Hard exclusive processes: some basics about theory

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Introduction

Exclusive processes at high energy in QCD

- A tremendous effort is being performed to extract exclusive data. They are now coming with increasing precision (DVCS, meson production, polarized experiments, ...) at moderate and high energy
- Since a decade, there have been much theoretical developpements in hard exclusive processes.
  - form factors, Distribution Amplitudes $\longrightarrow$ Generalized Distribution Amplitudes
  - DVCS $\longrightarrow$ Generalized Parton Distributions, Transition Distribution Amplitudes
- The key tool is the collinear factorization

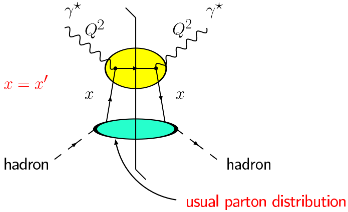
Introduction

Extensions from DIS

- DIS: inclusive process $\longrightarrow$ forward amplitude $(t = 0)$

Structure Function

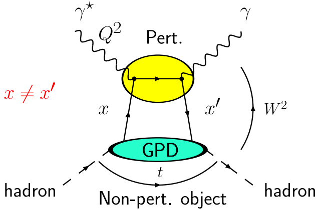
= Coefficient Function (hard) $\otimes$ Parton Distribution Function (soft)



- DVCS: exclusive process $\longrightarrow$ non forward amplitude $(-t \ll s = W^2)$

Amplitude

= Coefficient Function (hard) $\otimes$ Generalized Parton Distribution (soft)



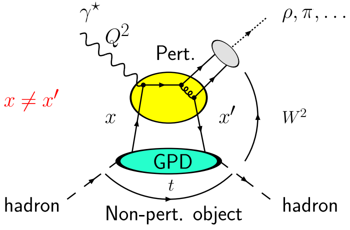
Müller et al. '91 - '94; Radyushkin '96; Ji '97

Introduction
Extensions from GPD

- Meson production: γ replaced by $\rho,\ \pi, \cdots$

Amplitude

= GPD (soft) $\otimes$ CF (hard) $\otimes$ Distribution Amplitude (soft)

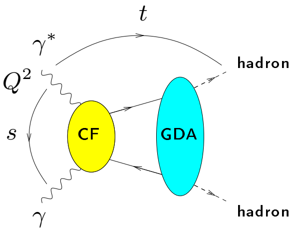


Collins, Frankfurt, Strikman '97; Radyushkin '97

- Crossed process: $s \ll -t$

Amplitude

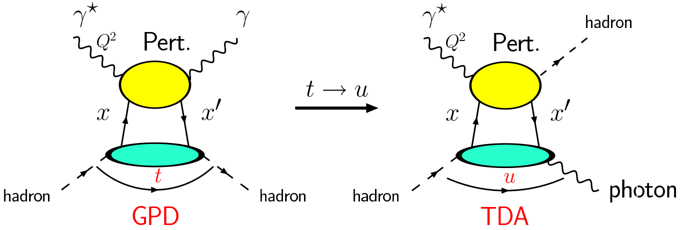
= Coefficient Function (hard) $\otimes$ Generalized Distribution Amplitude (soft)



Diehl, Gousset, Pire, Teryaev '98

Introduction
Extensions from GPD

- starting from usual DVCS, one allows initial hadron ≠ final hadron
example:



Pire, Szymanowski '05

which can be further extended by replacing the outoing γ by any hadronic state

Amplitude	=	Transition Distribution Amplitude (soft)	⊗	CF (hard)	⊗	DA (soft)

Lansberg, Pire, Szymanowski '06

(see talk of J-P.Lansberg)

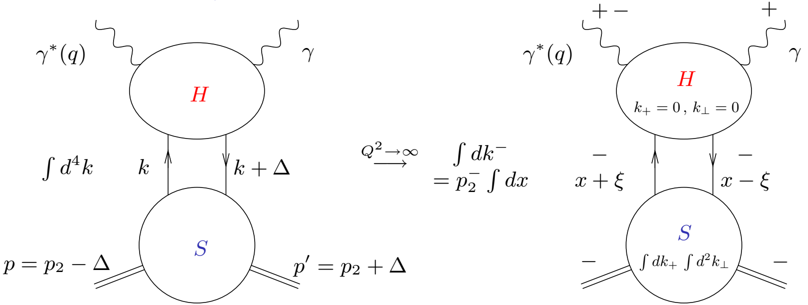
$\rho-$electroproduction

DVCS and GPD

Two steps for factorization

- momentum factorization: use Sudakov decomposition

$$\underset{}{k} = \underset{+}{\alpha\, p_1} + \underset{-}{\beta\, p_2} + \underset{\perp}{k_\perp} \quad (p_1^2 = p_2^2 = 0,\; 2p_1 \cdot p_2 = s)$$



$$\int d^4k\; S(k,\, k+\Delta)\, H(q,\, k,\, k+\Delta) \;=\; \int dk^- \int dk^+ d^2k_\perp\; S(k,\, k+\Delta)\;\; H(q,\, k^-,\, k^- + \Delta^-)$$

- supplement with Fierz identity in spinor + color space

$$\Rightarrow \qquad \mathcal{M} = \mathrm{GPD} \otimes \text{Hard part}$$

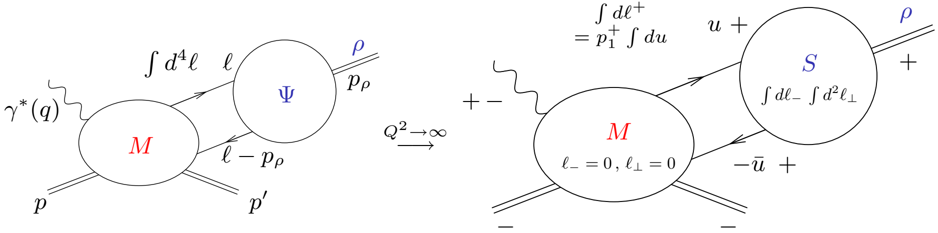
Müller et al. '91 - '94; Radyushkin '96; Ji '97

$\rho-$electroproduction

$\rho-$meson production: from the wave function to the DA

What is a $\rho-$meson in QCD?

