Some recent developments in the theory of hard exclusive processes

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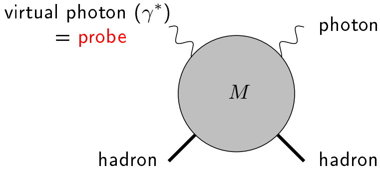
Exclusive processes are theoretically challenging

How to deal with QCD?

example: Compton scattering



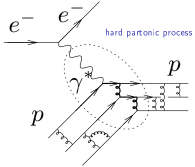
- Aim: describe M by separating:
  - quantities non-calculable perturbatively
    some tools:
    - Discretization of QCD on a 4-d lattice: numerical simulations
    - AdS/CFT $\Rightarrow$ AdS/QCD : $AdS_5 \times S^5 \leftrightarrow$ QCD
      Polchinski, Strassler '01
      for some issues related to Deep Inelastic Scattering (DIS):
      B. Pire, L. Szymanowski, C. Roiesnel, S. W. Phys.Lett.B670 (2008) 84-90
      for some issues related to Deep Virtual Compton Scattering (DVCS):
      C. Marquet, C. Roiesnel, S. W. JHEP 1004:051 (2010) 1-26
  - pertubatively calculable quantities
- We will here focus on theory and phenomenology of exclusive processes for which the dynamics is governed by QCD in the perturbative regime

Exclusive processes are phenomenologically challenging

Key question of QCD:
how to obtain and understand the tri-dimensional structure of hadrons
in terms of quarks and gluons?

Can this be achieved using hard exclusive processes?

- The aim is to reduce the process to interactions involving a small number of *partons* (quarks, gluons), despite confinement
- This is possible if the considered process is driven by short distance phenomena ($d \ll 1\,\mathrm{fm}$)
$\Longrightarrow \alpha_s \ll 1$: Perturbative methods
- One should hit strongly enough a hadron
Example: electromagnetic probe and form factor

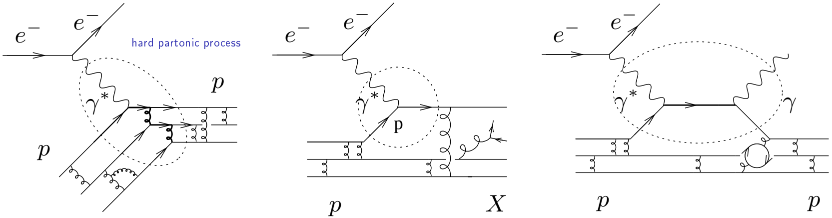


$\tau_{\text{electromagnetic interaction}} \sim \tau_{\text{parton life time after interaction}}$
$\ll \tau_{\text{caracteristic time of strong interaction}}$

To get such situations in exclusive reactions is very challenging phenomenologically: the cross sections are very small

Hard processes in QCD

- This is justified if the process is governed by a hard scale:
  - virtuality of the electromagnetic probe
    in elastic scattering $e^\pm\, p \to e^\pm\, p$
    in Deep Inelastic Scattering (DIS) $e^\pm\, p \to e^\pm\, X$
    in Deep Virtual Compton Scattering (DVCS) $e^\pm\, p \to e^\pm\, p\, \gamma$
  - Total center of mass energy in $e^+ e^- \to X$ annihilation
  - t-channel momentum exchange in meson photoproduction $\gamma\, p \to M\, p$
- A precise treatment relies on factorization theorems
- The scattering amplitude is described by the convolution of the partonic amplitude with the non-perturbative hadronic content



Introduction
Counting rules and limitations

The partonic point of view... and its limitations

- Counting rules:

$$F_n(q^2) \simeq \frac{C}{(Q^2)^{n-1}} \qquad n = \text{number of minimal constituents: } \begin{cases} \text{meson: } n = 2 \\ \text{baryon: } n = 3 \end{cases}$$

Brodsky, Farrar '73

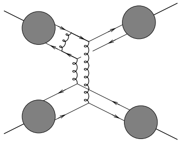
- Large angle (i.e. $s \sim t \sim u$ large) elastic processes $h_a\, h_b \to h_a\, h_b$

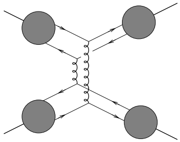
e.g. : $\pi\pi \to \pi\pi$ or $p\,p \to p\,p$

$$\frac{d\sigma}{dt} \sim \left(\frac{\alpha_S(p_\perp^2)}{s}\right)^{n-2} \quad n = \#\text{ of external fermionic lines } (n = 8 \text{ for } \pi\pi \to \pi\pi)$$

Brodsky, Lepage '81

Other contributions might be significant, even at large angle: e.g. $\pi\pi \to \pi\pi$





Brodsky Lepage mecanism: $\dfrac{d\sigma_{BL}}{dt} \sim \left(\dfrac{1}{s}\right)^6$

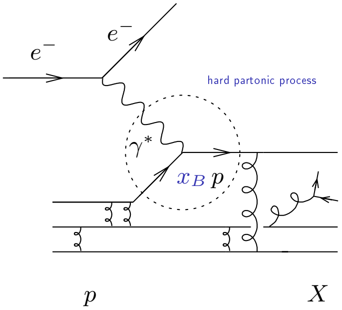
Landshoff '74 mecanism: $\dfrac{d\sigma_{L}}{dt} \sim \left(\dfrac{1}{s}\right)^5$

absent with at least one $\gamma^{(*)}$ (point-like coupling)

Accessing the perturbative proton content using inclusive processes

no $1/Q$ suppression

example: DIS



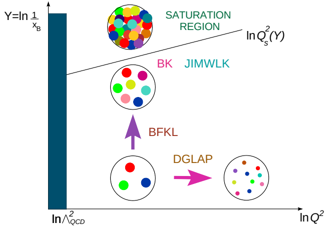
$$s_{\gamma^* p} = (q_\gamma^* + p_p)^2 = 4\, E^2_{\rm c.m.}$$

$$Q^2 \equiv -q^2_{\gamma^*} > 0$$

$$x_B = \frac{Q^2}{2\, p_p \cdot q_\gamma^*} \simeq \frac{Q^2}{s_{\gamma^* p}}$$

- x_B = proton momentum fraction carried by the scattered quark
- $1/Q$ = transverse resolution of the photonic probe $\ll 1/\Lambda_{QCD}$

The various regimes governing the perturbative content of the proton



- "usual" regime: x_B moderate ($x_B \gtrsim .01$):
  Evolution in Q governed by the QCD renormalization group
  (Dokshitser, Gribov, Lipatov, Altarelli, Parisi equation)

$$\underbrace{\sum_n (\alpha_s \ln Q^2)^n}_{\text{LLQ}} + \underbrace{\alpha_s \sum_n (\alpha_s \ln Q^2)^n}_{\text{NLLQ}} + \cdots$$

- perturbative Regge limit: $s_{\gamma^* p} \to \infty$ i.e. $x_B \sim Q^2/s_{\gamma^* p} \to 0$
  in the perturbative regime (hard scale Q^2)
  (Balitski Fadin Kuraev Lipatov equation)

$$\underbrace{\sum_n (\alpha_s \ln s)^n}_{\text{LLs}} + \underbrace{\alpha_s \sum_n (\alpha_s \ln s)^n}_{\text{NLLs}} + \cdots$$

From inclusive to exclusive processes

Experimental effort

- Inclusive processes are not $1/Q$ suppressed (e.g. DIS);
  Exclusive processes are suppressed
- Going from inclusive to exclusive processes is difficult
- High luminosity accelerators and high-performance detection facilities

HERA (H1, ZEUS), HERMES, JLab@6 GeV (Hall A, CLAS), BaBar, Belle, BEPC-II (BES-III)
future: LHC, COMPASS-II, JLab@12 GeV, LHeC, EIC, ILC

- What to do, and where?
  - Proton form factor: JLab@6 GeV future: PANDA (timelike proton form factor through $p\bar{p} \to e^+e^-$)
  - e^+e^- in $\gamma^*\gamma$ single-tagged channel: Transition form factor $\gamma^*\gamma \to \pi$, exotic hybrid meson production BaBar, Belle, BES,...
  - Deep Virtual Compton Scattering (GPD)
    HERA (H1, ZEUS), HERMES, JLab@6 GeV
    future: JLab@12GeV, COMPASS-II, EIC, LHeC
  - Non exotic and exotic hybrid meson electroproduction (GPD and DA), etc...
    NMC (CERN), E665 (Fermilab), HERA (H1, ZEUS), COMPASS, HERMES, CLAS (JLab)
  - TDA (PANDA at GSI)
  - TMDs (BaBar, Belle, COMPASS, ...)
  - Diffractive processes, including ultraperipheral collisions
    LHC (with or without fixed targets), ILC, LHeC

From inclusive to exclusive processes

Theoretical efforts

Very important theoretical developments during the last decade

- Key words:

  DAs, GPDs, GDAs, TDAs ... TMDs

- Fundamental tools:
  - At medium energies:

    JLab, HERMES, COMPASS, BaBar, Belle, PANDA, EIC

    collinear factorization

  - At asymptotical energies:

    HERA, Tevatron, LHC, LHeC, ILC (EIC and COMPASS at the boundary)

    k_T-factorization

We will now explain and illustrate these concepts, and discuss issues and possible solutions...

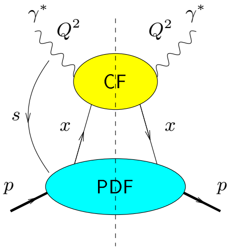
Extensions from DIS

- DIS: inclusive process $\rightarrow$ forward amplitude ($t = 0$) *(optical theorem)*

  (DIS: Deep Inelastic Scattering)

  ex: $e^{\pm}p \rightarrow e^{\pm}X$ at HERA

  $x \Rightarrow$ 1-dimensional structure



Structure Function

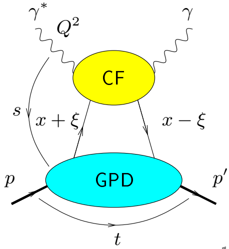
= Coefficient Function (hard) $\otimes$ Parton Distribution Function (soft)

- DVCS: exclusive process $\rightarrow$ non forward amplitude ($-t \ll s = W^2$)

  (DVCS: Deep Vitual Compton Scattering)

  Fourier transf.: $t \leftrightarrow$ impact parameter

  $(x, t) \Rightarrow$ 3-dimensional structure



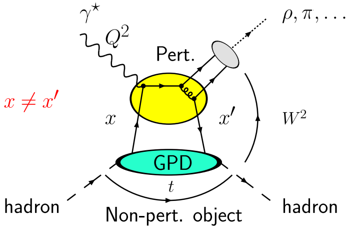
Amplitude

= Coefficient Function (hard) $\otimes$ Generalized Parton Distribution (soft)

Müller et al. '91 - '94; Radyushkin '96; Ji '97

Extensions from DVCS

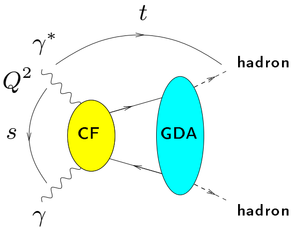
- Meson production: γ replaced by $\rho,\ \pi, \cdots$



Amplitude = GPD (soft) ⊗ CF (hard) ⊗ Distribution Amplitude (soft)

Collins, Frankfurt, Strikman '97; Radyushkin '97

- Crossed process: $s \ll -t$



Amplitude = Coefficient Function (hard) ⊗ Generalized Distribution Amplitude (soft)

Diehl, Gousset, Pire, Teryaev '98

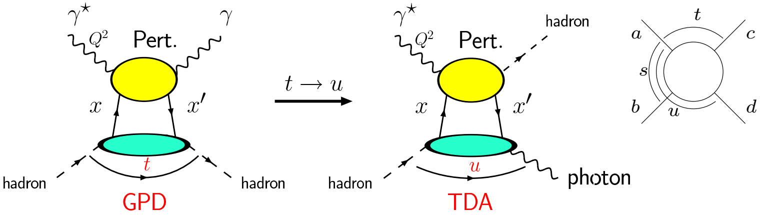
Extensions from DVCS

- Starting from usual DVCS, one allows: initial hadron $\neq$ final hadron (in the same octuplet): transition GPDs

Even less diagonal:

baryonic number $_{\text{(initial state)}}$ $\neq$ baryonic number $_{\text{(final state)}}$ $\rightarrow$ TDA

