-x}{x+\xi} \log \left( \frac{\xi-x}{2\xi} - i\epsilon \right) \right\} \right] \right. \\ \left. + C_{coll}^q \log \frac{Q^2}{\mu_F^2} \right) - (x \rightarrow -x)$$

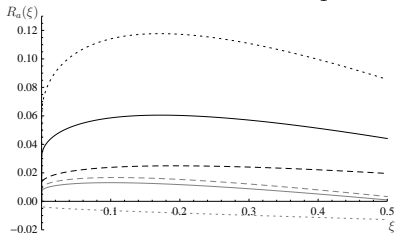
These resummed formulas differ through logarithmic contributions which are beyond the precision of our study.

# Threshold effects for DVCS and TCS

## Phenomenological implications

- We use a Double Distribution based model
  - S. V. Goloskokov and P. Kroll, *Eur. Phys. J. C* **50**, 829 (2007)
- Blind integral in the whole  $x$ -range: amplitude = NLO result  $\pm 1\%$
- To respect the domain of applicability of our resummation procedure:
  - restrict the use of our formula to  $\xi - a\gamma < |x| < \xi + a\gamma$
  - width  $a\gamma$  defined through  $|D \log(\gamma/(2\xi))| = 1$
  - theoretical uncertainty evaluated by varying  $a$
  - a more precise treatment is beyond the leading logarithmic approximation

$$R_a(\xi) = \frac{[\int_{\xi-a\gamma}^{\xi+a\gamma} + \int_{-\xi-a\gamma}^{-\xi+a\gamma}] dx (C^{\text{res}} - C_0 - C_1) H(x, \xi, 0)}{|\int_{-1}^1 dx (C_0 + C_1) H(x, \xi, 0)|}$$



$Re[R_a(\xi)]$  : black upper curves

$Im[R_a(\xi)]$  : grey lower curves

$a = 1$  (solid)

$a = 1/2$  (dotted)

$a = 2$  (dashed)

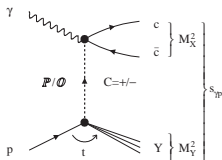


## Finding the hard Odderon

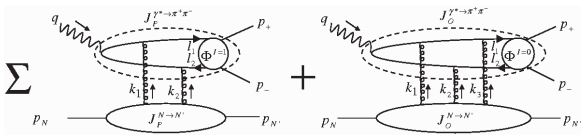
- colorless gluonic exchange
  - $C = +1$  : Pomeron, in pQCD described by **BFKL** equation
  - $C = -1$  : Odderon, in pQCD described by **BJKP** equation
- best but still weak evidence for  $\mathbb{O}$ :  $pp$  and  $p\bar{p}$  data at **ISR**
- no evidence for perturbative  $\mathbb{O}$

# Finding the hard Odderon

- exchange much weaker than  $\mathbb{P} \Rightarrow$  two strategies in QCD
  - consider **processes**, where  $\mathbb{P}$  vanishes due to  $C$ -parity conservation:
    - exclusive  $\eta, \eta_c, f_2, a_2, \dots$  in  $ep$ ;  $\gamma\gamma \rightarrow \eta_c \eta_c \sim |\mathcal{M}_0|^2$  Braunewell, Ewerz '04
    - exclusive  $J/\Psi, \Upsilon$  in  $pp$  ( $\mathbb{P}\mathbb{O}$  fusion, not  $\mathbb{P}\mathbb{P}$ ) Bzdak, Motyka, Szymanowski, Cudell '07
  - consider **observables** sensitive to the **interference** between  $\mathbb{P}$  and  $\mathbb{O}$  (open charm in  $ep$ ;  $\pi^+\pi^-$  in  $ep$ )  $\sim \text{Re} \mathcal{M}_{\mathbb{P}} \mathcal{M}_{\mathbb{O}}^* \Rightarrow$  observable **linear** in  $\mathcal{M}_0$



Brodsky, Rathsman, Merino '99



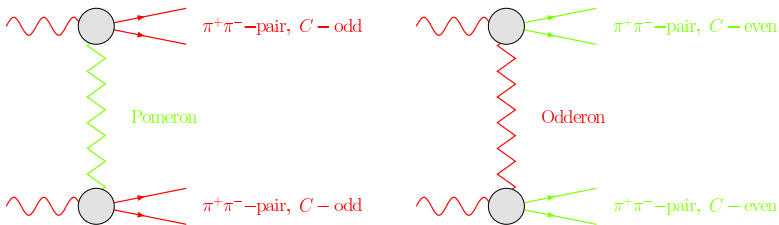
Ivanov, Nikolaev, Ginzburg '01 in photo-production

Hägler, Pire, Szymanowski, Teryaev '02 in electro-production

# Finding the hard Odderon

$\mathbb{P} - \mathbb{O}$  interference in double UPC

$\mathbb{P} - \mathbb{O}$  interference in  $\gamma\gamma \rightarrow \pi^+ \pi^- \pi^+ \pi^-$



Hard scale =  $t$

B. Pire, F. Schwennsen, L. Szymanowski, S. W.  
 Phys.Rev.D78:094009 (2008)

pb at LHC: pile-up!