

Diffractive processes and non-perturbative matrix elements beyond leading twist: ρ_T -meson production

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Introduction: phenomenology of exclusive processes within collinear factorization

- Experimental tests are possible in **fixed target** experiments
 - $e^{\pm}p, \mu^{\pm}p$: **HERA (HERMES), JLab, COMPASS...**
- as well as in **colliders, mainly for medium s**
 - $e^{\pm}p$ colliders: **HERA (H1, ZEUS)**
 - e^+e^- colliders: **LEP, Belle, BaBar, BEPC**
- **Collinear factorization** has been proven only for specific cases:
e.g.: ρ_T production cannot directly be factorized (appearance of **end point singularities**)
 \Rightarrow improvement needed for a consistent approach of exclusive processes

QCD in the perturbative Regge limit

- At the same time, **at large s** , the interest for phenomenological tests of **hard Pomeron** and related resummed approaches has become pretty wide:
 - **inclusive** tests (total cross-section) and semi-inclusive tests (diffraction, forward jets, ...)
 - **exclusive** tests (meson production)
- These tests concern all type of collider experiments:
 - $e^\pm p$: **HERA**: (H1, ZEUS)
 - $p\bar{p}$ and pp : **TEVATRON** (CDF, D0); **LHC** (CMS, ATLAS, ALICE)
 - e^+e^- : (LEP, ILC)
- These high energy exclusive processes in the perturbative **Regge** limit may provide new ideas when dealing with collinear factorization

Introduction

Exclusive ρ -production

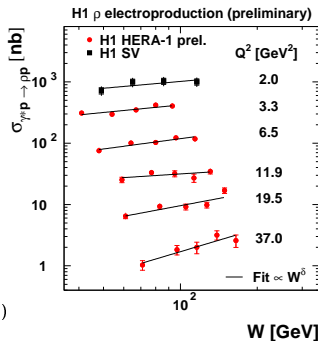
Our studies attempt to describe exclusive processes involving the production of ρ -mesons in diffraction-type experiment. We choose $t = t_{min}$ for simplicity.

- $\gamma^*(q) + \gamma^*(q') \rightarrow \rho_T(p_1) + \rho(p_2)$ process in $e^+ e^- \rightarrow e^+ e^- \rho_T(p_1) + \rho(p_2)$ with double tagged lepton at ILC
- $\gamma^*(q) + P \rightarrow \rho_T(p_1) + P$ at HERA

This process was studied by H1 and ZEUS

- the total cross-section strongly **decreases with Q^2**
- dramatic **increase with $W^2 = s_{\gamma^* P}$** (transition from soft to hard regime governed by Q^2)

(from X. Janssen (H1), DIS 2008)



Introduction

Exclusive ρ -productionPolarization effects in $\gamma^* P \rightarrow \rho P$ at HERA

- one can experimentally measure all spin density matrix elements
- at $t = t_{min}$ one can experimentally distinguish

$$\begin{cases} \gamma_L^* \rightarrow \rho_L : & \text{dominates} & (\text{twist 2 dominance}) \\ \gamma_T^* \rightarrow \rho_T : & \text{sizable} & (\text{twist 3}) \end{cases}$$

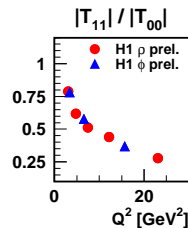
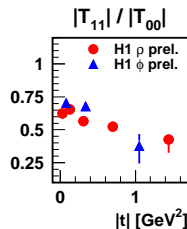
- S-channel helicity conservation:

$$\begin{cases} \gamma_L^* \rightarrow \rho_L & (\equiv T_{00}) \\ \gamma_T^* \rightarrow \rho_T, \end{cases}$$

Dominate with respect to all other transitions.

Experimentally, $\gamma_T^* \rightarrow \rho_T$ is dominated

by $\gamma_{T(-)}^* \rightarrow \rho_{T(-)}$ and $\gamma_{T(+)}^* \rightarrow \rho_{T(+)} (\equiv T_{11})$



(from X. Janssen (H1), DIS 2008)

Introduction

Exclusive ρ -production

The processes with vector particle such as rho-meson probe deeper into the fine features of QCD.