Example:



Pire, Szymanowski '05

which can be further extended by replacing the outoing γ by any hadronic state

Amplitude	=	Transition Distribution Amplitude (soft)	$\otimes$	CF (hard)	$\otimes$	DA (soft)

Lansberg, Pire, Szymanowski '06

Collinear factorization

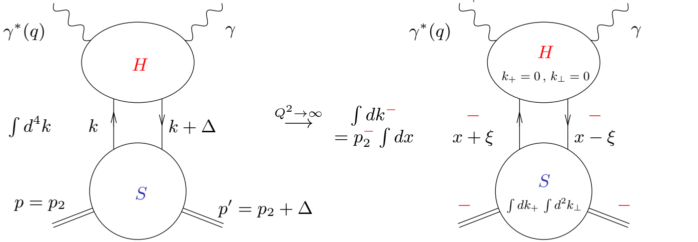
A bit more technical: DVCS and GPDs

Two steps for factorization

- momentum factorization: light-cone vector dominance for $Q^2 \to \infty$

$p_1,\, p_2$: the two light-cone directions $\begin{cases} p_1 = \frac{\sqrt{s}}{2}(1, 0_\perp, 1) & p_1^2 = p_2^2 = 0 \\ p_2 = \frac{\sqrt{s}}{2}(1, 0_\perp, -1) & 2\, p_1 \cdot p_2 = s \sim s_{\gamma^* p} \gtrsim Q^2 \end{cases}$

Sudakov decomposition: $\underset{}{k} = \underset{+}{\alpha\, p_1} + \underset{-}{\beta\, p_2} + \underset{\perp}{k_\perp}$



$$\int d^4k\; S(k,\, k+\Delta)\, H(q,\, k,\, k+\Delta) \;=\; \int dk^- \int dk^+ d^2k_\perp\; S(k,\, k+\Delta)\;\; H(q,\, k^-,\, k^- + \Delta^-)$$

- Quantum numbers factorization (Fierz identity: spinors + color)

$$\Rightarrow \qquad \mathcal{M} = \text{GPD} \otimes \text{Hard part}$$

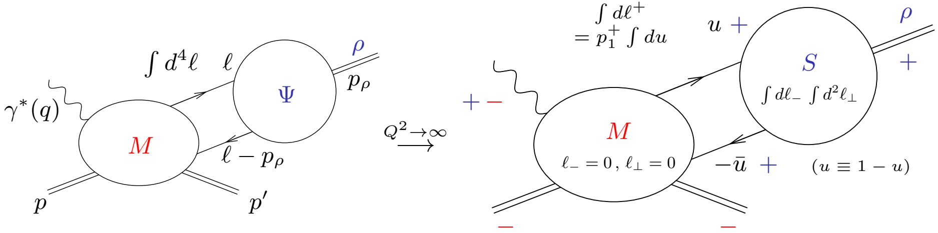
Müller et al. '91 - '94; Radyushkin '96; Ji '97

Collinear factorization

$\rho-$meson production: from the wave function to the DA

What is a $\rho-$meson in QCD?

It is described by its wave function Ψ which reduces in hard processes to its Distribution Amplitude



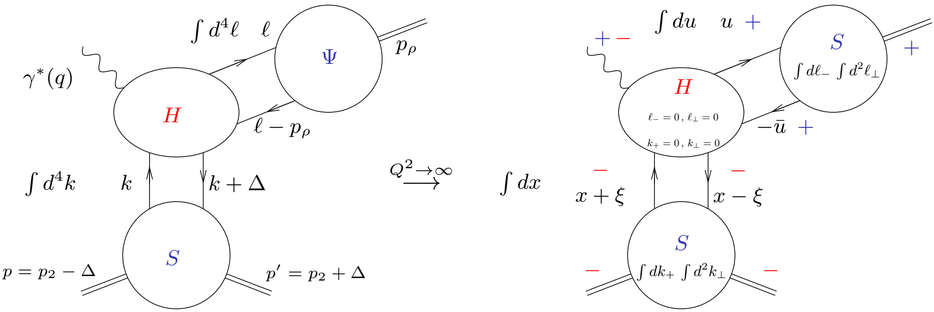
$$\int d^4\ell \; M(q,\,\ell,\,\ell-p_\rho)\Psi(\ell,\,\ell-p_\rho) \;=\! \int d\ell^+ \, M(q,\,\ell^+,\,\ell^+ - p_\rho^+) \;\int d\ell^- \int^{|\ell_\perp^2|\,<\,\mu_F^2} d^2\ell_\perp \, \Psi(\ell,\,\ell-p_\rho)$$

Hard part — DA $\Phi(u,\mu_F^2)$

(see Chernyak, Zhitnitsky '77; Brodsky, Lepage '79; Efremov, Radyushkin '80; ... in the case of form-factors studies)

Collinear factorization

Meson electroproduction: factorization with a GPD and a DA



$\int d^4k\, d^4\ell$ $\quad S(k,\ k+\Delta)$ $\quad H(q,\ k,\ k+\Delta)$ $\quad \Psi(\ell,\ \ell-p_\rho)$

$$= \int dk^- d\ell^+ \int dk^+ \int^{|k_\perp^2|<\mu_{F_2}^2} d^2k_\perp\, S(k,\ k+\Delta)\, H(q;\ k^-,\ k^-+\Delta^-;\ell^+,\ \ell^+-p_\rho^+) \int d\ell^- \int^{|\ell_\perp^2|<\mu_{F_1}^2} d^2\ell_\perp\, \Psi(\ell,\ \ell-p_\rho)$$

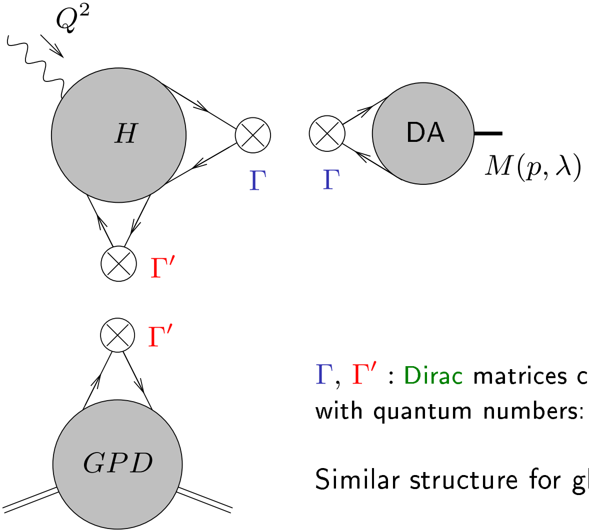
GPD $F(x,\ \xi, t, \mu_{F_2}^2)$ Hard part $T(x/\xi, u, \mu_{F_1}^2, \mu_{F_2}^2, \mu_R^2)$ DA $\Phi(u, \mu_{F_1}^2)$

Collins, Frankfurt, Strikman '97; Radyushkin '97

Collinear factorization

Meson electroproduction: factorization with a GPD and a DA

The building blocks



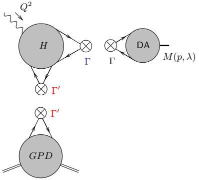
Γ, Γ' : Dirac matrices compatible with quantum numbers: C, P, T, chirality

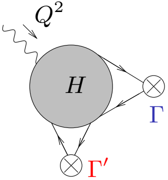
Similar structure for gluon exchange

Collinear factorization

Meson electroproduction: factorization with a GPD and a DA

The building blocks





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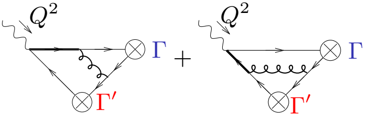
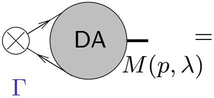
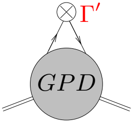


hand-bag diagrams



$= \quad \langle M(p,\lambda)|\mathcal{O}(\Psi,\,\bar{\Psi}\,A)|0\rangle$

matrix element of a non-local light-cone operator

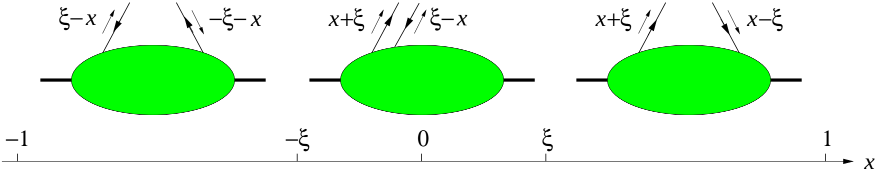


$= \quad \langle N(p')|\mathcal{O}'(\Psi,\,\bar{\Psi}\,A)|N(p)\rangle$

matrix element of a non-local light-cone operator

Collinear factorization
Twist 2 GPDs

Physical interpretation for GPDs



Emission and reabsoption of an antiquark	Emission of a quark and emission of an antiquark	Emission and reabsoption of a quark
$\sim$ PDFs for antiquarks	$\sim$ meson exchange	$\sim$ PDFs for quarks
DGLAP-II region	ERBL region	DGLAP-I region

Collinear factorization

Twist 2 GPDs

Classification of twist 2 GPDs

- For quarks, one should distinguish the exchanges
  - without helicity flip (chiral-even Γ' matrices): 4 chiral-even GPDs:
    $H^q \xrightarrow{\xi=0,t=0}$ PDF q, E^q, $\tilde{H}^q \xrightarrow{\xi=0,t=0}$ polarized PDFs Δq, $\tilde{E}^q$

$$F^q = \frac{1}{2}\int \frac{dz^+}{2\pi}\, e^{ixP^-z^+} \langle p'|\,\bar{q}(-\tfrac{1}{2}z)\,\gamma^-\,q(\tfrac{1}{2}z)\,|p\rangle\Big|_{z^-=0,\,z_\perp=0}$$
$$= \frac{1}{2P^-}\left[H^q(x,\xi,t)\,\bar{u}(p')\gamma^-u(p) + E^q(x,\xi,t)\,\bar{u}(p')\frac{i\,\sigma^{-\alpha}\Delta_\alpha}{2m}u(p)\right],$$
$$\tilde{F}^q = \frac{1}{2}\int \frac{dz^+}{2\pi}\, e^{ixP^-z^+} \langle p'|\,\bar{q}(-\tfrac{1}{2}z)\,\gamma^-\gamma_5\,q(\tfrac{1}{2}z)\,|p\rangle\Big|_{z^-=0,\,z_\perp=0}$$
$$= \frac{1}{2P^-}\left[\tilde{H}^q(x,\xi,t)\,\bar{u}(p')\gamma^-\gamma_5u(p) + \tilde{E}^q(x,\xi,t)\,\bar{u}(p')\frac{\gamma_5\,\Delta^-}{2m}u(p)\right].$$

  - with helicity flip (chiral-odd Γ' mat.): 4 chiral-odd GPDs:
    $H_T^q \xrightarrow{\xi=0,t=0}$ quark transversity PDFs $\Delta_T q$, E_T^q, $\tilde{H}_T^q$, $\tilde{E}_T^q$

$$\frac{1}{2}\int \frac{dz^+}{2\pi}\, e^{ixP^-z^+} \langle p'|\,\bar{q}(-\tfrac{1}{2}z)\,i\,\sigma^{-i}\,q(\tfrac{1}{2}z)\,|p\rangle\Big|_{z^-=0,\,z_\perp=0}$$
$$= \frac{1}{2P^-}\bar{u}(p')\left[H_T^q\,i\sigma^{-i} + \tilde{H}_T^q\,\frac{P^-\Delta^i-\Delta^-P^i}{m^2} + E_T^q\,\frac{\gamma^-\Delta^i-\Delta^-\gamma^i}{2m} + \tilde{E}_T^q\,\frac{\gamma^-P^i-P^-\gamma^i}{m}\right]$$

Collinear factorization
Twist 2 GPDs

Classification of twist 2 GPDs

- analogously, for gluons:
  - 4 gluonic GPDs without helicity flip:

    $H^g \xrightarrow{\xi=0,t=0}$ PDF $x\, g$
    E^g
    $\tilde{H}^g \xrightarrow{\xi=0,t=0}$ polarized PDF $x\, \Delta g$
    $\tilde{E}^g$

  - 4 gluonic GPDs with helicity flip:

    H_T^g
    E_T^g
    $\tilde{H}_T^g$
    $\tilde{E}_T^g$
    (no forward limit reducing to gluons PDFs here: a change of 2 units of helicity cannot be compensated by a spin 1/2 target)

A few applications

Production of an exotic hybrid meson in hard processes

Quark model and meson spectroscopy

- spectroscopy: $\vec{J} = \vec{L} + \vec{S}$; neglecting any spin-orbital interaction
  $\Rightarrow S,\ L =$ additional quantum numbers to classify hadron states

$$\vec{J}^{\,2} = J(J+1)\,, \quad \vec{S}^{\,2} = S(S+1)\,, \quad \vec{L}^{\,2} = L(L+1),$$

  with $J = |L-S|\,, \cdots, L+S$
- In the usual quark-model: meson $= q\bar{q}$ bound state with

$$C = (-)^{L+S} \quad \text{and} \quad P = (-)^{L+1}.$$

- Thus:

$$\begin{array}{llll}
S=0\,, & L=J, & J=0,\,1,\,2,\ldots: & J^{PC} = 0^{-+}(\pi,\eta),\ 1^{+-}(h_1,b_1),\ 2^{-+},\ 3^{+-},\ \ldots \\
S=1\,, & L=0\,, & J=1: & J^{PC} = 1^{--}(\rho,\omega,\phi) \\
 & L=1\,, & J=0,\,1,\,2: & J^{PC} = 0^{++}(f_0,a_0),\ 1^{++}(f_1,a_1),\ 2^{++}(f_2,a_2) \\
 & L=2\,, & J=1,\,2,\,3: & J^{PC} = 1^{--},\ 2^{--},\ 3^{--}
\end{array}$$

...