It is described by its wave function Ψ which reduces in hard processes to its Distribution Amplitude



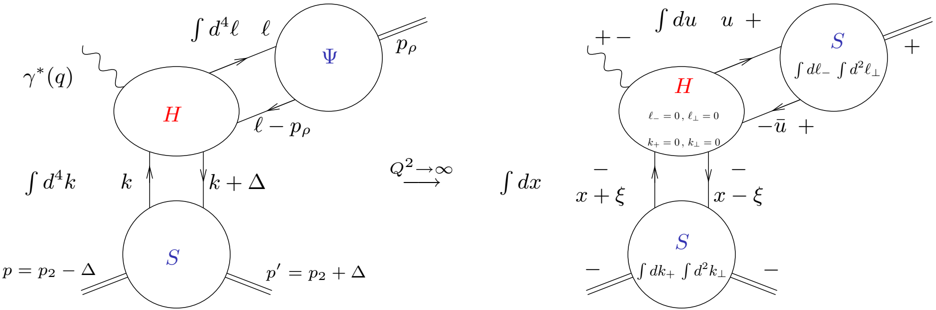
$$\int d^4\ell\; M(q,\,\ell,\,\ell-p_\rho)\Psi(\ell,\,\ell-p_\rho) \;=\int d\ell^+\, M(q,\,\ell^+,\,\ell^+-p_\rho^+)\;\int d\ell^- \int^{|k_\perp^2|<\mu_F^2} d^2\ell_\perp\,\Psi(\ell,\,\ell-p_\rho)$$

Hard part DA $\Phi(u,\mu_F^2)$

(see Chernyak, Zhitnitsky '77; Brodsky, Lepage '79; Efremov, Radyushkin '80; ... in the case of form-factors studies)

$\rho-$electroproduction

$\rho-$meson production: factorization with a GPD and a DA



$\int d^4k\, d^4\ell \qquad S(k,\, k+\Delta) \qquad H(q,\, k,\, k+\Delta) \qquad \Psi(\ell,\, \ell - p_\rho)$

$$= \int dk^- d\ell^+ \int dk^+ \int^{|k_\perp^2| < \mu_{F_2}^2} d^2k_\perp\, S(k,\, k+\Delta)\, H(q;\, k^-,\, k^- + \Delta^-;\, \ell^+,\, \ell^+ - p_\rho^+) \int d\ell^- \int^{|\ell_\perp^2| < \mu_{F_1}^2} d^2\ell_\perp\, \Psi(\ell,\, \ell - p_\rho)$$

GPD $F(x,\, \xi, t, \mu_{F_2}^2)$ Hard part $T(x/\xi, u, \mu_{F_1}^2, \mu_{F_2}^2)$ DA $\Phi(u, \mu_{F_1}^2)$

Collins, Frankfurt, Strikman '97; Radyushkin '97

$\rho-$electroproduction
Chiral-even DA

Two-particles DAs

$\int d\ell^- \int d\ell_\perp \Rightarrow$ one deal with non-local correlators between fields separated by a light-like distance z (along p_2, conjugated to $+$ direction by Fourier transf.)

$$\langle 0|\bar{u}(z)\gamma_\mu d(-z)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho \left[p_\mu \frac{e^{(\lambda)}\cdot z}{p\cdot z}\int_0^1 du\, e^{i(u-\bar{u})p\cdot z}\phi_\parallel(u,\mu_F^2)\right.$$

$$\left. + e^{(\lambda)}_{\perp\mu}\int_0^1 du\, e^{i(u-\bar{u})p\cdot z} g_\perp^{(v)}(u,\mu_F^2) - \frac{1}{2} z_\mu \frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2} m_\rho^2 \int_0^1 du\, e^{i(u-\bar{u})p\cdot z} g_3(u,\mu_F^2)\right]$$

twists: 2 (ρ_L) + 3 $(\rho_\perp)$ + 4 $\qquad p = p_1,\ P = p_\rho$

$$\langle 0|\bar{u}(z)\gamma_\mu\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle = \frac{1}{2}\left[f_\rho - f_\rho^T \frac{m_u+m_d}{m_\rho}\right] m_\rho \epsilon_\mu{}^{\nu\alpha\beta} e^{(\lambda)}_{\perp\nu} p_\alpha z_\beta \int_0^1 du\, e^{i\xi p\cdot z} g_\perp^{(a)}(u,\mu_F^2)$$

twists: 3 $(\rho_\perp)$

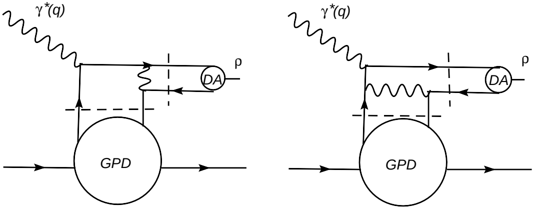
normalization: from local limit

$$\langle 0|\bar{u}(0)\gamma_\mu d(0)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho e_\mu^{(\lambda)}, \qquad \langle 0|\bar{u}(0)\sigma_{\mu\nu} d(0)|\rho^-(P,\lambda)\rangle = i f_\rho^T (e_\mu^{(\lambda)} P_\nu - e_\nu^{(\lambda)} P_\mu)$$

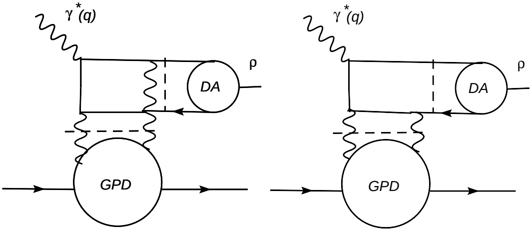
All four functions $\phi = \{\phi_\parallel, g_\perp^{(v)}, g_\perp^{(a)}, g_3\}$ are normalized as $\int_0^1 du\, \phi(u) = 1$.