It deserves theoretical developpement to describe HERA data in its special kinematical range:

- large $s_{\gamma^* P} \Rightarrow$ small-x effects expected, within k_t -factorization
- large $Q^2 \Rightarrow$ hard scale \Rightarrow perturbative approach and collinear factorization \Rightarrow the ρ can be described through its chiral even Distribution Amplitudes

$$\left\{ \begin{array}{ll} \rho_L & \text{twist 2} \\ \rho_T & \text{twist 3} \end{array} \right.$$

The main ingredient is the $\gamma^* \rightarrow \rho$ impact factor

- For ρ_T , special care is needed: a pure 2-body description would violate gauge invariance.
- We show that:
 - Including in a consistent way all twist 3 contributions, i.e. 2-body and 3-body correlators, gives a gauge invariant impact factor
 - Our treatment is free of end-point singularities and does not violates the QCD factorization

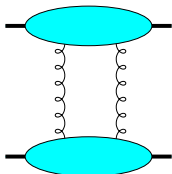
Impact factor for exclusive processes

Theoretical motivations

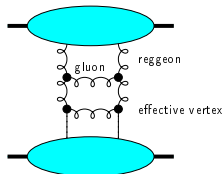
QCD in perturbative Regge limit

- In this limit, the dynamics is dominated by gluons (dominance of spin 1 exchange in t channel)
- BFKL (and extensions: NLL, saturations effects, ...) is expected to dominate with respect to Born order at large relative rapidity.

Born order:



BFKL ladder:



Impact factor for exclusive processes

 k_T factorization

impact representation

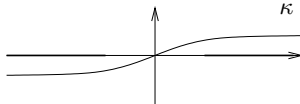
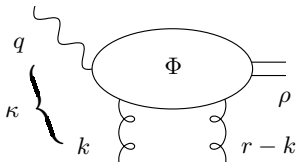
 $\underline{k} = \text{Eucl.} \leftrightarrow k_{\perp} = \text{Mink.}$

$$\mathcal{M} = is \int \frac{d^2 \underline{k}}{(2\pi)^2 \underline{k}^2 (\underline{r} - \underline{k})^2} \Phi^{\gamma^*(q_1) \rightarrow \rho(p_1^{\rho})}(\underline{k}, \underline{r} - \underline{k}) \Phi^{\gamma^*(q_2) \rightarrow \rho(p_2^{\rho})}(-\underline{k}, -\underline{r} + \underline{k})$$

The $\gamma_{L,T}^*(q)g(k_1) \rightarrow \rho_{L,T}g(k_2)$ **impact factor** is normalized as

$$\Phi^{\gamma^* \rightarrow \rho}(\underline{k}^2) = e^{\gamma^* \mu} \frac{1}{2s} \int \frac{d\kappa}{2\pi} \text{Disc}_{\kappa} \mathcal{S}_{\mu}^{\gamma^* g \rightarrow \rho g}(\underline{k}^2),$$

with $\kappa = (q+k)^2 = \beta s - Q^2 - \underline{k}^2$

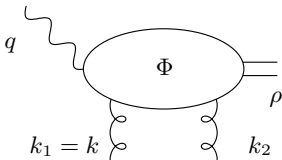


Impact factor for exclusive processes

Gauge invariance within subleading twists

Gauge invariance

- **QCD gauge invariance** (probes are colorless)
 \Rightarrow impact factor should **vanish** when $\underline{k} \rightarrow 0$ or $\underline{r} - \underline{k} \rightarrow 0$
- In the following we will restrict ourselves to the case $t = t_{min}$, i.e. to $\underline{r} = 0$



$$k_1 = \frac{\kappa + Q^2 + \underline{k}^2}{s} p_2 + k_\perp$$

$$k_2 = \frac{\kappa + \underline{k}^2}{s} p_2 + k_\perp,$$

$$k_1^2 = k_2^2 = -\underline{k}^2$$

This kinematics takes into account **skewedness effects** along p_2
 $t = t_{min} \Rightarrow$ restriction to the transitions

$$\begin{cases} 0 & \rightarrow & 0 & \text{(twist 2)} \\ (+ \text{ or } -) & \rightarrow & (+ \text{ or } -) & \text{(twist 3)} \end{cases}$$

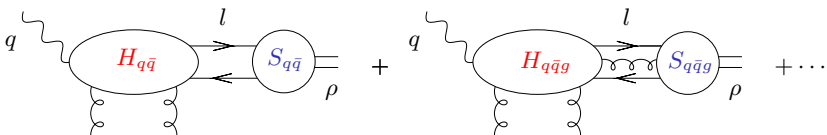
- At twist 3 level (for $\gamma_T^* \rightarrow \rho_T$ transition), gauge invariance is a non trivial statement which **requires 2 and 3 body correlators**