- $\Rightarrow$ the exotic mesons with $J^{PC} = 0^{--},\ 0^{+-},\ 1^{-+},\ \cdots$ are forbidden

A few applications

Production of an exotic hybrid meson in hard processes

Experimental candidates for light hybrid mesons (1)

three candidates:

- $\pi_1(1400)$
  - GAMS '88 (SPS, CERN): in $\pi^- \, p \to \eta \, \pi^0 \, n$ (through $\eta \, \pi^0 \to 4\gamma$ mode)
    M= 1406 ± 20 MeV $\Gamma = 180 \pm 30$ MeV
  - E852 '97 (BNL): $\pi^- \, p \to \eta \, \pi^- \, p$
    M=1370 ± 16 MeV $\Gamma = 385 \pm 40$ MeV
  - VES '01 (Protvino) in $\pi^- \, Be \to \eta \, \pi^- \, Be$, $\pi^- \, Be \to \eta' \, \pi^- \, Be$,
    $\pi^- \, Be \to b_1 \, \pi^- \, Be$
    M = 1316 ± 12 MeV $\Gamma = 287 \pm 25$ MeV
    but resonance hypothesis ambiguous
  - Crystal Barrel (LEAR, CERN) '98 '99 in $\bar{p} \, n \to \pi^- \, \pi^0 \, \eta$ and $\bar{p} \, p \to 2\pi^0 \, \eta$
    (through $\pi\eta$ resonance)
    M=1400 ± 20 MeV $\Gamma = 310 \pm 50$ MeV
    and M=1360 ± 25 MeV $\Gamma = 220 \pm 90$ MeV

A few applications

Production of an exotic hybrid meson in hard processes

Experimental candidates for light hybrid mesons (2)

- $\pi_1(1600)$
  - E852 (BNL): in peripheral $\pi^- p \to \pi^+\pi^-\pi^- p$ (through $\rho\pi^-$ mode) '98 '02, M = 1593 ± 8 MeV $\Gamma = 168 \pm 20$ MeV $\pi^- p \to \pi^+\pi^-\pi^-\pi^0\pi^0 p$ (in $b_1(1235)\pi^- \to (\omega\pi^0)\pi^- \to (\pi^+\pi^-\pi^0)\pi^0\pi^-$ '05 and $f_1(1285)\pi^-$ '04 modes), in peripheral $\pi^- p$ through $\eta'\pi^-$ '01
    M = 1597 ± 10 MeV $\Gamma = 340 \pm 40$ MeV
    but E852 (BNL) '06: no exotic signal in $\pi^- p \to (3\pi)^- p$ for a larger sample of data!
  - VES '00 (Protvino): in peripheral $\pi^- p$ through $\eta'\pi^-$ '93, '00, $\rho(\pi^+\pi^-)\pi^-$ '00, $b_1(1235)\pi^- \to (\omega\pi^0)\pi^-$ '00
  - Crystal Barrel (LEAR, CERN) '03 $\bar{p}p \to b_1(1235)\pi\pi$
  - COMPASS '10 (SPS, CERN): diffractive dissociation of π^- on Pb target through Primakov effect $\pi^-\gamma \to \pi^-\pi^-\pi^+$ (through $\rho\pi^-$ mode)
    M = 1660 ± 10 MeV $\Gamma = 269 \pm 21$ MeV
- $\pi_1(2000)$: seen only at E852 (BNL) '04 '05 (through $f_1(1285)\pi^-$ and $b_1(1235)\pi^-$)

A few applications

Production of an exotic hybrid meson in hard processes

What about hard processes?

- Is there a hope to see such states in hard processes, with high counting rates, and to exhibit their light-cone wave-function?
- hybrid mesons $= q\bar{q}g$ states T. Barnes '77; R. L. Jaffe, K. Johnson, and Z. Ryzak, G. S. Bali
- popular belief: $H = q\bar{q}g \Rightarrow$ higher Fock-state component $\Rightarrow$ twist-3 $\Rightarrow$ hard electroproduction of H versus ρ suppressed as $1/Q$
- This is not true!! Electroproduction of hybrid is similar to electroproduction of usual $\rho-$meson: it is twist 2 dominated

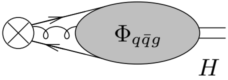
  I. V. Anikin, B. Pire, O. V. Teryaev, L. Szymanowski, S.W. '04

A few applications

Production of an exotic hybrid meson in hard processes

Distribution amplitude of exotic hybrid mesons at twist 2

- One may think that to produce $|q\bar{q}g\rangle$, the fields Ψ, $\bar{\Psi}$, A should appear explicitly in the non-local operator $\mathcal{O}(\Psi,\ \bar{\Psi}\ A)$



- If one tries to produce $H = 1^{-+}$ from a local operator, the dominant operator should be $\bar{\Psi}\gamma^\mu G_{\mu\nu}\Psi$ of twist = dimension - spin = 5 - 1 = 4
- It means that there should be a $1/Q^2$ suppression in the production amplitude of H versus the usual ρ-production (which is twist 2 dominated)
- But collinear approach describes hard exclusive processes in terms of non-local light-cone operators, among which are the twist 2 operator

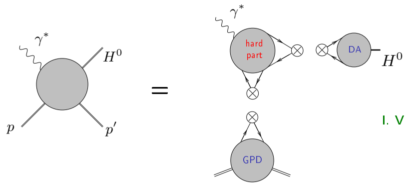
$$\bar{\psi}(-z/2)\gamma_\mu[-z/2; z/2]\psi(z/2)$$

where $[-z/2; z/2]$ is a Wilson line, necessary to fullfil gauge invariance (i.e. a "color tube" between q and $\bar{q}$) which thus hides gluonic degrees of freedom: the needed gluon is there, at twist 2. This does not requires to introduce explicitely A!

A few applications
Production of an exotic hybrid meson in hard processes

Accessing the partonic structure of exotic hybrid mesons

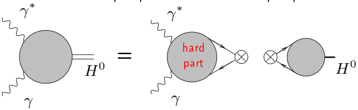
- Electroproduction $\gamma^* p \to H^0\, p$: JLab, COMPASS, EIC



prediction: $\frac{d\sigma^H}{d\sigma^\rho} \approx 15\%$

I. V. Anikin, B. Pire, O. V. Teryaev, L. Szymanowski, S.W.
Phys.Rev.D70 (2004) 011501
Phys.Rev.D71 (2005) 034021
Eur.Phys.J.C42 (2005) 163

- Channels $\gamma^*\gamma \to H$ and $\gamma^*\gamma \to \pi\eta$: BaBar, Belle, BES-III



prediction: $\frac{\left|M^{\gamma^*\gamma\to H}\right|^2}{\left|M^{\gamma^*\gamma\to\pi^0}\right|^2} \approx 20\%$

I. V. Anikin, B. Pire, O. V. Teryaev, L. Szymanowski, S.W.
Eur.Phys.J.C47 (2006)

[backup]

$\Longrightarrow$ the partonic content of exotic hybrid meson is experimentally accessible

A few applications
Spin transversity in the nucleon

What is transversity?

- Tranverse spin content of the proton:

$$|\uparrow\rangle_{(x)} \sim |\rightarrow\rangle + |\leftarrow\rangle$$
$$|\downarrow\rangle_{(x)} \sim |\rightarrow\rangle - |\leftarrow\rangle$$

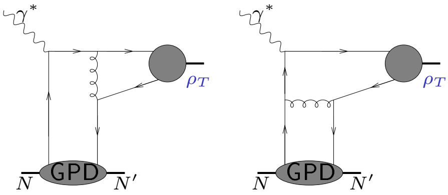
spin along x helicity state

- An observable sensitive to helicity spin flip gives thus access to the transversity $\Delta_T q(x)$, which is very badly known (first data have recently been obtained by COMPASS)
- The transversity GPDs are completely unknown
- Chirality: $q_\pm(z) \equiv \frac{1}{2}(1 \pm \gamma^5) q(z)$ with $q(z) = q_+(z) + q_-(z)$
  Chiral-even: chirality conserving
  $\bar{q}_\pm(z)\gamma^\mu q_\pm(-z)$ and $\bar{q}_\pm(z)\gamma^\mu\gamma^5 q_\pm(-z)$
  Chiral-odd: chirality reversing
  $\bar{q}_\pm(z)\cdot 1 \cdot q_\mp(-z)$, $\bar{q}_\pm(z)\cdot\gamma^5\cdot q_\mp(-z)$ and $\bar{q}_\pm(z)[\gamma^\mu,\gamma^\nu]q_\mp(-z)$
- For a massless (anti)particle, chirality = (-)helicity
- Transversity is thus a chiral-odd quantity
- QCD and QED are chiral even $\Rightarrow \mathcal{A} \sim (\text{Ch.-odd})_1 \otimes (\text{Ch.-odd})_2$

A few applications
Spin transversity in the nucleon

How to get access to transversity?

- The dominant DA for ρ_T is of twist 2 and chiral-odd ($[\gamma^\mu, \gamma^\nu]$ coupling)
- Unfortunately $\gamma^* \, N^\uparrow \to \rho_T \, N' = 0$
  - this is true at any order in perturbation theory (i.e. corrections as powers of α_s), since this would require a transfer of 2 units of helicity from the proton: impossible!
    Diehl, Gousset, Pire '99; Collins, Diehl '00
  - diagrammatic argument at Born order:



vanishes: $\gamma^\alpha [\gamma^\mu, \gamma^\nu] \gamma_\alpha = 0$

A few applications
Spin transversity in the nucleon

Can one circumvent this vanishing?

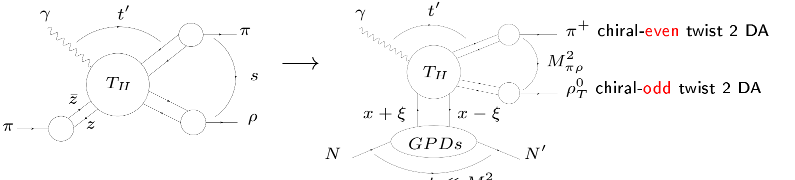
- This vanishing is true only a twist 2
- At twist 3 this process does not vanish
- However processes involving twist 3 DAs may face problems with factorization (end-point singularities: see later)
- The problem of classification of twist 3 chiral-odd GPDs is still open: Pire, Szymanowski, S.W. in progress, in the spirit of our Light-Cone Collinear Factorization framework recently developped (Anikin, Ivanov, Pire, Szymanowski, S. W.)