$\rho-$electroproduction

Chiral-even DA: the hard part



with quark GPDs



with gluonic GPDs

$\rho-$electroproduction
Selection rules and factorization status

- chirality = helicity for a particule, chirality = -helicity for an antiparticule
- for massless quarks: QED and QCD vertices = chiral even (no chirality flip during the interaction)
  $\Rightarrow$ the total helicity of a $q\bar{q}$ produced by a γ^* should be 0
  $\Rightarrow$ helicity of the γ^* $= L_z^{q\bar{q}}$ (z projection of the $q\bar{q}$ angular momentum)
- in the pure collinear limit (i.e. twist 2), $L_z^{q\bar{q}}$=0 $\Rightarrow$ γ_L^*
- at $t = 0$, no source of orbital momentum from the proton coupling $\Rightarrow$ helicity of the meson = helicity of the photon
- in the collinear factorization approach, $t \neq 0$ change nothing from the hard side $\Rightarrow$ the above selection rule remains true
- thus: 2 transitions possible ($s-$channel helicity conservation (SCHC)):
  - $\gamma_L^* \to \rho_L$ production: QCD factorization holds at t=2 at any order (i.e. LL, NLL, etc...)

    Collins, Frankfurt, Strikman '97
  - $\gamma_T^* \to \rho_T$ production: QCD factorization has problems at t=3

    Mankiewicz-Piller '00 $\int_0^1 \frac{du}{u}$ or $\int_0^1 \frac{du}{1-u}$ diverge (end-point singularity)

$\rho-$electroproduction

Some solutions to factorization breaking? Add contribution of 3-particle DAs for ρ_T

addition of 3-particle DAs for ρ Anikin, Teryaev '03 (not enough for ρ_T)

Chiral-even three-particle DAs of ρ

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - g[A_\mu, A_\nu]$$

$$\langle 0|\bar{u}(z) g \widetilde{G}_{\mu\nu}(vz)\gamma_\alpha\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho p_\alpha[p_\nu e^{(\lambda)}_{\perp\mu} - p_\mu e^{(\lambda)}_{\perp\nu}]\, \mathcal{A}(v,pz)$$
$$+ f_\rho m_\rho^3 \frac{e^{(\lambda)}\cdot z}{pz}[p_\mu g^\perp_{\alpha\nu} - p_\nu g^\perp_{\alpha\mu}]\, \widetilde{\Phi}(v,pz) + f_\rho m_\rho^3 \frac{e^{(\lambda)}\cdot z}{(pz)^2} p_\alpha[p_\mu z_\nu - p_\nu z_\mu]\, \widetilde{\Psi}(v,pz)$$

$$\langle 0|\bar{u}(z) g G_{\mu\nu}(vz) i\gamma_\alpha d(-z)|\rho^-(P)\rangle = f_\rho m_\rho p_\alpha[p_\nu e^{(\lambda)}_{\perp\mu} - p_\mu e^{(\lambda)}_{\perp\nu}]\, \mathcal{V}(v,pz)$$
$$+ f_\rho m_\rho^3 \frac{e^{(\lambda)}\cdot z}{pz}[p_\mu g^\perp_{\alpha\nu} - p_\nu g^\perp_{\alpha\mu}]\Phi(v,pz) + f_\rho m_\rho^3 \frac{e^{(\lambda)}\cdot z}{(pz)^2} p_\alpha[p_\mu z_\nu - p_\nu z_\mu]\Psi(v,pz)$$

twists: 3 + 4

$$\mathcal{A}(v,pz) = \int \mathcal{D}\underline{\alpha}\, e^{-ipz(\alpha_u - \alpha_d + v\alpha_g)} \mathcal{A}(\underline{\alpha})$$

$\underline{\alpha}$ is the set of three mom. fractions $\underline{\alpha} = \{\alpha_d, \alpha_u, \alpha_g\}$

$$\int \mathcal{D}\underline{\alpha} \equiv \int_0^1 d\alpha_d \int_0^1 d\alpha_u \int_0^1 d\alpha_g\, \delta(1 - \sum \alpha_i)$$

Ball, Braun, Koike, Tanaka '98

$\rho-$electroproduction

Some solutions to factorization breaking? Transverse momenta $\ell_\perp$ as a regulator

Improved collinear approximation

- keep a transverse $\ell_\perp$ dependency in the q, $\bar{q}$ momenta, used to regulate end-point singularities
- soft and collinear gluon exchange between the valence quark are responsible for large double-logarithmic effects which exponentiate
- this is made easier when using the impact parameter space $b_\perp$ conjugated to $\ell_\perp$ $\Rightarrow$ Sudakov factor

$$\exp[-S(u, b, Q)]$$

- S diverges when $b_\perp \sim O(1/\Lambda_{QCD})$ (large transverse separation, i.e. small transverse momenta) or $u \sim O(\Lambda_{QCD}/Q)$ Botts, Sterman '89
$\Rightarrow$ regularization of end-point singularities for $\pi \to \pi\gamma^*$ and $\gamma\gamma^*\pi^0$ form factors, based on the factorization approach Li, Sterman '92
- combining this perturbative resummation tail effect with an ad-hoc non-perturbative gaussian ansatz for the DAs

$$\exp[-a^2\,|k_\perp^2|/(u\bar{u})]$$

which gives back the usual asymptotic DA $6u\bar{u}$ when integrating over $k_\perp$
$\Rightarrow$ practical tools for phenomenology of meson electroproduction
Goloskokov, Kroll '05