Collinear factorization

Light-Cone Collinear approach

- The impact factor can be written as

$$\Phi = \int d^4l \dots \text{tr}[\underbrace{H(l \dots)}_{\text{hard part}} \underbrace{S(l \dots)}_{\text{soft part}}]$$



- At the 2-body level:

$$S_{q\bar{q}}(l) = \int d^4z e^{-il \cdot z} \langle \rho(p) | \psi(0) \bar{\psi}(z) | 0 \rangle,$$

- H and S are related by $\int d^4l$ and by the **summation over spinor indices**

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

1 - Momentum factorization (1)

- Use **Sudakov** decomposition in the form ($p = p_1$, $n = 2p_2/s \Rightarrow p \cdot n = 1$)

$$l_\mu = y p_\mu + l_\mu^\perp + (l \cdot p) n_\mu, \quad y = l \cdot n$$

$$\text{scaling:} \quad 1 \quad 1/Q \quad 1/Q^2$$

- decompose $H(k)$ around the p direction:

$$H(l) = H(y p) + \frac{\partial H(l)}{\partial l_\alpha} \Big|_{l=y p} (l - y p)_\alpha + \dots \quad \text{with } (l - y p)_\alpha \approx l_\alpha^\perp$$

- In **Fourier** space, the **twist 3** term l_α^\perp turns into a derivative of the **soft term**

$$\Rightarrow \text{one will deal with } \int d^4 z e^{-i l \cdot z} \langle \rho(p) | \psi(0) i \overleftrightarrow{\partial}_{\alpha^\perp} \bar{\psi}(z) | 0 \rangle$$

Collinear factorization

Light-Cone Collinear approach: **2 steps of factorization** (2-body case)

1 - Momentum factorization (2)

- write

$$d^4l \longrightarrow d^4l \delta(\mathbf{y} - l \cdot \mathbf{n}) \, d\mathbf{y}$$

- $\int d^4l \delta(\mathbf{y} - l \cdot \mathbf{n})$ is then absorbed in the soft term:

$$\begin{aligned} (\tilde{S}_{q\bar{q}}, \partial_{\perp} \tilde{S}_{q\bar{q}}) &\equiv \int d^4l \delta(\mathbf{y} - l \cdot \mathbf{n}) \int d^4z e^{-il \cdot z} \langle \rho(p) | \psi(0) (1, i \overrightarrow{\partial}_{\perp}) \bar{\psi}(z) | 0 \rangle \\ (\delta(\mathbf{y} - l \cdot \mathbf{n}) = \int \frac{d\lambda}{2\pi} e^{-i\lambda(\mathbf{y} - l \cdot \mathbf{n})} \Rightarrow) &= \int \frac{d\lambda}{2\pi} e^{-i\lambda \mathbf{y}} \int d^4z \delta^{(4)}(z - \lambda \mathbf{n}) \langle \rho(p) | \psi(0) (1, i \overrightarrow{\partial}_{\perp}) \bar{\psi}(z) | 0 \rangle \\ &= \int \frac{d\lambda}{2\pi} e^{-i\lambda \mathbf{y}} \langle \rho(p) | \psi(0) (1, i \overrightarrow{\partial}_{\perp}) \bar{\psi}(\lambda \mathbf{n}) | 0 \rangle \end{aligned}$$

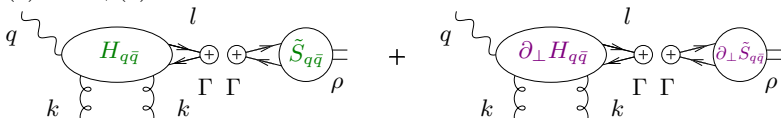
- $\int d\mathbf{y}$ performs the longitudinal momentum factorization

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

2 - Spinorial (and color) factorization

- Use Fierz decomposition of the Dirac (and color) matrices $\psi(0) \bar{\psi}(z)$ and $\psi(0) i \overleftrightarrow{\partial}_\perp \bar{\psi}(z)$:



- Φ has now the simple factorized form:

$$\Phi = \int dx \left\{ \text{tr} [H_{q\bar{q}}(xp) \Gamma] S_{q\bar{q}}^\Gamma(x) + \text{tr} [\partial_\perp H_{q\bar{q}}(xp) \Gamma] \partial_\perp S_{q\bar{q}}^\Gamma(x) \right\}$$