A few applications

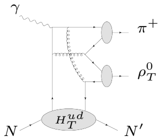
Spin transversity in the nucleon

$\gamma N \to \pi^+ \rho^0_T N'$ gives access to transversity

- Factorization à la Brodsky Lepage of $\gamma + \pi \to \pi + \rho$ at large s and fixed angle (i.e. fixed ratio $t'/s, u'/s$)
  $\Longrightarrow$ factorization of the amplitude for $\gamma + N \to \pi + \rho + N'$ at large $M^2_{\pi\rho}$



- a typical non-vanishing diagram:



M. El Beiyad, P. Pire, M. Segond, L. Szymanowski, S.W
Phys.Lett.B688:154-167,2010

see also, at large s, with $\mathbb{P}$omeron exchange:

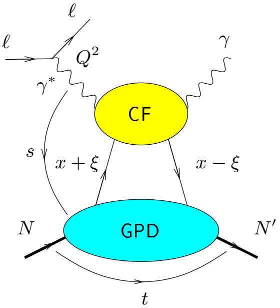
R. Ivanov, B. Pire, L. Symanowski, O. Teryaev '02

R. Enberg, B. Pire, L. Symanowski '06

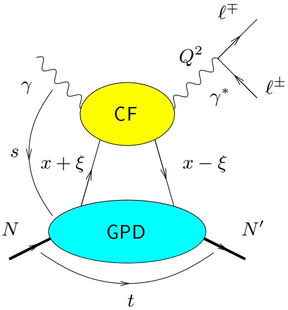
- These processes with 3 body final state can give access to all GPDs: $M^2_{\pi\rho}$ plays the role of the γ^* virtuality of usual DVCS (here in the time-like domain) JLab, COMPASS

Threshold effects for DVCS and TCS

DVCS and TCS



Deeply Virtual Compton Scattering
$lN \to l'N'\gamma$



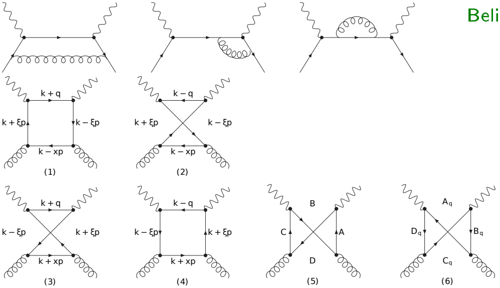
Timelike Compton Scattering
$\gamma N \to l^+l^-N'$

- TCS versus DVCS:
  - universality of the GPDs
  - another source for GPDs (special sensitivity on real part)
  - spacelike-timelike crossing and understanding the structure of the NLO corrections
- Where to measure TCS? In Ultra Peripheral Collisions
  LHC, JLab, COMPASS, AFTER

Threshold effects for DVCS and TCS

DVCS and TCS at NLO

One loop contributions to the coefficient function



Belitsky, Mueller, Niedermeier, Schafer, Phys.Lett.B474, 2000

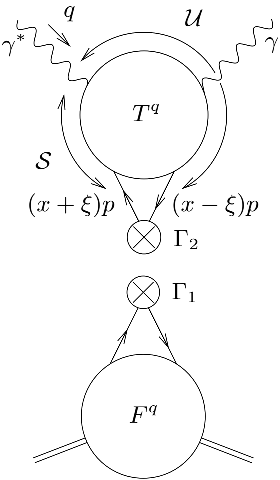
Pire, Szymanowski, Wagner Phys.Rev.D83, 2011

$$\mathcal{A}^{\mu\nu} = g_T^{\mu\nu} \int_{-1}^{1} dx \left[\sum_q^{n_F} T^q(x)\, F^q(x) + T^g(x)\, F^g(x) \right]$$

(symmetric part of the factorised amplitude)

Threshold effects for DVCS and TCS

Resummations effects are expected



- The renormalized quark coefficient functions T^q is

$$T^q = C_0^q + C_1^q + C_{coll}^q \log \frac{|Q^2|}{\mu_F^2}$$

$$C_0^q = e_q^2 \left(\frac{1}{x - \xi + i\varepsilon} - (x \to -x) \right)$$

$$C_1^q = \frac{e_q^2 \alpha_S C_F}{4\pi(x - \xi + i\varepsilon)} \left[\log^2\left(\frac{\xi - x}{2\xi} - i\varepsilon \right) + \ldots \right] - (x \to -x)$$

- Usual collinear approach: single-scale analysis w.r.t. Q^2
- Consider the invariants $\mathcal{S}$ and $\mathcal{U}$:

$$\mathcal{S} = \frac{x - \xi}{2\xi} Q^2 \quad \ll \quad Q^2 \quad \text{when } x \to \xi$$

$$\mathcal{U} = -\frac{x + \xi}{2\xi} Q^2 \quad \ll \quad Q^2 \quad \text{when } x \to -\xi$$

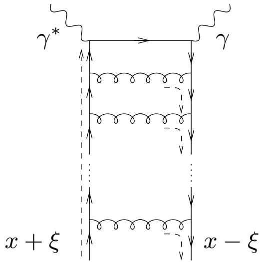
$\Rightarrow$ two scales problem; threshold singularities to be resummed

analogous to the $\log(x - x_{Bj})$ resummation for DIS coefficient functions

Threshold effects for DVCS and TCS
Resummation for Coefficient functions

Soft-collinear resummation effects for the coefficient function

- The resummation easier when using the axial gauge $p_1 \cdot A = 0$ $(p_\gamma \equiv p_1)$
- The dominant diagram are ladder-like [backup]



resummed formula (for DVCS), for $x \to \xi$:

$$(T^q)^{\text{res}} = \left(\frac{e_q^2}{x-\xi+i\epsilon} \left\{ \cosh\left[D \log\left(\frac{\xi-x}{2\xi} - i\epsilon \right) \right] - \frac{D^2}{2}\left[9 + 3\frac{\xi-x}{x+\xi} \log\left(\frac{\xi-x}{2\xi} - i\epsilon \right) \right] \right\} + C_{coll}^q \log \frac{Q^2}{\mu_F^2} \right) - (x \to -x) \quad \text{with} \quad D = \sqrt{\frac{\alpha_s C_F}{2\pi}}$$

T. Altinoluk, B. Pire, L. Szymanowski, S. W.
JHEP 1210 (2012) 49; [arXiv:1206.3115]

- Our analysis can be used for the gluon coefficient function [In progress].
- The measurement of the phenomenological impact of this procedure on the data analysis needs further analysis with the implementation of modeled generalized parton distributions [backup].

Problems

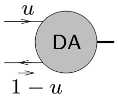
$\rho-$electroproduction: Selection rules and factorization status

- chirality = helicity for a particule, chirality = -helicity for an antiparticule
- for massless quarks: QED and QCD vertices = chiral even (no chirality flip during the interaction)
  $\Rightarrow$ the total helicity of a $q\bar{q}$ produced by a γ^* should be 0
  $\Rightarrow$ helicity of the $\gamma^* = L_z^{q\bar{q}}$ (z projection of the $q\bar{q}$ angular momentum)
- in the pure collinear limit (i.e. twist 2), $L_z^{q\bar{q}}$=0 $\Rightarrow \gamma_L^*$
- at $t = 0$, no source of orbital momentum from the proton coupling $\Rightarrow$ helicity of the meson = helicity of the photon
- in the collinear factorization approach, $t \neq 0$ change nothing from the hard side $\Rightarrow$ the above selection rule remains true
- thus: 2 transitions possible ($s-$channel helicity conservation (SCHC)):
  - $\gamma_L^* \to \rho_L$ transition: QCD factorization holds at t=2 at any order in perturbation (i.e. LL, NLL, etc...)

    Collins, Frankfurt, Strikman '97 Radyushkin '97
  - $\gamma_T^* \to \rho_T$ transition: QCD factorization has problems at t=3

    Mankiewicz-Piller '00

    $\int_0^1 \frac{du}{u}$ or $\int_0^1 \frac{du}{1-u}$ diverge (end-point singularity)



Problems

$\rho-$electroproduction: Selection rules and factorization status

Improved collinear approximation: a solution?

- keep a transverse $\ell_\perp$ dependency in the q, $\bar{q}$ momenta, used to regulate end-point singularities
- soft and collinear gluon exchange between the valence quark are responsible for large double-logarithmic effects which are conjectured to exponentiate
- this is made easier when using the impact parameter space $b_\perp$ conjugated to $\ell_\perp$ $\Rightarrow$ Sudakov factor

$$\exp[-S(u,b,Q)]$$

- S diverges when $b_\perp \sim O(1/\Lambda_{QCD})$ (large transverse separation, i.e. small transverse momenta) or $u \sim O(\Lambda_{QCD}/Q)$ Botts, Sterman '89
  $\Rightarrow$ regularization of end-point singularities for $\pi \to \pi\gamma^*$ and $\gamma\gamma^*\pi^0$ form factors, based on the factorization approach Li, Sterman '92
- it has been proposed to combine this perturbative resummation tail effect with an ad-hoc non-perturbative gaussian ansatz for the DAs

$$\exp[-a^2\,|k_\perp^2|/(u\bar{u})]$$

which gives back the usual asymptotic DA $6u\bar{u}$ when integrating over $k_\perp$
$\Rightarrow$ practical tools for meson electroproduction phenomenology
Goloskokov, Kroll '05

QCD at large s

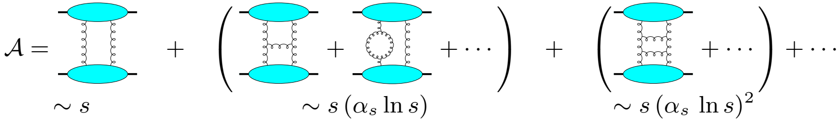
Theoretical motivations

A particular regime for QCD:
The perturbative Regge limit $s \to \infty$

Consider the diffusion of two hadrons h_1 and h_2:

- $\sqrt{s}\,(= E_1 + E_2$ in the center-of-mass system) $\gg$ other scales (masses, transfered momenta, ...) eg $x_B \to 0$ in DIS
- other scales comparable (virtualities, etc...) $\gg \Lambda_{QCD}$

regime $\alpha_s \ln s \sim 1 \implies$ dominant sub-series:



$$\implies \sigma_{tot}^{h_1\, h_2 \to tout} = \frac{1}{s} Im\,\mathcal{A} \sim s^{\alpha_{\mathbb{P}}(0)-1}$$

with $\alpha_{\mathbb{P}}(0) - 1 = C\,\alpha_s \; (C > 0)$ hard Pomeron (Balitsky, Fadin, Kuraev, Lipatov)

- This result violates QCD S matrix unitarity
  ($S\,S^\dagger = S^\dagger\,S = 1$ i.e. $\sum Prob. = 1$)
- Until when this result could be applicable, and how to improve it?
- How to test this dynamics experimentally, in particular based on exclusive processes?