$\rho-$electroproduction

Chiral-odd sector: Chiral-even versus chiral-odd DAs

Chirality

Define

$$q_\pm(z) \equiv \frac{1}{2}(1 \pm \gamma^5) q(z) \qquad q(z) = q_+(z) + q_-(z)$$

Chiral-even: conserve chirality

$$\bar{q}_\pm(z)\gamma^\mu q_\pm(-z) \qquad \text{or} \qquad \bar{q}_\pm(z)\gamma^\mu\gamma^5 q_\pm(-z)$$

Chiral-odd: change chirality

$$\bar{q}_\pm(z)\cdot 1\cdot q_\mp(-z), \quad \bar{q}_\pm(z)\cdot\gamma^5\cdot q_\mp(-z) \quad \text{or} \quad \bar{q}_\pm(z)[\gamma^\mu,\gamma^\nu]q_\mp(-z)$$

QCD conserves chirality $\Longrightarrow \mathcal{A} \sim$ (Ch.-odd)$_1 \otimes$ (Ch.-odd)$_2$

chiral-odd objects appear in pairs

$\rho-$electroproduction
Chiral-odd DAs

Chiral-odd two-particles DAs

$$\sigma_{\mu\nu} \equiv \frac{i}{2}[\gamma_\mu, \gamma_\nu]$$

$$\langle 0|\bar{u}(z)\sigma_{\mu\nu}d(-z)|\rho^-(P,\lambda)\rangle = i f_\rho^T \Big[(e^{(\lambda)}_{\perp\mu} p_\nu - e^{(\lambda)}_{\perp\nu} p_\mu) \int_0^1 du\, e^{i\xi p\cdot z} \phi_\perp(u,\mu^2)$$
$$+ (p_\mu z_\nu - p_\nu z_\mu) \frac{e^{(\lambda)} \cdot z}{(p\cdot z)^2} m_\rho^2 \int_0^1 du\, e^{i\xi p\cdot z} h_\parallel^{(t)}(u,\mu^2)$$
$$+ \frac{1}{2}(e^{(\lambda)}_{\perp\mu} z_\nu - e^{(\lambda)}_{\perp\nu} z_\mu) \frac{m_\rho^2}{p\cdot z} \int_0^1 du\, e^{i\xi p\cdot z} h_3(u,\mu^2) \Big]$$

$$\langle 0|\bar{u}(z)d(-z)|\rho^-(P,\lambda)\rangle =$$
$$- i \left(f_\rho^T - f_\rho \frac{m_u + m_d}{m_\rho} \right) (e^{(\lambda)} \cdot z) m_\rho^2 \int_0^1 du\, e^{i\xi p\cdot z} h_\parallel^{(s)}(u,\mu^2)$$

twists:

- $\phi_\perp$ of ρ_T is twist-2
- $h_\parallel^{(s)}$ and $h_\parallel^{(t)}$ of ρ_L are twist-3
- h_3 of ρ_T is twist-4

$\rho-$electroproduction Chiral-odd DAs

Chiral-odd three-particles DAs

$$\begin{aligned}
&\langle 0|\bar{u}(z)\sigma_{\alpha\beta}\, g\, G_{\mu\nu}(vz)d(-z)|\rho^-(P,\lambda)\rangle \\
&= f_\rho^T m_\rho^2 \frac{e^{(\lambda)}\cdot z}{2(p\cdot z)}[p_\alpha p_\mu g^\perp_{\beta\nu} - p_\beta p_\mu g^\perp_{\alpha\nu} - p_\alpha p_\nu g^\perp_{\beta\mu} + p_\beta p_\nu g^\perp_{\alpha\mu}]\,\mathcal{T}(v,pz) \\
&\quad +4 \text{ DAs involving } \rho_T
\end{aligned}$$

twists:

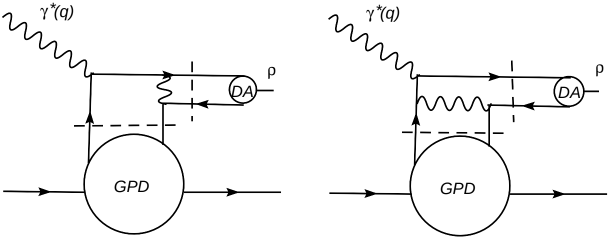
- $\mathcal{T}$ of ρ_L is twist 3
- the 4 DA of ρ_T are twist 4

$$\begin{aligned}
\langle 0|\bar{u}(z) g\, G_{\mu\nu}(vz)d(-z)|\rho^-(P,\lambda)\rangle &= i f_\rho^T m_\rho^2 [e^{(\lambda)}_{\perp\mu}p_\nu - e^{(\lambda)}_{\perp\nu}p_\mu]\, S(v,pz) \\
\langle 0|\bar{u}(z) i g\, \widetilde{G}_{\mu\nu}(vz)\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle &= i f_\rho^T m_\rho^2 [e^{(\lambda)}_{\perp\mu}p_\nu - e^{(\lambda)}_{\perp\nu}p_\mu]\, \widetilde{S}(v,pz)
\end{aligned}$$

twists: S and $\widetilde{S}$ of ρ_T are twist 4

$\rho-$electroproduction
Accessing GPD of transversity with ρ_T-electroproduction