$\Gamma = \gamma^\mu$ and $\gamma^\mu \gamma^5$ matrices

$$S_{q\bar{q}}^\Gamma(x) = \int \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle \rho(p) | \bar{\psi}(\lambda n) \Gamma \psi(0) | 0 \rangle$$

$$\partial_\perp S_{q\bar{q}}^\Gamma(x) = \int \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle \rho(p) | \bar{\psi}(\lambda n) \Gamma i \overleftrightarrow{\partial}_\perp \psi(0) | 0 \rangle$$

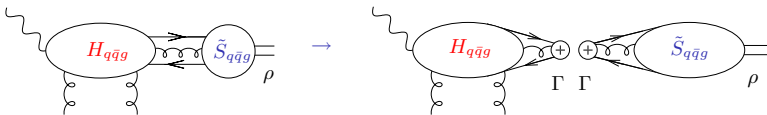
- choose axial gauge condition for gluons, i.e. $n \cdot A = 0 \Rightarrow$ no Wilson line

Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (3-body case)

Factorization of 3-body contributions

- 3-body contributions start at **genuine twist 3**
 ⇒ no need for **Taylor** expansion
- Momentum factorization goes in the same way as for 2-body case
- Spinorial (and color) factorization is similar:



Collinear factorization

Parametrization of vacuum-to- ρ -meson matrix elements (DAs): 2-body correlators

2-body **non-local** correlators

ρ_L

twist 2

- vector correlator

$$\langle \rho(p) | \bar{\psi}(z) \gamma_\mu \psi(0) | 0 \rangle \stackrel{\mathcal{F}}{=} m_\rho f_\rho \left[\varphi_1(y) (e^* \cdot n) p_\mu + \varphi_3(y) e_\mu^{*T} \right]$$

ρ_T

kinematical twist 3 (WW)

genuine twist 3

genuine + kinematical twist 3

- axial correlator

$$\langle \rho(p) | \bar{\psi}(z) \gamma_5 \gamma_\mu \psi(0) | 0 \rangle \stackrel{\mathcal{F}}{=} m_\rho f_\rho i \varphi_A(y) \varepsilon_{\mu\lambda\beta\delta} e_\lambda^{*T} p_\beta n_\delta$$

- vector correlator with transverse derivative

$$\langle \rho(p) | \bar{\psi}(z) \gamma_\mu i \overleftrightarrow{\partial}_\alpha^\perp \psi(0) | 0 \rangle \stackrel{\mathcal{F}}{=} m_\rho f_\rho \varphi_1^T(y) p_\mu e_\alpha^{*T}$$

- axial correlator with transverse derivative

$$\langle \rho(p) | \bar{\psi}(z) \gamma_5 \gamma_\mu i \overleftrightarrow{\partial}_\alpha^\perp \psi(0) | 0 \rangle \stackrel{\mathcal{F}}{=} m_\rho f_\rho i \varphi_A^T(y) p_\mu \varepsilon_{\alpha\lambda\beta\delta} e_\lambda^{*T} p_\beta n_\delta,$$

where y ($\bar{y} \equiv 1 - y$) = momentum fraction along $p \equiv p_1$ of the quark (antiquark) and

$$\stackrel{\mathcal{F}}{=} \int_0^1 dy \exp[i y p \cdot z], \text{ with } z = \lambda n$$

⇒ 5 2-body DAs

Collinear factorization

Parametrization of vacuum-to-rho-meson matrix elements: 3-body correlators

3-body non-local correlators

genuine twist 3

- vector correlator

$$\langle \rho(p) | \bar{\psi}(z_1) \gamma_\mu g A_\alpha^T(z_2) \psi(0) | 0 \rangle \stackrel{\mathcal{F}_2}{\equiv} m_\rho f_3^V B(y_1, y_2) p_\mu e_\alpha^{*T},$$

- axial correlator

$$\langle \rho(p) | \bar{\psi}(z_1) \gamma_5 \gamma_\mu g A_\alpha^T(z_2) \psi(0) | 0 \rangle \stackrel{\mathcal{F}_2}{\equiv} m_\rho f_3^A i D(y_1, y_2) p_\mu \varepsilon_{\alpha\lambda\beta\delta} e_\lambda^{*T} p_\beta n_\delta,$$

where $y_1, \bar{y}_2, y_2 - y_1 =$ quark, antiquark, gluon momentum fraction

and $\stackrel{\mathcal{F}_2}{\equiv} \int_0^1 dy_1 \int_0^1 dy_2 \exp[i y_1 p \cdot z_1 + i(y_2 - y_1) p \cdot z_2]$, with $z_{1,2} = \lambda n$