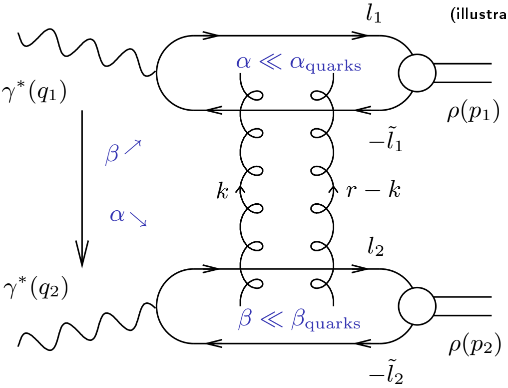
QCD at large s

k_T factorization

$\gamma^* \gamma^* \to \rho\,\rho$ as an example

- Use Sudakov decomposition $k = \alpha\, p_1 + \beta\, p_2 + k_\perp$ $(p_1^2 = p_2^2 = 0,\ 2p_1 \cdot p_2 = s)$
- write $d^4k = \frac{s}{2}\, d\alpha\, d\beta\, d^2k_\perp$
- $t-$channel gluons with non-sense polarizations $(\epsilon_{NS}^{up} = \frac{2}{s}\, p_2,\ \epsilon_{NS}^{down} = \frac{2}{s}\, p_1)$ dominate at large s

(illustration for 2-body case)



$\Rightarrow$ set $\alpha = 0$ and $\int d\beta$

$\int d^2k_\perp$

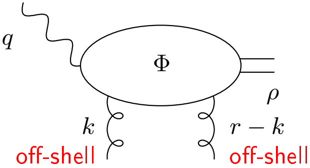
$\Rightarrow$ set $\beta = 0$ and $\int d\alpha$

QCD at large s
k_T factorization

Impact representation for exclusive processes $\underline{k}$ = Eucl. $\leftrightarrow$ $k_\perp$ = Mink.

$$\mathcal{M} = is \int \frac{d^2\,\underline{k}}{(2\pi)^2 \underline{k}^2\,(\underline{r}-\underline{k})^2} \Phi^{\gamma^*(q_1)\to\rho(p_1^\rho)}(\underline{k}, \underline{r}-\underline{k})\; \Phi^{\gamma^*(q_2)\to\rho(p_2^\rho)}(-\underline{k}, -\underline{r}+\underline{k})$$

$\Phi^{\gamma^*(q_1)\to\rho(p_1^\rho)}$: $\gamma^*_{L,T}(q) g(k_1) \to \rho_{L,T}\, g(k_2)$ impact factor



Gauge invariance of QCD:

- probes are color neutral
  $\Rightarrow$ their impact factor should vanish when $\underline{k} \to 0$ or $\underline{r}-\underline{k} \to 0$
- At twist-3 level (for the $\gamma^*_T \to \rho_T$ transition), gauge invariance is a non-trivial constraint when combining 2- *and* 3-body correlators

QCD at large s

Phenomenological applications: Meson production at HERA

Diffractive meson production at HERA

HERA (DESY, Hambourg): first and single $e^{\pm}p$ collider (1992-2007)

- The "easy" case (from factorization point of view): J/Ψ production ($u \sim 1/2$: non-relativistic limit for bound state) combined with k_T-factorisation
  Ryskin '93; Frankfurt, Koepf, Strikman '98; Ivanov, Kirschner, Schäfer, Szymanowski '00; Motyka, Enberg, Poludniowski '02
- Exclusive vector meson photoproduction at large t (= hard scale):
  $\gamma(q) + P \to \rho_{L,T}(p_1) + P$
  based on k_T-factorization:
  Forshaw, Ryskin '95; Bartels, Forshaw, Lotter, Wüsthoff '96; Forshaw, Motyka, Enberg, Poludniowski '03
  - H1, ZEUS data seems to favor BFKL
  - but end-point singularities for ρ_T are regularized with a quark mass: $m = m_\rho/2$
  - the spin density matrix is badly described
- Exclusive electroproduction of vector meson
  $\gamma^*_{L,T}(q) + P \to \rho_{L,T}(p_1) + P$ Goloskokov, Kroll '05
  based on improved collinear factorization for the coupling with the meson DA and collinear factorization for GPD coupling

QCD at large s

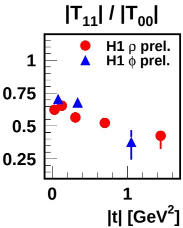
Phenomenological applications: Meson production at HERA

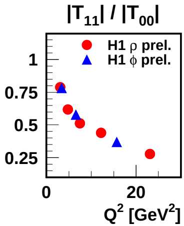
Polarization effects in $\gamma^* P \to \rho P$ at HERA

- Very precise experimental data on the spin density matrix (i.e. correlations between γ^* and ρ polarizations)
- for $t = t_{min}$ one can experimentally distinguish

$$\begin{cases} \gamma_L^* \to \rho_L : \text{dominates} \quad (\text{“twist 2”: amplitude } |\mathcal{A}| \sim \frac{1}{Q}) \\ \gamma_T^* \to \rho_T : \text{ visible} \qquad (\text{“twist 3”: amplitude } |\mathcal{A}| \sim \frac{1}{Q^2}) \end{cases}$$

- How to calculate the $\gamma_T^* \to \rho_T$ transition from first principles?

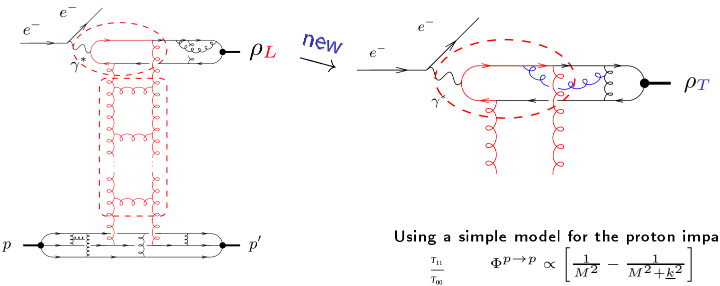




QCD at large s

Phenomenological applications: Meson production at HERA

Diffractive exclusive process $e^- p \to e^- \, p \, \rho_{L,T}$



first description combining beyond leading twist

- collinear factorisation
- k_T −factorisation

I. V. Anikin, D .Yu. Ivanov, B. Pire, L. Szymanowski, S.W.

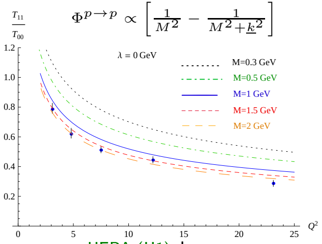
Phys.Lett.B682 (2010) 413-418

Nucl.Phys.B828 (2010) 1-68

HERA, EIC, LHeC, AFP@LHC

Using a simple model for the proton impact factor:

$$\Phi^{p \to p} \propto \left[\frac{1}{M^2} - \frac{1}{M^2 + \underline{k}^2}\right]$$



HERA (H1) data

I. V. Anikin, A. Besse, D .Yu. Ivanov, B. Pire, L. Szymanowski, S.W.
Phys.Rev. D84 (2011) 054004

QCD at large s

Phenomenological applications: exclusive processes at Tevatron, RHIC, LHC, ILC

Exclusive $\gamma^{(*)}\gamma^{(*)}$ processes = gold place for testing QCD at large s

Proposals in order to test perturbative QCD in the large s limit
(t-structure of the hard $\mathbb{P}$omeron, saturation, $\mathbb{O}$dderon...)

- $\gamma^{(*)}(q) + \gamma^{(*)}(q') \to J/\Psi\, J/\Psi$ Kwiecinski, Motyka '98
- $\gamma^*_{L,T}(q) + \gamma^*_{L,T}(q') \to \rho_L(p_1) + \rho_L(p_2)$ process in
  $e^+ e^- \to e^+ e^- \rho_L(p_1) + \rho_L(p_2)$ with double tagged lepton at ILC

  Pire, Szymanowski, S. W. '04; Pire, Szymanowski, Enberg, S. W. '06; Ivanov, Papa '06; Segond, Szymanowski, S. W. '07

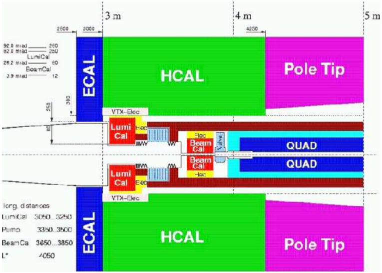
  conclusion: feasible at ILC (high energy and high luminosity); BFKL NLL enhancement with respect to Born and DGLAP contributions
- What about the $\mathbb{O}$dderon? C-parity of $\mathbb{O}$dderon = -1
  consider $\gamma + \gamma \to \pi^+\pi^-\pi^+\pi^-$: $\pi^+\pi^-$ pair has no fixed C-parity
  $\Rightarrow$ Odderon and Pomeron can interfere
  $\Rightarrow$ Odderon appears linearly in the charge asymmetry
  Pire, Schwennsen, Szymanowski, S. W. '07
  = example of possibilities offered by ultraperipheral exclusive processes at LHC [backup]
  (p, $\bar{p}$ or A as effective sources of photon)
  but the distinction with pure QCD processes (with gluons intead of a photon) is tricky...

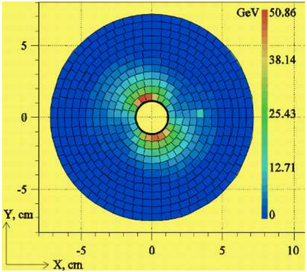
QCD at large s

Phenomenological applications: exclusive test of $\mathbb{P}$omeron

An example of realistic exclusive test of $\mathbb{P}$omeron: $\gamma^{(*)}\gamma^{(*)} \to \rho\,\rho$
as a subprocess of $e^-\,e^+ \to e^-\,e^+\,\rho^0_L\,\rho^0_L$

- ILC should provide $\begin{cases} \text{very large } \sqrt{s}\ (= 500 \text{ GeV}) \\ \text{very large luminosity } (\simeq 125 \text{ fb}^{-1}/\text{year}) \end{cases}$
- detectors are planned to cover the very forward region, close from the beampipe (directions of out-going e^+ and e^- at large s)





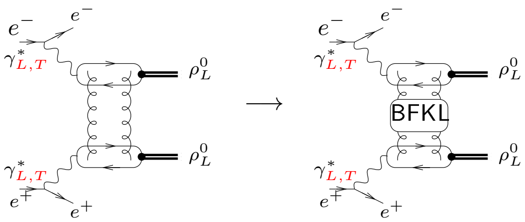
good efficiency of tagging for outgoing $e^\pm$ for $E_e > 100$ GeV and $\theta > 4$ mrad (illustration for LDC concept)

- could be equivalently done at LHC based on the AFP project

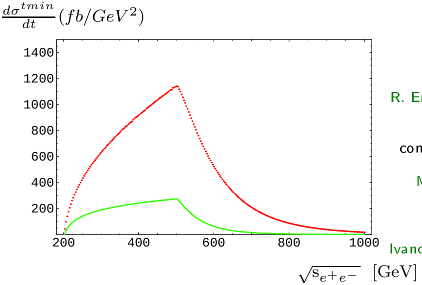
QCD at large s

Phenomenological applications: exclusive test of ℙomeron

QCD effects in the Regge limit on $\gamma^{(*)}\gamma^{(*)} \to \rho\,\rho$



$\simeq 4.10^3$ events/year $\simeq 2.10^4$ events/year



proof of feasibility:
B. Pire, L. Szymanowski and S. W.
Eur.Phys.J.C44 (2005) 545

proof of visible BFKL enhancement:
R. Enberg, B. Pire, L. Szymanowski and S. W.
Eur.Phys.J.C45 (2006) 759

comprensive study of γ^* polarization effects and event rates:
M. Segond, L. Szymanowski and S. W.
Eur. Phys. J. C **52** (2007) 93

NLO BFKL study:
Ivanov, Papa '06 '07; Caporale, Papa, Vera '08

Beyond leading twist

Light-Cone Collinear Factorization versus Covariant Collinear Factorization

- The Light-Cone Collinear Factorization, a new self-consistent method, while non-covariant, is very efficient for practical computations
  Anikin, Ivanov, Pire, Szymanowski, S.W. '09
  - inspired by the inclusive case
    Ellis, Furmanski, Petronzio '83; Efremov, Teryaev '84
  - axial gauge
  - parametrization of matrix element along a light-like prefered direction $z = \lambda n$ $(n = 2\,p_2/s)$.
  - non-local correlators are defined along this prefered direction, with contributions arising from Taylor expansion up to needed term for a given twist order computation
  - their number is then reduced to a minimal set combining equations of motion and $n-$independency condition
- Another approach (Braun, Ball), fully covariant but much less convenient when practically computing coefficient functions, can equivalently be used
- We have established the dictionnary between these two approaches
- This as been explicitly checked for the $\gamma^*_T \to \rho_T$ impact factor at twist 3

  Anikin, Ivanov, Pire, Szymanowski, S.W.