Electroproduction of ρ_T on linearly polarized N



with the chiral-odd $\langle \rho_T | \bar{q}\, \sigma^{\mu\nu}\, q | 0 \rangle$ DA

AND

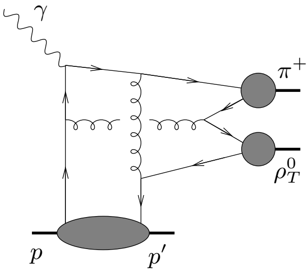
with the chiral-odd $\langle p' \uparrow | \bar{q}\, \sigma^{\mu\nu}\, q | p \uparrow \rangle$ GPD

Result: $\mathcal{A} = 0$ at the leading twist 2!!! since $\gamma^\alpha \, [\gamma^\mu, \gamma^\nu] \, \gamma_\alpha = 0$

Can we avoid the vanishing of $\mathcal{M}$ at the leading twist 2?

$\rho-$electroproduction
Accessing GPD of transversity with ρ_T-electroproduction

Electroproduction of π^+ AND ρ_T^0 on a linearly polarized N



chiral-even twist 2 DA

chiral-odd twist 2 DA

chiral-odd twist 2 GPD

$+ \sim 60$ diagrams

hard scale $= p_\perp$ of π^+ and ρ_T^0

Our proposition for JLab and Compass

M. El Beiyad, B. Pire, M.Segond, L.Szymanowski, S. W., in preparation

Generic results for DAs

Gauge invariance: application to exotic hybrids mesons

- The above non-local correlators fullfill gauge invariance:

$$\langle 0|\bar{\Psi}(z)\gamma_\mu \Psi(-z)|\rho\rangle$$

  should be understood as

$$\langle 0|\bar{\Psi}(z)\gamma_\mu [z,\,-z]\Psi(-z)|\rho\rangle$$

  where $[,]$ is a Wilson line along p_2
- Thus, even at twist 2, gluons are there, although hidden!
- Taylor expansion with respect to z involves the covariant derivative $\overleftrightarrow{D}_\mu$
  $\Rightarrow$ this can be used for studying hard electroproduction of exotic (non $q\bar{q}$ quantum numbers) hybrids mesons $|q\bar{q}g\rangle$ with $J^{PC} = 1^{-+}$
- Thus, $\gamma^* p \to H^0 p$ is not suppressed: it is twist 2. Expected order of magnitude of the cross-section comparable with ρ-electroproduction.
  Anikin, Pire, Szymanowski, Teryaev, S.W. '04, '05
  Tests at JLab, Compass ? n.b.: H^0 could be the $\pi_1(1400)$ candidate
- same conclusion for the process $\gamma\gamma^* \to H^0$ with the advantage of avoiding the mixing with GPD ibid. '06
  Tests at BaBar, BELLE, Bepc-II ?

n.b.: $H^0 \to \pi\eta$

Generic results for DAs

Equations of motion

Equations of motion

- Dirac equation leads to

$$\langle i(\overrightarrow{\not{D}}(0)\psi(0))_\alpha\,\bar{\psi}_\beta(z)\rangle = 0 \qquad (i\,\overrightarrow{D}_\mu = i\,\overrightarrow{\partial}_\mu + A_\mu)$$

- Apply the Fierz decomposition to the above 2 and 3-body correlators

$$-\langle\psi(x)\,\bar{\psi}(z)\rangle = \frac{1}{4}\langle\bar{\psi}(z)\gamma_\mu\psi(x)\rangle\gamma_\mu + \frac{1}{4}\langle\bar{\psi}(z)\gamma_5\gamma_\mu\psi(x)\rangle\gamma_\mu\gamma_5.$$

- $\Rightarrow$ Equation of motion which relates the various 2 and 3-body DAs.

Generic results for DAs

Renormalization group equations

Back to the factorization of the process in term of a DA:

$$\mathcal{M}(Q^2) = \Phi^*(x, \mu_F^2) \otimes T_H(x, Q^2, \mu_F^2) \,.$$

The DA $\Phi(u,\, \mu_F^2)$ satisfies the Efremov-Radyushkin,Brodsky-Lepage equation:

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \Phi(x, \mu_F^2) = V(x, u, \mu_F^2) \otimes \Phi(u, \mu_F^2) \,,$$

Generic results for DAs

Collinear conformal invariance: Generalities

- the full conformal group $SO(4,2)$ is defined as transformations which only change the scale of the metric
- $Q^2 \to \infty$: hadron states are replaced by a bunch of partons that are collinear to p_1, which thus lives along $p_2 \Rightarrow z$ variable only
- transformations which map the light-ray in p_2 direction into itself = collinear subgroup of the full conformal group $SO(4,2)$ $= SL(2,\mathbb{R})$:
  - translations $z \to z + c$
  - dilatation $z \to \lambda z$
  - special conformal transformation

$$z \to z' = \frac{z}{1 + 2\,a\,z}$$

- algebra of $SL(2,\mathbb{R}) = O(2,1)$
- one remaining additional generator commutes with the 3 previous one: the collinear-twist operator

Generic results for DAs

Collinear conformal invariance: Applications

- the light-cone operators which enters the definition of DAs can be expressed in terms of a basis of conformal operators
- conformal transformations commute with exact Equations Of Motion (EOM are not renormalized) $\Rightarrow$ EOM can be solved exactly (with an expansion in terms of the conformal spin $n+2$). Ex.: twist 2 DA for ρ_L :

$$\phi_{\|}(u,\,\mu_0) = 6u\bar{u}\sum_{n=0}^{\infty} a_n^{\|}(\mu)\, C_n^{3/2}(u-\bar{u}) \qquad C_n^{3/2} = \text{Gegenbauer polynomial}$$