⇒ 2 3-body DAs

Collinear factorization

Symmetry properties

From **C-conjugation** on the previous correlators, one gets:

- 2-body correlators:

$$\varphi_1(y) = \varphi_1(1-y)$$

$$\varphi_3(y) = \varphi_3(1-y)$$

$$\varphi_A(y) = -\varphi_A(1-y)$$

$$\varphi_1^T(y) = -\varphi_1^T(1-y)$$

$$\varphi_A^T(y) = \varphi_A^T(1-y)$$

- 3-body correlators:

$$B(y_1, y_2) = -B(1-y_2, 1-y_1)$$

$$D(y_1, y_2) = D(1-y_2, 1-y_1)$$

Collinear factorization

Equations of motion

Equations of motion

twist 2
kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- Dirac equation leads to

$$\langle i \overrightarrow{\mathcal{D}}(0) \psi(0) \rangle_{\alpha} \bar{\psi}_{\beta}(z) = 0 \quad (i \overrightarrow{D}_{\mu} = i \overrightarrow{\partial}_{\mu} + A_{\mu})$$

- Apply the Fierz decomposition to the above 2 and 3-body correlators

$$-\langle \psi(x) \bar{\psi}(z) \rangle = \frac{1}{4} \langle \bar{\psi}(z) \gamma_{\mu} \psi(x) \rangle \gamma_{\mu} + \frac{1}{4} \langle \bar{\psi}(z) \gamma_5 \gamma_{\mu} \psi(x) \rangle \gamma_{\mu} \gamma_5.$$

- \Rightarrow 2 Equations of motion:

$$\bar{y}_1 \varphi_3(y_1) + \bar{y}_1 \varphi_A(y_1) + \varphi_1^T(y_1) + \varphi_A^T(y_1) + \int dy_2 \left[\zeta_3^V B(y_1, y_2) + \zeta_3^A D(y_1, y_2) \right] = 0 \quad \text{and} \quad (\bar{y}_1 \leftrightarrow y_1)$$

- In WW approximation: genuine twist 3 = 0 i.e. $B = D = 0$

$$\begin{cases} \varphi_A^T(y) = \frac{1}{2} [(y - \bar{y}) \varphi_A^{WW}(y) - \varphi_3^{WW}(y)] \\ \varphi_1^T(y) = \frac{1}{2} [(y - \bar{y}) \varphi_3^{WW}(y) - \varphi_A^{WW}(y)] \end{cases}$$

Collinear factorization

n -independence

A minimal set of DAs

- The non-perturbative correlators cannot be obtained from perturbative QCD (!)
- one should reduce them to a minimal set before using any model
- this can be achieved by using an additional condition:
independency of the full amplitude with respect to the light-cone direction n

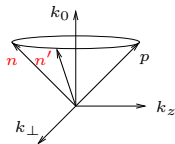
⇒ we prove that 3 independent Distribution Amplitudes are needed:

$\phi_1(y)$ ← 2 body twist 2 correlator

$B(y_1, y_2)$ ← 3 body genuine twist 3 vector correlator

$D(y_1, y_2)$ ← 3 body genuine twist 3 axial correlator

Collinear factorization

 n -independence n -independence in practice

- ρ_T polarization: $e_{\mu}^{*T} = e_{\mu}^* - p_{\mu} e^* \cdot n$ keeping $n \cdot p = 1$
- for the full factorized amplitude:

$$\mathcal{A} = H \otimes S \quad \frac{d\mathcal{A}}{dn^{\mu}} = 0, \quad \text{where} \quad \frac{d}{dn^{\mu}} = \frac{\partial}{\partial n^{\mu}} + e_{\mu}^* \frac{\partial}{\partial (e^* \cdot n)}$$

- rewrite hard terms in one single form, of 2-body type: use Ward identities
Example: hard 3-body \longrightarrow hard 2-body

$$\text{tr} [H_{3\rho}(y_1, y_2) p^{\rho} \not{p}] B(y_1, y_2) = \frac{1}{y_1 - y_2} (\text{tr} [H_2(y_1) \not{p}] - \text{tr} [H_2(y_2) \not{p}]) B(y_1, y_2),$$

$$(y_1 - y_2) \text{Diagram} = \text{Diagram} - \text{Diagram}$$

The diagram shows a wavy line entering a grey circle from the left. From the top of the circle, two lines emerge: one labeled y_1 and another labeled $y_2 - y_1$. From the bottom of the circle, two lines emerge: one labeled $\bar{1} - y_2$ and another labeled $\bar{1} - y_1$. The equation states that this is equal to the difference of two similar diagrams. In the first diagram, the top line is y_1 and the bottom line is $\bar{1} - y_1$. In the second diagram, the top line is y_2 and the bottom line is $\bar{1} - y_2$.