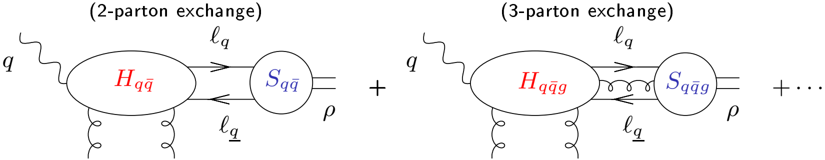
  Nucl.Phys.B 828 (2010) 1-68; Phys.Lett.B682 (2010) 413

Beyond leading twist
Light-Cone Collinear Factorization

- The impact factor $\Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)}$ can be written as

$$\Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)} = \int d^4\ell \cdots \mathrm{tr}[H^{(\lambda_\gamma)}(\ell\cdots) \quad S^{(\lambda_\rho)}(\ell\cdots)]$$

hard part — soft part



- Soft parts:

$$S_{q\bar{q}}(\ell_q) = \int d^4z\, e^{-i\ell_q\cdot z}\langle\rho(p)|\psi(0)\,\bar{\psi}(z)|0\rangle$$

$$S_{q\bar{q}q}(\ell_q,\ell_g) = \int d^4z_1 \int d^4z_2\, e^{-i(\ell_q\cdot z_1+\ell_g\cdot z_2)}\langle\rho(p)|\psi(0)\,gA_\alpha^\perp(z_2)\bar{\psi}(z_1)|0\rangle$$

Beyond leading twist
Light-Cone Collinear Factorization

Light-Cone Collinear Factorization

- Sudakov expansion in the basis $p \sim p_\rho,\ n$ ($p^2 = n^2 = 0$ and $p \cdot n = 1$)

$$l_\mu \;=\; u\, p_\mu \;+\; l^\perp_\mu \;+\; (l \cdot p)\, n_\mu, \qquad u = l \cdot n$$

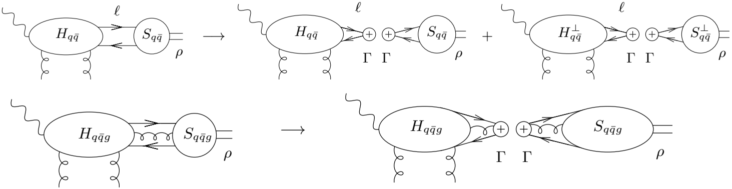
$$1 \qquad 1/Q \qquad 1/Q^2$$

- Taylor expansion of the hard part $H(\ell)$ along the collinear direction p:

$$H(\ell) = H(up) \;+\; \left.\frac{\partial H(\ell)}{\partial \ell_\alpha}\right|_{\ell = up} (\ell - u\,p)_\alpha + \ldots \quad \text{with} \quad (\ell - u\,p)_\alpha \approx \ell^\perp_\alpha$$

- $l^\perp_\alpha \xrightarrow{Fourier}$ derivative of the soft term: $\int d^4z\; e^{-i\ell\cdot z} \langle \rho(p)|\psi(0)\, i\, \overleftrightarrow{\partial_{\alpha\perp}}\, \bar\psi(z)|0\rangle$

- Color + spinor factorization = Fierz transforms:



Beyond leading twist
Light-Cone Collinear Factorization

2-body non-local correlators

ρ_L twist 2

ρ_T kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- vector correlator

$$\langle\rho(p)|\bar{\psi}(z)\gamma_\mu\psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, \left[\varphi_1(y)\, (e^* \cdot n) p_\mu + \varphi_3(y)\, e^{*T}_\mu\right]$$

- axial correlator

$$\langle\rho(p)|\bar{\psi}(z)\gamma_5\gamma_\mu\psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, i\, \varphi_A(y)\, \varepsilon_{\mu\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta\, n_\delta$$

- vector correlator with transverse derivative

$$\langle\rho(p)|\bar{\psi}(z)\gamma_\mu\, i\, \overleftrightarrow{\partial^\perp_\alpha}\, \psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, \varphi^T_1(y)\, p_\mu e^{*T}_\alpha$$

- axial correlator with transverse derivative

$$\langle\rho(p)|\bar{\psi}(z)\gamma_5\gamma_\mu\, i\, \overleftrightarrow{\partial^\perp_\alpha}\, \psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, i\, \varphi^T_A(y)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta\, n_\delta,$$

where y ($\bar{y} \equiv 1-y$) = momentum fraction along $p \equiv p_1$ of the quark (antiquark) and $\stackrel{\mathcal{F}}{=} \int_0^1 dy\, \exp\,[i\, y\, p \cdot z]$, with $z = \lambda n$

$\Rightarrow$ 5 2-body DAs

Beyond leading twist
Light-Cone Collinear Factorization

3-body non-local correlators **genuine twist 3**

- vector correlator

$$\langle\rho(p)|\bar{\psi}(z_1)\gamma_\mu g A_\alpha^T(z_2)\psi(0)|0\rangle \stackrel{\mathcal{F}_2}{=} m_\rho\, f_3^V\; B(y_1,y_2)\, p_\mu\, e_\alpha^{*T},$$

- axial correlator

$$\langle\rho(p)|\bar{\psi}(z_1)\gamma_5\gamma_\mu g A_\alpha^T(z_2)\psi(0)|0\rangle \stackrel{\mathcal{F}_2}{=} m_\rho\, f_3^A\; i\, D(y_1,y_2)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e_\lambda^{*T}\, p_\beta\, n_\delta,$$

where y_1, $\bar{y}_2$, $y_2 - y_1$ = quark, antiquark, gluon momentum fraction

and $\stackrel{\mathcal{F}_2}{=} \int_0^1 dy_1 \int_0^1 dy_2 \; \exp\left[i\, y_1\, p\cdot z_1 + i(y_2-y_1)\, p\cdot z_2\right]$, with $z_{1,2} = \lambda n$

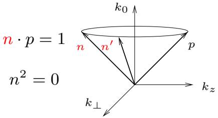
$\Rightarrow$ 2 3-body DAs

Beyond leading twist
Light-Cone Collinear Factorization

Minimal set of DAs

- Number of non-perturbative quantities: a priori 7 at twist 3
  (5 2-parton DA and 2 2-parton DA)
- Non-perturbative correlators cannot be obtained perturbatively!
- One should reduce their number to a minimal set before any use of a model or any measure on the QCD lattice
- independence w.r.t the choice of the vector n defining
  - the light-cone direction z: $z = \lambda n$
  - the ρ_T polarization vector: $e_T \cdot n = 0$
  - the axial gauge: $n \cdot A = 0$

$$\mathcal{A} = H \otimes S \qquad \frac{d\mathcal{A}}{dn_\perp^\mu} = 0 \Rightarrow S \text{ are related}$$



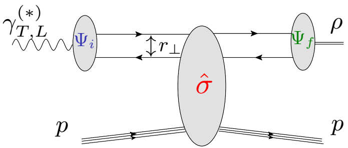
- We have proven that 3 independent Distribution Amplitudes are necessary:

$$\left\{ \begin{array}{ll} \text{QCD equations of motion} & \text{2 equations} \\ \text{Arbitrariness in the choice of } n & \text{2 equations} \end{array} \right.$$

$\varphi_1(y)$ $\leftarrow$ 2-body twist 2 correlator
$B(y_1, y_2)$ $\leftarrow$ 3-body genuine twist 3 vector correlator
$D(y_1, y_2)$ $\leftarrow$ 3-body genuine twist 3 axial correlator

Beyond leading twist
Dipole representation and saturation effects

The dipole picture at high energy



Nikolaev, Zakharov '91

- Initial Ψ_i and final Ψ_f states wave functions of projectiles
- Primitive picture: proton = color dipole
  scattering amplitude for two $t-$ channel exchanged gluons:

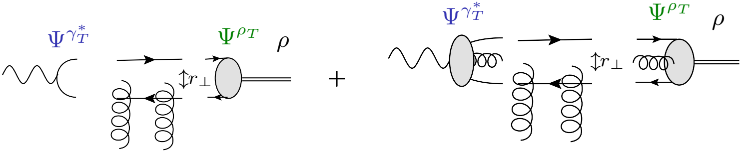
$$\mathcal{N}(\underline{r}, \underline{k}) = \frac{4\pi\alpha_s}{N_c}\left(1 - e^{i\underline{k}\cdot\underline{r}}\right)\left(1 - e^{-i\underline{k}\cdot\underline{r}}\right)$$

- Real proton: $\mathcal{N} \to \hat{\sigma}_{\text{dipole-target}}$ = universal scattering amplitude
  Golec-Biernat Wusthoff '98
  - color transparency for small $r_\perp$: $\hat{\sigma}_{\text{dipole-target}} \sim r_\perp^2$
  - saturation for large $r_\perp \sim 1/Q_{\text{sat}}$: $T \lesssim 1$
- Data for ρ production calls for models encoding saturation
  Munier, Stasto, Mueller '04; Kowalski, Motyka, Watt '06
- The dipole representation is consistent with the twist 2 collinear factorization

Beyond leading twist
Dipole representation and saturation effects

A dipole picture beyond leading twist?

- New: the dipole picture is still consistent with collinear factorization at higher twist order:



twist 2 + kinematical twist 3   genuine twist 3

A. Besse, L. Szymanowski, S. W., NPB 867 (2013) 19-60

- key ideas:
  - reformulate the Light-Cone Collinear Factorization in the Fourier conjugated coordinate space: $\ell_\perp \leftrightarrow r_\perp$
  - use QCD equations of motion

Beyond leading twist

Factorization in coordinate space: the 2-parton contribution

Light-Cone Collinear Factorization in the coordinate space

- Recall: impact factors $\Phi_{q\bar{q}}^{\gamma^* \to \rho} = -\frac{1}{4}\int d^4\ell \; Tr(H_{q\underline{q}}\,\Gamma)(\ell)\; S_{q\underline{q}\Gamma}(\ell)$
- Collinear approximation $\Rightarrow$ expansion around $\ell_\perp = 0$:

Gives the moments of $S_{q\underline{q}\Gamma}$

$$Tr(H_{q\underline{q}}\,\Gamma)(\ell) = \int \frac{d^2 r_\perp}{2\pi} \tilde{H}^{\Gamma}_{q\underline{q}}(y, r_\perp)\, e^{-i\ell_\perp \cdot r_\perp} = \int \frac{d^2 r_\perp}{2\pi} \underbrace{\tilde{H}^{\Gamma}_{q\underline{q}}(y, r_\perp)}_{\text{factorizes out}} \overbrace{(\underbrace{1 - i\ell_\perp \cdot r_\perp}_{\text{twist 2 and 3}} + \cdots)}$$

- 2-parton impact factor up to twist 3 (Wandzura-Wilczek (WW) approximation):

$$\Phi_{q\bar{q}}^{\gamma^* \to \rho} = -\frac{1}{4} m_\rho f_\rho \int dy \int \frac{d^2 r_\perp}{(2\pi)} \Big\{ \tilde{H}^{\gamma,\mu}_{q\underline{q}}(y, \underline{r}) \Big(\varphi_3(y)\, e^*_{\rho\mu} + i\,\varphi_1^T(y)\, p_{1\mu}(\underline{e}^*_\rho \cdot \underline{r})\Big)$$
$$+ \tilde{H}^{\gamma_5\gamma,\mu}_{q\underline{q}}(y, \underline{r}) \Big(i\,\varphi_A(y)\, \varepsilon_{\mu e^*_\rho p_1 n} + \varphi_A^T(y)\, p_{1\mu}\, \varepsilon_{r_\perp e^*_\rho p_1 n}\Big)\Big\}$$

- The Fourier transform of the hard part gives:

Cancels due to EOM in WW approx.

$$\Phi_{q\underline{q}}^{\gamma^* \to \rho} = \int dy \int d^2\underline{r}\; \psi_{(q\underline{q})}^{\gamma^*_T \to \rho_T} \times \mathcal{N}(\underline{r}, \underline{k}) + \text{Hard Terms} \times \overbrace{(2y\bar{y}\varphi_3(y) + (y - \bar{y})\varphi_1^T(y) + \varphi_A^T(y))}$$

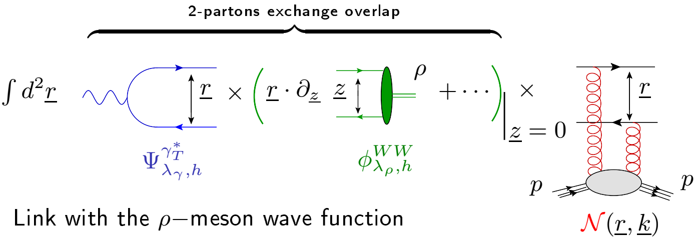
$\Rightarrow$ dipole picture!