Ohrndorf '82; Braun, Filyanov '90 — but $a_n^{\|}(\mu)$ are unpredicted

- the Leading Order renormalization of the conformal operators is diagonal in the conformal spin (counterterms are tree level at this accuracy $\Rightarrow$ they respect the conformal symetry of the classical theory)

$$\phi_{\|}(u,\,\mu) = 6u\bar{u}\sum_{n=0}^{\infty} a_n^{\|}(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_n^{(0)}/\beta_0} C_n^{3/2}(u-\bar{u}) \overset{\mu\to\infty}{\longrightarrow} 6u\bar{u} \text{ asymp. DA}$$

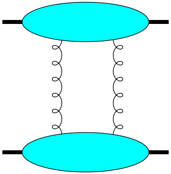
with the anomalous dimensions $\gamma_n^{(0)} = C_F\left(1 - \frac{2}{(n+1)(n+2)} + 4\sum_{m=2}^{n+1}\frac{1}{m}\right)$

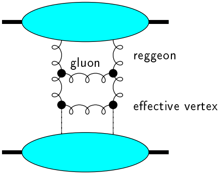
- at Next to Leading Order conformal symetry is broken; studying conformal anomalies provides the NLO anomalous dimensions and corresponding ERBL kernels Belitsky, Freund, Müller '99 '00

The specific case of QCD at large s

QCD in perturbative Regge limit

- In this limit, the dynamics is dominated by gluons (dominance of spin 1 exchange in t channel)
- BFKL (and extensions: NLL, saturations effects, ...) is expected to dominate with respect to Born order at large relative rapidity.

Born order: 

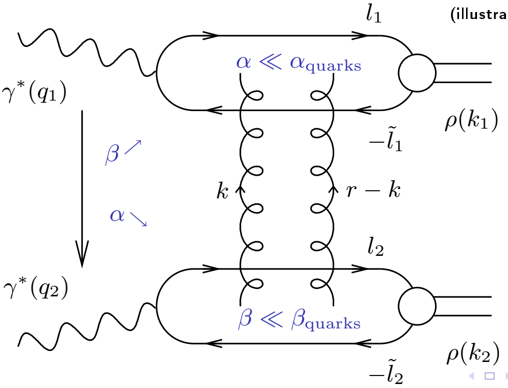
BFKL ladder: 

The specific case of QCD at large s
k_T factorization

$\gamma^* \gamma^* \to \rho\,\rho$ as an example

- Use Sudakov decomposition $k = \alpha\, p_1 + \beta\, p_2 + k_\perp$ $(p_1^2 = p_2^2 = 0,\ 2p_1 \cdot p_2 = s)$
- write $d^4k = \frac{s}{2}\, d\alpha\, d\beta\, d^2k_\perp$
- $t-$channel gluons with non-sense polarizations $(\epsilon_{NS}^{up} = \frac{2}{s}\, p_2,\ \epsilon_{NS}^{down} = \frac{2}{s}\, p_1)$ dominate at large s

(illustration for 2-body case)



$\Rightarrow$ set $\alpha = 0$ and $\int d\beta$

$\int d^2k_\perp$

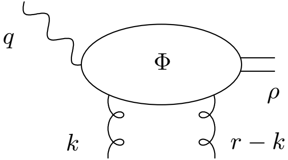
$\Rightarrow$ set $\beta = 0$ and $\int d\alpha$

The specific case of QCD at large s
k_T factorization

Impact representation for exclusive processes $\underline{k}$ = Eucl. $\leftrightarrow$ $k_\perp$ = Mink.

$$\mathcal{M} = is \int \frac{d^2\,\underline{k}}{(2\pi)^2 \underline{k}^2\, (\underline{r}-\underline{k})^2} \Phi^{\gamma^*(q_1)\to\rho(p_1^\rho)}(\underline{k}, \underline{r}-\underline{k})\; \Phi^{\gamma^*(q_2)\to\rho(p_2^\rho)}(-\underline{k}, -\underline{r}+\underline{k})$$

$\Phi^{\gamma^*(q_1)\to\rho(p_1^\rho)}$: $\gamma^*_{L,T}(q)g(k_1) \to \rho_{L,T}\, g(k_2)$ impact factor



QCD gauge invariance:

- probes are colorless
  $\Rightarrow$ impact factor should vanish when $\underline{k} \to 0$ or $\underline{r}-\underline{k} \to 0$
- At twist 3 level (for $\gamma^*_T \to \rho_T$ transition), gauge invariance is a non trivial statement which requires 2 *and* 3 body correlators

The specific case of QCD at large s

Phenomenological applications: Data for meson production at HERA

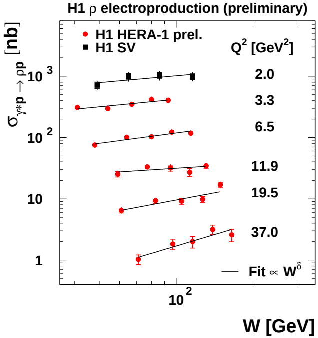
Recently, a whole bunch of very nice results, with high precision, have been obtained, in particular for vector mesons with detailled polarization studies.