- thus, symbolically,

$$\frac{dS}{dn^{\mu}} = 0$$

Collinear factorization

n -independence

Constraints from n -independence

twist 2
kinematical twist 3 (WW)
genuine twist 3
genuine + kinematical twist 3

- vector correlators

$$\frac{d}{dy_1} \varphi_1^T(y_1) = -\varphi_1(y_1) + \varphi_3(y_1)$$
$$-\zeta_3^V \int_0^1 \frac{dy_2}{y_2 - y_1} (B(y_1, y_2) + B(y_2, y_1))$$

- axial correlators

$$\frac{d}{dy_1} \varphi_A^T(y_1) = \varphi_A(y_1) - \zeta_3^A \int_0^1 \frac{dy_2}{y_2 - y_1} (D(y_1, y_2) + D(y_2, y_1))$$

Collinear factorization

A set of independent non-perturbative correlators

Solution

twist 2

kinematical twist 3 (WW)

genuine twist 3

genuine + kinematical twist 3

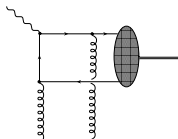
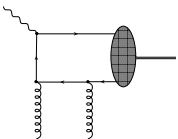
- the set of 4 equations (2 EOM + 2 n -independence relations) can be solved analytically
- 7 \longrightarrow 3 independent DAs

Computation and results

Computation of the hard part

2-body diagrams

- without derivative



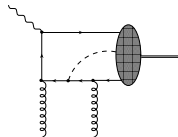
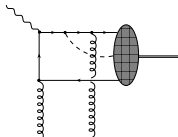
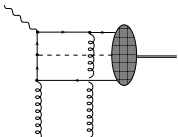
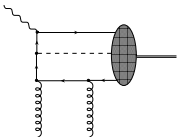
twist 2 $(\gamma_L^* \rightarrow \rho_L)$

twist 3 $(\gamma_T^* \rightarrow \rho_T)$

- practical trick for computing $\partial_\perp H$: use the **Ward identity**

$$\frac{\partial}{\partial p_\mu} \longrightarrow p = \longrightarrow p \quad \bullet \quad \longrightarrow p$$

$$\text{where } \longrightarrow p = \frac{1}{m - \not{p} - i\epsilon}$$

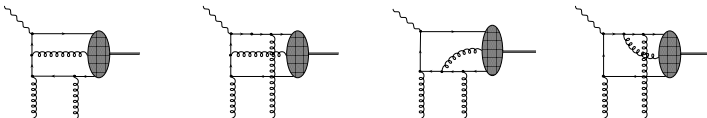


Computation and results

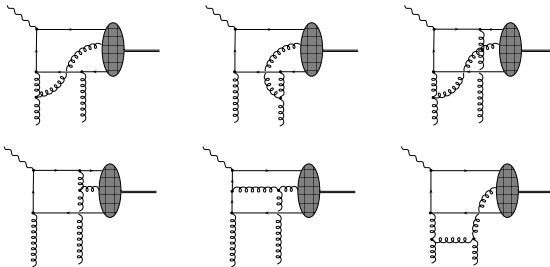
Computation of the hard part

3-body diagrams

- “abelian” type



- “non-abelian” type



Computation and results

Recall: $\gamma_L^* \rightarrow \rho_L$ impact factor

$\gamma_L^* \rightarrow \rho_L$ impact factor

$$\Phi^{\gamma_L^* \rightarrow \rho_L}(\underline{k}^2) = \frac{2 e g^2 f_\rho}{Q} \frac{\delta^{ab}}{2 N_c} \int dy \varphi_1(y) \frac{\underline{k}^2}{y \bar{y} Q^2 + \underline{k}^2}$$

pure twist 2 scaling (from ρ -factorization point of view)

Computation and results

Results: $\gamma_T^* \rightarrow \rho_T$ impact factor $\gamma_T^* \rightarrow \rho_T$ impact factor:

Spin Non-Flip/Flip separation appears

$$\Phi^{\gamma_T^* \rightarrow \rho_T}(\underline{k}^2) = \Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T}(\underline{k}^2) T_{n.f.} + \Phi_{f.}^{\gamma_T^* \rightarrow \rho_T}(\underline{k}^2) T_f.$$

where

$$T_{n.f.} = -(e_\gamma \cdot e^*) \quad \text{and} \quad T_f = \frac{(e_\gamma \cdot k)(e^* k)}{\underline{k}^2} + \frac{(e_\gamma \cdot e^*)}{2}$$

non-flip transitions $\begin{cases} + \rightarrow + \\ - \rightarrow - \end{cases}$ flip transitions $\begin{cases} + \rightarrow - \\ - \rightarrow + \end{cases}$

Computation and results

Results: $\gamma_T^* \rightarrow \rho_T$ impact factorpure twist 3 scaling (from ρ -factorization point of view)

$$\begin{aligned} & \Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T}(\underline{k}^2) \\ &= -\frac{e g^2 m_\rho f_\rho}{2\sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \left\{ -2 \int dy_1 \frac{(\underline{k}^2 + 2 Q^2 y_1 (1 - y_1)) \underline{k}^2}{y_1 (1 - y_1) (\underline{k}^2 + Q^2 y_1 (1 - y_1))^2} \left[(2y_1 - 1) \varphi_1^T(y_1) + \varphi_A^T(y_1) \right] \right. \\ &+ 2 \int dy_1 dy_2 \left[\zeta_3^V B(y_1, y_2) - \zeta_3^A D(y_1, y_2) \right] \frac{y_1 (1 - y_1) \underline{k}^2}{\underline{k}^2 + Q^2 y_1 (1 - y_1)} \left[\frac{(2 - N_c/C_F) Q^2}{\underline{k}^2 (y_1 - y_2 + 1) + Q^2 y_1 (1 - y_2)} \right. \\ &- \left. \frac{N_c}{C_F} \frac{Q^2}{y_2 \underline{k}^2 + Q^2 y_1 (y_2 - y_1)} \right] - 2 \int dy_1 dy_2 \left[\zeta_3^V B(y_1, y_2) + \zeta_3^A D(y_1, y_2) \right] \left[\frac{2 + N_c/C_F}{1 - y_1} \right. \\ &+ \frac{y_1 Q^2}{\underline{k}^2 + Q^2 y_1 (1 - y_1)} \left(\frac{(2 - N_c/C_F) y_1 \underline{k}^2}{\underline{k}^2 (y_1 - y_2 + 1) + Q^2 y_1 (1 - y_2)} - 2 \right) \\ &\left. \left. + \frac{N_c (y_1 - y_2) (1 - y_2)}{C_F} \frac{Q^2}{\underline{k}^2 (1 - y_1) + Q^2 (y_2 - y_1) (1 - y_2)} \right] \right\} \end{aligned}$$

and

$$\begin{aligned} & \Phi_{f.}^{\gamma_T^* \rightarrow \rho_T}(\underline{k}^2) = -\frac{e g^2 m_\rho f_\rho}{2\sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \left\{ 4 \int dy_1 \frac{\underline{k}^2 Q^2}{(\underline{k}^2 + Q^2 y_1 (1 - y_1))^2} \left[\varphi_A^T(y_1) - (2y_1 - 1) \varphi_1^T(y_1) \right] \right. \\ &- 4 \int dy_1 dy_2 \frac{y_1 \underline{k}^2}{\underline{k}^2 + Q^2 y_1 (1 - y_1)} \left[\zeta_3^A D(y_1, y_2) (-y_1 + y_2 - 1) + \zeta_3^V B(y_1, y_2) (y_1 + y_2 - 1) \right] \\ &\left. \times \left[\frac{(2 - N_c/C_F) Q^2}{\underline{k}^2 (y_1 - y_2 + 1) + Q^2 y_1 (1 - y_2)} - \frac{N_c}{C_F} \frac{Q^2}{y_2 \underline{k}^2 + Q^2 y_1 (y_2 - y_1)} \right] \right\} \end{aligned}$$