Beyond leading twist

Factorization in coordinate space: the 2-parton contribution

WW approximation: interpretation

- Scanning the $\rho-$meson wave function:



- Link with the $\rho-$meson wave function

$$\Psi^{\rho\, q\underline{q}}_{\lambda_\rho, h} = \text{Spinor part} \times \varphi^{(q\underline{q})}_{\lambda_\rho}$$

$$\underbrace{\phi^{WW}_{\lambda_\rho, h}(y, \underline{r})}_{\sim \text{combination of DAs}} \quad \propto (\underline{e}^{(\lambda_\rho)} \cdot \underline{r}) \frac{y\delta_{h,\lambda_\rho} + \bar{y}\delta_{h,-\lambda_\rho}}{y\bar{y}} \int^{|\ell_\perp|<\mu_F} d^2\ell_\perp\, \ell_\perp^2\, \varphi^{(q\underline{q})}_{\lambda_\rho}(y, \ell_\perp)$$

Beyond leading twist

Factorization in coordinate space: the complete twist 3 contribution

- The 3-parton amplitude in transverse coordinate space at twist 3:

$$\Phi^{\gamma^*\to\rho}_{q\underline{q}g} = -\frac{im_\rho f_\rho}{4}\int dy_1 dy_g \int \frac{d^2 r_{1\perp}}{(2\pi)^2}\frac{d^2 r_{g\perp}}{(2\pi)^2}\Big[\zeta^V_{3\rho} B(y_1,y_2) p_\mu e_{\rho\perp\alpha}\ \tilde{H}^{\alpha,\gamma^\mu}_{q\underline{q}g}(y_1,y_g,r_{1\perp},r_{g\perp}) + \zeta^A_{3\rho} iD(y_1,y_2)\, p_\mu\, \varepsilon_{\alpha e_{\rho\perp} pn}\ \tilde{H}^{\alpha,\gamma^\mu\gamma_5}_{q\underline{q}g}(y_1,y_g,r_{1\perp},r_{g\perp})\Big]$$

- 3-partons exchanged; however, no quadrupole structure involved
  (even at finite N_c, beyond the 't Hooft limit)
- 3-partons results:

$$\Phi^{\gamma_T^*\to\rho_T}_{q\underline{q}g} \propto \int dy_1 \int dy_2 \int d^2\underline{r}\,\psi^{\gamma_T^*\to\rho_T}_{(q\underline{q}g)}(y_1,y_2,\underline{r}) \times \mathcal{N}(\underline{r},\underline{k}) + \int dy_1\, dy_2\, \frac{2\,S(y_1,y_2)}{\bar{y}_1}$$

$(S(y_1,y_2) = \zeta^V_\rho(\mu^2)B(y_1,y_2;\mu^2) + \zeta^A_\rho(\mu^2)D(y_1,y_2;\mu^2))$

- Full twist 3 impact factor:

$$\Phi^{\gamma_T^*\to\rho_T} = \Phi^{\gamma_T^*\to\rho_T}_{q\underline{q}} + \Phi^{\gamma_T^*\to\rho_T}_{q\underline{q}g} \propto \int dy_i \int d^2\underline{r}\,\mathcal{N}(\underline{r},\underline{k})\left(\psi^{\gamma_T^*\to\rho_T}_{(q\underline{q})}(y,\underline{r}) + \psi^{\gamma_T^*\to\rho_T}_{(q\underline{q}g)}(y_1,y_2,\underline{r})\right)$$
$$+\underbrace{\int \frac{dy}{y\bar{y}}\Big(2y\bar{y}\varphi_3(y) + (y-\bar{y})\varphi_1^T(y) + \varphi_A^T(y)\Big) + \int dy_1\, dy_2\, \frac{2\,S(y_1,y_2)}{\bar{y}_1}}_{\text{Cancel due to EOM of QCD}}$$

$\Rightarrow$ dipole picture again!

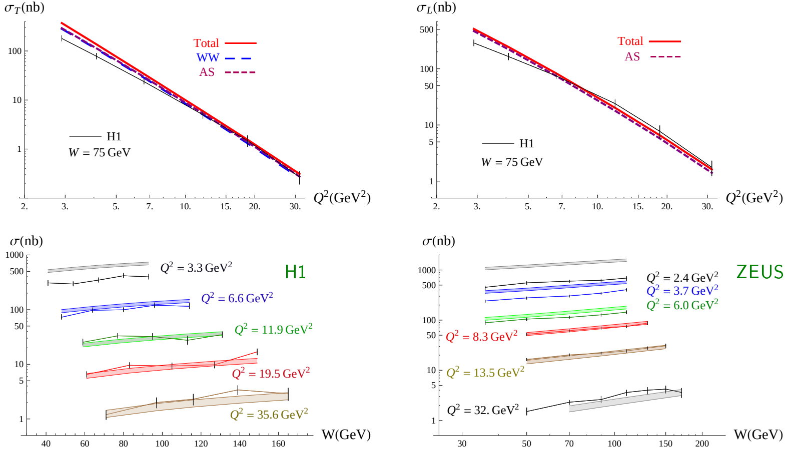
Beyond leading twist

Comparison with data

Comparison with H1 and ZEUS data

A. Besse, L. Szymanowski, S.W. [arXiv:1302.1766]

We use a model for the dipole cross-section $\hat{\sigma}$:
running coupling Balitsky Kovchegov numerical solution (i.e. include saturation effects at Leading Order) Albacete, Armesto, Milhano, Quiroga Arias, Salgado, 2011



Conclusion

- Since a decade, there have been much progress in the understanding of hard exclusive processes
  - at medium energies, there is now a conceptual framework starting from first principle, allowing to describe a huge number of processes
  - at high energy, the impact representation is a powerful tool for describing exclusive processes in diffractive experiments; they are and will be essential for studying QCD in the hard Regge limit ($\mathbb{P}$omeron, $\mathbb{O}$dderon, saturation...)
- Still, some problems remain:
  - proofs of factorization have been obtained only for very few processes (ex.: $\gamma^* p \to \gamma p$, $\gamma_L^* p \to \rho_L p$)
  - for some other processes factorization is highly plausible, but not fully demonstrated at any order (ex.: processes involving GDAs and TDAs)
  - some processes explicitly show sign of breaking of factorization (ex.: $\gamma_T^* p \to \rho_T p$ which has end-point singularities at Leading Order)
  - models and results from the lattice or from AdS/QCD for the non-perturbative correlators entering GPDs, DAs, GDAs, TDAs are needed, even at a qualitative level!
  - the effect of QCD evolution, the NLO corrections, choice of renormalization/factorization scale, power corrections, threshold resummations will be very relevant to interpret and describe the forecoming data
- Links between theoretical and experimental communities are very fruitful

  HERA, HERMES, Tevatron, LHC, JLab, Compass, BaBar, BELLE, EIC, LHeC, ILC

Distribution amplitude and quantum numbers: C-parity

- Define the H DA as (for long. pol.)

$$\langle H(p,0)|\bar{\psi}(-z/2)\gamma_\mu[-z/2;z/2]\psi(z/2)|0\rangle\Big|_{\substack{z^2=0\\ z+=0\\ z_\perp=0}} = i f_H M_H e_\mu^{(0)}\int_0^1 dy\, e^{i(\bar{y}-y)p\cdot z/2}\phi_L^H(y)$$

- Expansion in terms of local operators

$$\langle H(p,\lambda)|\bar{\psi}(-z/2)\gamma_\mu[-z/2;z/2]\psi(z/2)|0\rangle = \sum_n \frac{1}{n!} z_{\mu_1}..z_{\mu_n}\langle H(p,\lambda)|\bar{\psi}(0)\gamma_\mu \overset{\leftrightarrow}{D}_{\mu_1} .. \overset{\leftrightarrow}{D}_{\mu_n} \psi(0)|0\rangle,$$

- $C-$parity: $\begin{cases} H \text{ selects the odd-terms:} & C_H = (-) \\ \rho \text{ selects even-terms:} & C_\rho = (-) \end{cases}$

$$\langle H(p,\lambda)|\bar{\psi}(-z/2)\gamma_\mu[-z/2;z/2]\psi(z/2)|0\rangle = \sum_{n\ odd} \frac{1}{n!} z_{\mu_1}..z_{\mu_n}\langle H(p,\lambda)|\bar{\psi}(0)\gamma_\mu \overset{\leftrightarrow}{D}_{\mu_1} .. \overset{\leftrightarrow}{D}_{\mu_n} \psi(0)|0\rangle$$

- Special case $n=1$: $\mathcal{R}_{\mu\nu} = \mathsf{S}_{(\mu\nu)}\bar{\psi}(0)\gamma_\mu \overset{\leftrightarrow}{D}_\nu \psi(0)$

$\mathsf{S}_{(\mu\nu)}$ = symmetrization operator: $\mathsf{S}_{(\mu\nu)}T_{\mu\nu} = \frac{1}{2}(T_{\mu\nu}+T_{\nu\mu})$

Non perturbative imput for the hybrid DA

- We need to fix f_H (the analogue of f_ρ)
- This is a non-perturbative imput
- Lattice does not yet give information
- The operator $\mathcal{R}_{\mu\nu}$ is related to quark energy-momentum tensor $\Theta_{\mu\nu}$:

$$\mathcal{R}_{\mu\nu} = -i\,\Theta_{\mu\nu}$$

- Rely on QCD sum rules: resonance for $M \approx 1.4$ GeV
  I. I. Balitsky, D. Diakonov, and A. V. Yung

$$f_H \approx 50\,\mathrm{MeV}$$

$f_\rho = 216$ MeV

- Note: f_H evolves according to the γ_{QQ} anomalous dimension

$$f_H(Q^2) = f_H \left(\frac{\alpha_S(Q^2)}{\alpha_S(M_H^2)}\right)^{K_1} \qquad K_1 = \frac{2\,\gamma_{QQ}(1)}{\beta_0} \,,$$

A few applications

Electroproduction of an exotic hybrid meson

Counting rates for H versus ρ electroproduction: order of magnitude

- Ratio:

$$\frac{d\sigma^H(Q^2,x_B,t)}{d\sigma^\rho(Q^2,x_B,t)} = \left|\frac{f_H}{f_\rho}\,\frac{(e_u\mathcal{H}^-_{uu}-e_d\mathcal{H}^-_{dd})\,\mathcal{V}^{(H,-)}}{(e_u\mathcal{H}^+_{uu}-e_d\mathcal{H}^+_{dd})\,\mathcal{V}^{(\rho,+)}}\right|^2$$

- Rough estimate:
  - neglect $\bar{q}$ *i.e.* $x \in [0,1]$

    $\Rightarrow ImA_H$ and ImA_ρ are equal up to the factor $\mathcal{V}^M$
  - Neglect the effect of $Re\mathcal{A}$

$$\frac{d\sigma^H(Q^2,x_B,t)}{d\sigma^\rho(Q^2,x_B,t)} \approx \left(\frac{5f_H}{3f_\rho}\right)^2 \approx 0.15$$

- More precise study based on *Double Distributions* to model GPDs + effects of varying μ_R: order of magnitude unchanged

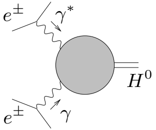
- The range around 1400 MeV is dominated by the $a_2(1329)(2^{++})$ resonance
  - possible interference between H and a_2
  - identification through the $\pi\eta$ GDA, main decay mode for the $\pi_1(1400)$ candidate, through angular asymmetry in θ_π in the $\pi\eta$ cms

A few applications

Electroproduction of an exotic hybrid meson

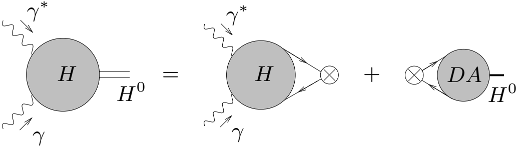
Hybrid meson production in e^+e^- colliders

- Hybrid can be copiously produced in $\gamma^*\gamma$, i.e. at e^+e^- colliders with one tagged out-going electron

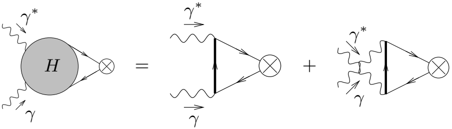


BaBar, Belle

- This can be described in a hard factorization framework:



with



Counting rates for H^0 versus π^0

- Factorization gives:

$$\mathcal{A}^{\gamma\gamma^* \to H^0}(\gamma\gamma^* \to H_L) = (\epsilon_\gamma \cdot \epsilon_\gamma^*) \frac{(e_u^2 - e_d^2) f_H}{2\sqrt{2}} \int_0^1 dz\, \Phi^H(z) \left(\frac{1}{\bar{z}} - \frac{1}{z}\right)$$