(see the talk of S. Kananov)

example: $\gamma^*_{L,T}(q) + P \to \rho_{L,T}(p_1) + P$

This process was studied by H1 and ZEUS

- the total cross-section strongly decreases with Q^2
- dramatic increase with $W^2 = s_{\gamma^* P}$ (transition from soft to hard regime governed by Q^2)



(from X. Janssen (H1), DIS 2008)

The specific case of QCD at large s

Phenomenological applications: Data for meson production at HERA

Polarization effects in $\gamma^* P \to \rho P$ at HERA

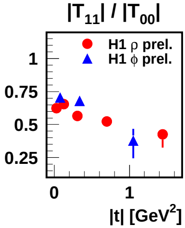
- one can experimentally measure all spin density matrix elements
- at $t = t_{min}$ one can experimentally distinguish

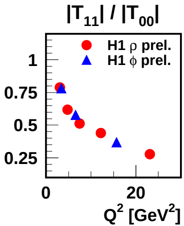
$$\begin{cases} \gamma_L^* \to \rho_L : & \text{dominates} & (\text{twist 2 dominance}) \\ \gamma_T^* \to \rho_T : & \text{sizable} & (\text{twist 3}) \end{cases}$$

- S-channel helicity conservation:

$$\begin{cases} \gamma_L^* \to \rho_L & (\equiv T_{00}) \\ \gamma_T^* \to \rho_T , & \end{cases}$$

Dominate with respect to all other transitions.
Experimentally, $\gamma_T^* \to \rho_T$ is dominated by $\gamma_{T(-)}^* \to \rho_{T(-)}$ and $\gamma_{T(+)}^* \to \rho_{T(+)}$ $(\equiv T_{11})$





(from X. Janssen (H1), DIS 2008)

The specific case of QCD at large s

Phenomenological applications: meson production at HERA

Production of mesons in diffraction-type experiment at HERA

- the safe case: J/Ψ production factorizes ($u \sim 1/2$: non-relativistic limit for a bound state) combined with k_T-factorization
  Ryskin '93; Frankfurt, Koepf, Strikman '98; Ivanov, Kirschner, Schäfer, Szymanowski '00; Motyka, Enberg, Poludniowski '02
- exclusive vector meson photoproduction at large t (=hard scale):
  $\gamma(q) + P \to \rho_{L,T}(p_1) + P$
  relying on k_T-factorization:
  Forshaw, Ryskin '95; Bartels, Forshaw, Lotter, Wüsthoff '96; Forshaw, Motyka, Enberg, Poludniowski '03
  - H1, ZEUS data seem to favor BFKL
  - but one needs to regularize end-point singularities for ρ_T: use of a quark mass $m = m_\rho/2$
  - rather poor understanding of the whole spin density matrix
- exclusive vector meson electroproduction
  $\gamma^*_{L,T}(q) + P \to \rho_{L,T}(p_1) + P$
  Goloskokov, Kroll '05
  based on the modified collinear factorization for DA coupling and collinear factorization with GPD

The specific case of QCD at large s

Phenomenological applications: $\rho-$meson production at HERA and twist 3

A full twist 3 treatment of ρ-electroproduction in k_T-factorisation

- we have computed the $\gamma_T^* \to \rho_T$ impact factor at twist 3
  Anikin, Ivanov, Pire, Szymanowski, S.W. to appear
- we show that:
  - Including in a consistent way all twist 3 contributions, i.e. 2-body and 3-body correlators, gives a gauge invariant impact factor
  - Our treatment is free of end-point singularities due to the presence of k_T and thus does not violates the QCD factorization
  - These points remain valid in the Wandzura-Wilczek approximation (i.e. 3-body correlators = 0, in which case twist 3 effects only arise due to kinematical effects and not from gluonic dynamical degrees of freedom)
- phenomenology remains to be done...

The specific case of QCD at large s

Phenomenological applications: exclusive processes at Tevatron, RHIC, LHC, ILC

Exclusive $\gamma^{(*)}\gamma^{(*)}$ processes = gold place for testing QCD at large s

Proposals in order to test perturbative QCD in the large s limit (t-structure of the hard Pomeron, saturation, Odderon...)

- $\gamma^{(*)}(q) + \gamma^{(*)}(q') \to J/\Psi \, J/\Psi$
  Kwiecinski, Motyka '98
- $\gamma^*_{L,T}(q) + \gamma^*_{L,T}(q') \to \rho_L(p_1) + \rho_L(p_2)$ process in $e^+ e^- \to e^+ e^- \rho_L(p_1) + \rho_L(p_2)$ with double tagged lepton at ILC

  Pire, Szymanowski, S. W. '04; Pire, Szymanowski, Enberg, S. W. '06; Ivanov, Papa '06; Segond, Szymanowski, S. W. '07

  conclusion: feasible at ILC (high energy and high luminosity); BFKL NLL enhancement with respect to Born and DGLAP
- What about the Odderon? C-parity of Odderon = -1
  consider $\gamma + \gamma \to \pi^+\pi^-\pi^+\pi^-$: $\pi^+\pi^-$ pair has no fixed C-parity
  $\Rightarrow$ Odderon and Pomeron can interfere $\Rightarrow$ Odderon appear linearly in the charge asymetry
  Pire, Schwennsen, Szymanowski, S. W. '07
  see the talk of F. Schwennsen
- other exclusive ultraperipheral processes: see the talk of J. Nystrand

Collinear factorization

Light-Cone Collinear approach versus Covariant approach

- The Light-Cone Collinear approach, which is self-consistent, while non-covariant, is very efficient for practical computations
  Anikin, Ivanov, Pire, Szymanowski, S.W. '09
  - inspired by the inclusive case
    Ellis, Furmanski, Petronzio '83; Efremov, Teryaev '84
  - axial gauge
  - parametrization of matrix element along a light-like prefered direction $z = \lambda\, n$ $(n = 2\, p_2/s)$.
  - Non-local correlators are defined around this prefered direction, with contributions arising from Taylor expansion up to needed term for a given twist order computation
  - their number is then reduced to a minimal set combining EOM and $n-$independency condition
- another approach (Braun, Ball), fully covariant but less convenient when practically computing coefficient functions, can equivalently be used
- we have established the dictionnary between the two approaches
- this as been explicitly checked for the $\gamma^*_T \to \rho_T$ impact factor at twist 3
  Anikin, Ivanov, Pire, Szymanowski, S.W. to appear