Computation and results

Results: $\gamma_T^* \rightarrow \rho_T$ impact factor

WW limit

- WW limit: keep only **twist 2** + **kinematical twist 3** terms (i.e $B = D = 0$)
- The only remaining contributions come from the two-body correlators
- non-flip transition

$$\Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T}(\underline{k}^2) = \frac{-e m_\rho f_\rho}{2\sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \int_0^1 dy \left\{ \frac{(y - \bar{y}) \varphi_1^{TWW}(y) + 2y\bar{y} \varphi_3^{WW}(y) + \varphi_A^{TWW}(y)}{y\bar{y}} - \frac{2\underline{k}^2 (\underline{k}^2 + 2Q^2 y\bar{y}) \left((y - \bar{y}) \varphi_1^{TWW}(y) + \varphi_A^{TWW}(y) \right)}{y\bar{y} (\underline{k}^2 + Q^2 y(1-y))^2} \right\}$$

which simplifies, using equation of motion:

$$\int dy [(y - \bar{y}) \varphi_1^{TWW}(y) + 2y\bar{y} \varphi_3^{WW}(y) + \varphi_A^{TWW}(y)] = 0$$

$$\Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T}(\underline{k}^2) = \frac{e m_\rho f_\rho}{\sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \int_0^1 dy \frac{2\underline{k}^2 (\underline{k}^2 + 2Q^2 y\bar{y})}{y\bar{y} (\underline{k}^2 + Q^2 y\bar{y})^2} [(2y - 1) \varphi_1^{TWW}(y) + \varphi_A^{TWW}(y)] .$$

- flip transition:

$$\Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T}(\underline{k}^2) = -\frac{e m_\rho f_\rho}{\sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \int_0^1 \frac{2\underline{k}^2 Q^2}{(\underline{k}^2 + Q^2 y\bar{y})^2} [(1 - 2y) \varphi_1^{TWW}(y) + \varphi_A^{TWW}(y)] .$$

Computation and results

Discussion: [gauge invariance](#)

- The obtained results are gauge invariant:

$$\Phi^{\gamma_T^* \rightarrow \rho_T} \rightarrow 0 \quad \text{when} \quad \underline{k} \rightarrow 0$$

- this is straightforward in the WW limit
- at the full twist 3 order:
 - the C_F part of the abelian 3-body contribution cancels the 2-body contribution **after using the equation of motion**
 - the N_c part of the abelian 3-body contribution cancels the 3-body non-abelian contribution
 - thus $\gamma_T^* \rightarrow \rho_T$ impact factor is **gauge-invariant only provided the 2 and 3-body contributions have been taken into account in a consistent way**

Computation and results

Discussion: **consistence with factorization**

- **Our results are free of end-point singularities**, in both **WW** approximation and full twist-3 order calculation:
 - the flip contribution obviously does not have any end-point singularity because of the \underline{k}^2 which regulates them
 - the potential end-point singularity for the non-flip contribution is spurious since $\varphi_A^T(y)$, $\varphi_1^T(y)$ vanishes at $y = 0, 1$ as well as $B(y_1, y_2)$ and $D(y_1, y_2)$.

Conclusions

1

- We have performed a full up to twist 3 computation of the $\gamma^* \rightarrow \rho$ impact factor, in the $t = t_{min}$ limit.
- Our result respects gauge invariance. This is achieved only after including 2 and 3 body correlators.
- It is free of end-point singularities (this should be contrasted with standard collinear treatment, at moderate s , where k_T -factorization is NOT applicable: see Mankiewicz-Piller).
- Phenomenological applications will be done in the near future.
- In this talk we relied on the Light-Cone Collinear approach (Ellis + Furmanski + Petronzio; Efremov + Teryaev; Anikin + Teryaev), which is non-covariant, but very efficient for practical computations.
- This Light-Cone Collinear approach is systematic, and can be extended to any process, including higher twist effects (but does not preclude potential end-point singularities)

Conclusions

2

- Comparison with a fully **covariant approach** by **Ball+Braun et al**:
The dictionary between the two approaches within a full twist 3 treatment is now established:

$$B(y_1, y_2) = -\frac{V(y_1, 1 - y_2, y_2 - y_1)}{y_2 - y_1},$$

$$D(y_1, y_2) = \frac{A(y_1, 1 - y_2, y_2 - y_1)}{y_2 - y_1}$$

$$\varphi_1(y) = f_\rho m_\rho \phi_{\parallel}(y)$$

$$\varphi_3(y) = f_\rho m_\rho g^{(v)}(y),$$

$$\varphi_A(y) = -\frac{1}{4} f_\rho m_\rho \frac{\partial g^{(a)}(y)}{\partial y}$$

- We also performed calculations of the same impact factor within the **covariant approach** by **Ball+Braun et al**: calculations proceed in quite different way : eg. no $\varphi_{1,A}^T$ -DAs but **Wilson** line effects are important !!
We got a full agreement with our approach