- Ratio H^0 versus π^0:

$$\frac{d\sigma^H}{d\sigma^{\pi^0}} = \left| \frac{f_H \int_0^1 dz\ \Phi^H(z) \left(\frac{1}{z} - \frac{1}{\bar{z}}\right)}{f_\pi \int_0^1 dz\ \Phi^\pi(z) \left(\frac{1}{z} + \frac{1}{\bar{z}}\right)} \right|^2$$

- This gives, with *asymptotical* DAs (i.e. limit $Q^2 \to \infty$):

$$\frac{d\sigma^H}{d\sigma^{\pi^0}} \approx 38\%$$

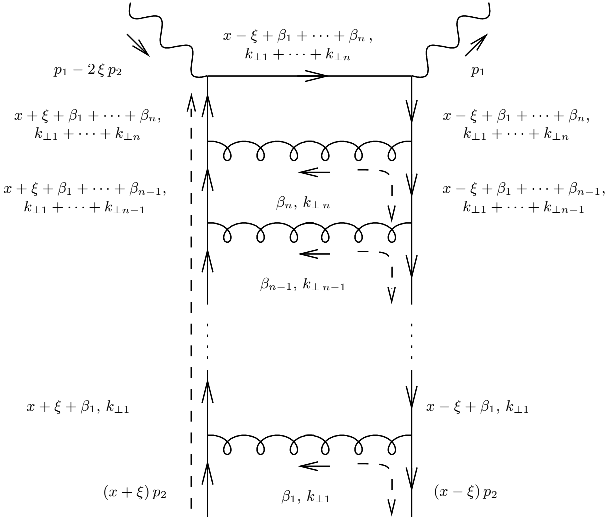
still larger than 20% at $Q^2 \approx 1$ GeV^2 (including kinematical twist-3 effects à la Wandzura-Wilczek for the H^0 DA) and similarly

$$\frac{d\sigma^H}{d\sigma^{\eta}} \approx 46\%$$

Threshold effects for DVCS and TCS

Resummation for Coefficient functions (1)

Computation of the n-loop ladder-like diagram



- All gluons are assumed to be on mass shell.
- Strong ordering in $\underline{k}_i$, α_i and β_i.
- The dominant momentum flows along p_2 are indicated

Computation of the n-loop ladder-like diagram (2)

- Strong ordering is given as :

$$|\underline{k}_n| \gg |\underline{k}_{n-1}| \gg \cdots \gg |\underline{k}_1| \quad , \quad 1 \gg |\alpha_n| \gg |\alpha_{n-1}| \gg \cdots \gg |\alpha_1|$$

$$x \sim \xi \gg |\beta_1| \sim |x-\xi| \gg |x-\xi+\beta_1| \sim |\beta_2| \gg \cdots \gg |x-\xi+\beta_1+\beta_2-\cdots+\beta_{n-1}| \sim |\beta_n|$$

- eikonal coupling on the left
- coupling on the right goes beyond eikonal
- Integral for n-loop:

$$I_n = \left(\frac{s}{2}\right)^n \int d\alpha_1\, d\beta_1\, d_2\underline{k}_1 \cdots \int d\alpha_n\, d\beta_n\, d_2\underline{k}_n\ (\mathrm{Num})_n \frac{1}{L_1^2} \cdots \frac{1}{L_n^2} \frac{1}{S^2} \frac{1}{R_1^2} \cdots \frac{1}{R_n^2} \frac{1}{k_1^2} \cdots \frac{1}{k_n^2}$$

- Numerator:

$$(\mathrm{Num})_2 = -4\,s \underbrace{\frac{-2\underline{k}_1^2\,(x+\xi)}{\beta_1}\left[1+\frac{2(x-\xi)}{\beta_1}\right]}_{\text{gluon 1}} \underbrace{\frac{-2\underline{k}_2^2\,(x+\xi)}{\beta_2}\left[1+\frac{2(\beta_1+x-\xi)}{\beta_2}\right]}_{\text{gluon 2}} \cdots \underbrace{\frac{-2\underline{k}_n^2\,(x+\xi)}{\beta_n}\left[1+\frac{2(\beta_{n-1}+\cdots+\beta_1+x-\xi)}{\beta_n}\right]}_{\text{gluon n}}$$

- Propagators:

$$L_1^2 = \alpha_1(x+\xi)s\,, \qquad R_1^2 = -\underline{k}_1^2 + \alpha_1(\beta_1+x-\xi)s\,,$$

$$L_2^2 = \alpha_2(x+\xi)s\,, \qquad R_2^2 = -\underline{k}_2^2 + \alpha_2(\beta_1+\beta_2+x-\xi)s\,,$$

$$\vdots$$

$$L_n^2 = \alpha_n(x+\xi)s\,, \qquad R_n^2 = -\underline{k}_n^2 + \alpha_n(\beta_1+\cdots+\beta_n+x-\xi)s\,,$$

Computation of the n-loop ladder-like diagram (3)

$$I_n = -4\frac{(2\pi i)^n}{x-\xi}\int_0^{\xi-x} d\beta_1 \cdots \int_0^{\xi-x-\beta_1-\cdots-\beta_{n-1}} d\beta_n \frac{1}{\beta_1+x-\xi}\cdots\frac{1}{\beta_1+\cdots+\beta_n+x-\xi}$$
$$\times \int_0^{\infty} d_N\underline{k}_n \cdots \int_{\underline{k}_2^2}^{\infty} d_N\underline{k}_1 \frac{1}{\underline{k}_1^2}\cdots\frac{1}{\underline{k}_{n-1}^2}\frac{1}{\underline{k}_n^2-(\beta_1+\cdots+\beta_n+x-\xi)s}$$

integration over $\underline{k}_i$ and β_i leads to our final result :

$$I_n^{\text{fin.}} = -4\frac{(2\pi i)^n}{x-\xi+i\epsilon}\frac{1}{(2n)!}\log^{2n}\left[\frac{\xi-x}{2\xi}-i\epsilon\right]$$

Resummation :

remember that $K_n = -\frac{1}{4}e_q^2\left(-i\,C_F\,\alpha_s\frac{1}{(2\pi)^2}\right)^n I_n$

$$\left(\sum_{n=0}^{\infty}K_n\right)-(x\to -x) = \frac{e_q^2}{x-\xi+i\epsilon}\cosh\left[D\log\left(\frac{\xi-x}{2\xi}-i\epsilon\right)\right]-(x\to -x)$$

where $D=\sqrt{\frac{\alpha_s C_F}{2\pi}}$

Threshold effects for DVCS and TCS

Resummed formula

Inclusion of our resummed formula into the NLO coefficient function

The inclusion procedure is not unique and it is natural to propose two choices:

- modifying only the Born term and the $\log^2$ part of the C_1^q and keeping the rest of the terms untouched :

$$(T^q)^{\text{res1}} = \left(\frac{e_q^2}{x-\xi+i\epsilon} \left\{ \cosh\left[D \log\left(\frac{\xi-x}{2\xi} - i\epsilon \right) \right] - \frac{D^2}{2}\left[9 + 3\frac{\xi-x}{x+\xi} \log\left(\frac{\xi-x}{2\xi} - i\epsilon \right) \right] \right\} + C_{coll}^q \log\frac{Q^2}{\mu_F^2} \right) - (x \to -x)$$

- the resummation effects are accounted for in a multiplicative way for C_0^q and C_1^q :

$$(T^q)^{\text{res2}} = \left(\frac{e_q^2}{x-\xi+i\epsilon} \cosh\left[D \log\left(\frac{\xi-x}{2\xi} - i\epsilon \right) \right] \left[1 - \frac{D^2}{2}\left\{ 9 + 3\frac{\xi-x}{x+\xi} \log\left(\frac{\xi-x}{2\xi} - i\epsilon \right) \right\} \right] + C_{coll}^q \log\frac{Q^2}{\mu_F^2} \right) - (x \to -x)$$

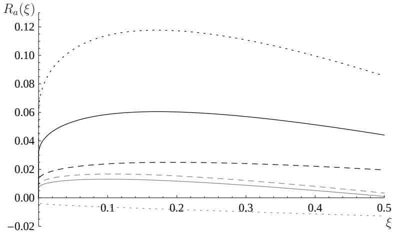
These resummed formulas differ through logarithmic contributions which are beyond the precision of our study.

Threshold effects for DVCS and TCS

Phenomenological implications

- We use a Double Distribution based model
  S. V. Goloskokov and P. Kroll, Eur. Phys. J. C **50**, 829 (2007)
- Blind integral in the whole $x-$range: amplitude $=$ NLO result $\pm\ 1\%$
- To respect the domain of applicability of our resummation procedure:
  - restrict the use of our formula to $\xi - a\gamma < |x| < \xi + a\gamma$
  - width $a\gamma$ defined through $|D\log(\gamma/(2\xi))| = 1$
  - theoretical uncertainty evaluated by varying a
  - a more precise treatment is beyond the leading logarithmic approximation

$$R_a(\xi) = \frac{\left[\int_{\xi-a\gamma}^{\xi+a\gamma} + \int_{-\xi-a\gamma}^{-\xi+a\gamma}\right] dx (C^{\text{res}} - C_0 - C_1) H(x,\xi,0)}{\left|\int_{-1}^{1} dx\, (C_0 + C_1) H(x,\xi,0)\right|}.$$



$Re[R_a(\xi)]$: black upper curves

$Im[R_a(\xi)]$: grey lower curves

$a = 1$ (solid)

$a = 1/2$ (dotted)

$a = 2$ (dashed)

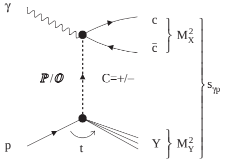
Finding the hard Odderon

- colorless gluonic exchange
  - $C = +1$: $\mathbb{P}$omeron, in pQCD described by BFKL equation
  - $C = -1$: $\mathbb{O}$dderon, in pQCD described by BJKP equation
- best but still weak evidence for $\mathbb{O}$: pp and $p\bar{p}$ data at ISR
- no evidence for perturbative $\mathbb{O}$

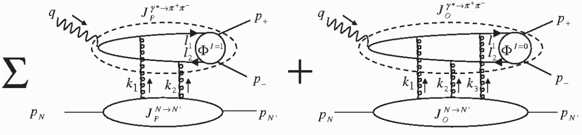
Finding the hard Odderon

$\mathbb{O}$ exchange much weaker than $\mathbb{P}$ $\Rightarrow$ two strategies in QCD

- consider processes, where $\mathbb{P}$ vanishes due to C-parity conservation:
  exclusive $\eta, \eta_c, f_2, a_2, \dots$ in ep; $\gamma\gamma \to \eta_c\eta_c \sim |\mathcal{M}_{\mathbb{O}}|^2$ Braunewell, Ewerz '04
  exclusive $J/\Psi, \Upsilon$ in pp ($\mathbb{P}\mathbb{O}$ fusion, not $\mathbb{P}\mathbb{P}$)) Bzdak, Motyka, Szymanowski, Cudell '07
- consider observables sensitive to the interference between $\mathbb{P}$ and $\mathbb{O}$
  (open charm in ep; $\pi^+\pi^-$ in ep)$\sim \mathrm{Re}\,\mathcal{M}_{\mathbb{P}}\mathcal{M}_{\mathbb{O}}^*$ $\Rightarrow$ observable linear in $\mathcal{M}_{\mathbb{O}}$



Brodsky, Rathsman, Merino '99

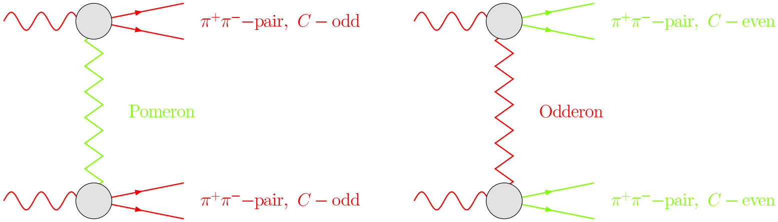


Ivanov, Nikolaev, Ginzburg '01 in photo-production
Hägler, Pire, Szymanowski, Teryaev '02 in electro-production

Finding the hard Odderon

$\mathbb{P} - \mathbb{O}$ interference in double UPC

$\mathbb{P} - \mathbb{O}$ interference in $\gamma\gamma \to \pi^+ \pi^- \pi^+ \pi^-$



Hard scale $= t$

B. Pire, F. Schwennsen, L. Szymanowski, S. W.
Phys.Rev.D78:094009 (2008)

pb at LHC: pile-up!