Collinear factorization

Light-Cone Collinear approach: factorization using Taylor expansion

- Use Sudakov decomposition in the form $(p = p_1,\ n = 2\,p_2/s \Rightarrow p \cdot n = 1)$

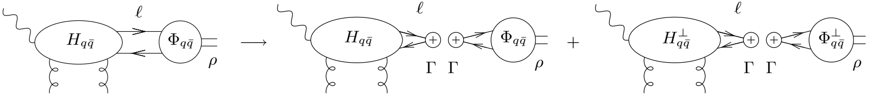
$$l_\mu \quad = \quad u\,p_\mu \quad + \quad l^\perp_\mu \quad + \quad (l \cdot p)\,n_\mu, \qquad u = l \cdot n$$

scaling: 1 $\quad 1/Q$ $\quad 1/Q^2$

- decompose $H(k)$ around the p direction:

$$H(\ell) = H(up) \quad + \quad \left.\frac{\partial H(\ell)}{\partial \ell_\alpha}\right|_{\ell=up} (\ell - u\,p)_\alpha + \dots \quad \text{with} \quad (\ell - u\,p)_\alpha \approx \ell^\perp_\alpha$$

- twist 3 term $l^\perp_\alpha \xrightarrow{Fourier}$ derivative of the soft term:
$\int d^4z\; e^{-i\ell\cdot z} \langle \rho(p)|\psi(0)\, i\, \overleftrightarrow{\partial}_{\alpha^\perp} \bar{\psi}(z)|0\rangle$
- after Fierz, this gives



- this leads to introduce 7 DAs at twist 3 (2 and 3 body DAs)

Collinear factorization

$n-$independence

A minimal set of DAs

- The non-perturbative correlators cannot be obtained from perturbative QCD (!)
- one should reduce them to a minimal set before using any model
- this can be achieved by using an additional condition:
  independency of the full amplitude with respect to the light-cone vector n

$\Rightarrow$ we prove that 3 independent Distribution Amplitudes are needed:
7 - 2 (=nb of EOM) - 2 (=nb of eq. from n-ind. cond.)

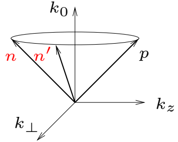
$\phi_1(y)$ $\leftarrow$ 2 body twist 2 correlator

$B(y_1,\, y_2)$ $\leftarrow$ 3 body genuine twist 3 vector correlator

$D(y_1,\, y_2)$ $\leftarrow$ 3 body genuine twist 3 axial correlator

Collinear factorization
$n-$independence

$n-$independence in practice

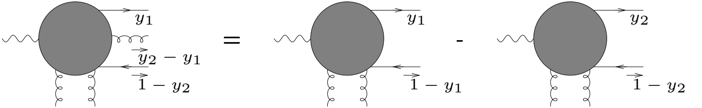


- ρ_T polarization: $e^{*T}_\mu = e^*_\mu - p_\mu \, e^* \cdot n$ keeping $n \cdot p = 1$
- for the full factorized amplitude:

$$\mathcal{A} = H \otimes S \qquad \frac{d\mathcal{A}}{dn^\mu} = 0\,, \qquad \text{where} \quad \frac{d}{dn^\mu} = \frac{\partial}{\partial n^\mu} + e^*_\mu \frac{\partial}{\partial (e^* \cdot n)}$$

- rewrite hard terms in one single form, of 2-body type: use Ward identities
  Example: hard 3-body $\longrightarrow$ hard 2-body

$$\text{tr}\left[H_{3\rho}(y_1, y_2)\, p^\rho \, \not{p}\right] \; B(y_1,\, y_2) = \frac{1}{y_1 - y_2}\left(\text{tr}\left[H_2(y_1)\, \not{p}\right] - \text{tr}\left[H_2(y_2)\, \not{p}\right]\right) B(y_1,\, y_2)\,,$$



- thus, symbolically,

example of the $\gamma^*_T \to \rho_T$ impact factor

$$\frac{dS}{dn^\mu} = 0$$

Conclusion

- since a decade, there have been much progress in the understanding of hard exclusive processes
  - at moderate energies, combined with GPD, there is now a framework starting from first principle to describe a huge number of processes
  - at high energy, the impact representation is a powerful tool for describing exclusive processes in diffractive experiments; they are and will be essential for studying QCD in the hard Regge limit (Pomeron, Odderon, saturation...)
- still, some problems remains:
  - proofs of factorization have been optained only for a very few processes (ex.: $\gamma^* \, p \to \gamma \, p \,, \, \gamma^*_L \, p \to \rho_L \, p \,, \, \gamma^* p \to J/\Psi \, p$)
  - for some other processes factorization is highly plausible, but not fully demonstrated at any order (ex.: processes involving TDAs)
  - some processes explicitly shows sign of breaking of factorization (ex.: $\gamma^*_T p \to \rho_T p$ which has end-point singularities at Leading Order)
  - models and results from the lattice for the non-perturbative correlators entering GPDs, DAs, GDAs, TDAs are needed, even at a qualitative level!
  - the effect of QCD evolution and renormalization/factorization scale might be relevant with the increasing precision of data
- links between theoretical and experimental communities are very fruitful
- message to experimentalists: high luminosity e^+e^- machine like BaBar, BELLE, BEPC-II are gold places for exclusive processes studies in $\gamma^*\gamma^{(*)}$ $\Rightarrow$ it is time to realize this and to use the potential of these experiments! We need your help!