

# QCD factorization beyond leading twist in exclusive processes: *p<sub>T</sub>*-meson production

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## Introduction

Exclusive  $\rho$ -production

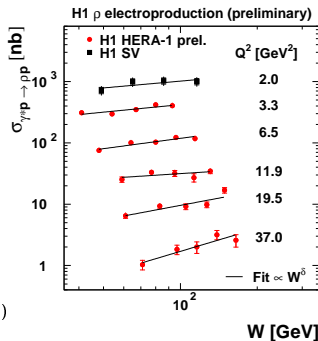
Our studies attempt to describe exclusive processes involving the production of  $\rho$ -mesons in diffraction-type experiment. We choose  $t = t_{min}$  for simplicity.

- $\gamma^*(q) + \gamma^*(q') \rightarrow \rho_T(p_1) + \rho(p_2)$  process in  $e^+ e^- \rightarrow e^+ e^- \rho_T(p_1) + \rho(p_2)$  with double tagged lepton at ILC
- $\gamma^*(q) + P \rightarrow \rho_T(p_1) + P$  at HERA

This process was studied by H1 and ZEUS

- the total cross-section strongly decreases with  $Q^2$
- dramatic increase with  $W^2 = s_{\gamma^*P}$  (transition from soft to hard regime governed by  $Q^2$ )

(from X. Janssen (H1), DIS 2008)



## Introduction

Exclusive  $\rho$ -productionPolarization effects in  $\gamma^* P \rightarrow \rho P$  at HERA

- one can experimentally measure all spin density matrix elements
- at  $t = t_{min}$  one can experimentally distinguish

$$\left\{ \begin{array}{ll} \gamma_L^* \rightarrow \rho_L : & \text{dominates (twist 2 dominance)} \\ \gamma_T^* \rightarrow \rho_T : & \text{sizable (twist 3)} \end{array} \right.$$

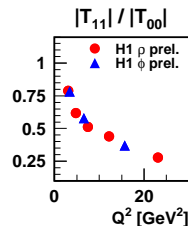
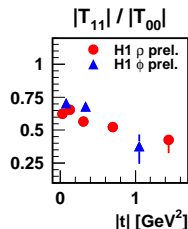
- S-channel helicity conservation:**

$$\left\{ \begin{array}{ll} \gamma_L^* \rightarrow \rho_L & (\equiv T_{00}) \\ \gamma_T^* \rightarrow \rho_T, & \end{array} \right.$$

Dominate with respect to all other transitions.

Experimentally,  $\gamma_T^* \rightarrow \rho_T$  is dominated

by  $\gamma_{T(-)}^* \rightarrow \rho_{T(-)}$  and  $\gamma_{T(+)}^* \rightarrow \rho_{T(+)}$  ( $\equiv T_{11}$ )



(from X. Janssen (H1), DIS 2008)

# Introduction

## Exclusive $\rho$ -production

The processes with vector particle such as  $\rho$ -meson probe deeper into the fine features of QCD.

It deserves theoretical development to describe HERA data in its special kinematical range:

- large  $s_{\gamma^*P}$   $\Rightarrow$  small- $x$  effects expected, within  $k_t$ -factorization
- large  $Q^2$   $\Rightarrow$  hard scale  $\Rightarrow$  perturbative approach and collinear factorization  $\Rightarrow$  the  $\rho$  can be described through its chiral even Distribution Amplitudes

$$\left\{ \begin{array}{ll} \rho_L & \text{twist 2} \\ \rho_T & \text{twist 3} \end{array} \right.$$

The main ingredient is the  $\gamma^* \rightarrow \rho$  impact factor

- For  $\rho_T$ , special care is needed: a pure 2-body description would violate gauge invariance.
- We show that:
  - Including in a consistent way all twist 3 contributions, i.e. 2-body and 3-body correlators, gives a gauge invariant impact factor
  - Our treatment is free of end-point singularities and does not violate the QCD factorization

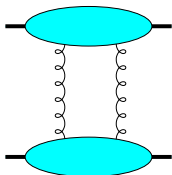
# Impact factor for exclusive processes

## Theoretical motivations

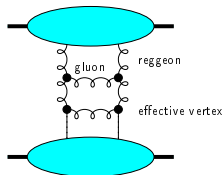
### QCD in perturbative Regge limit

- In this limit, the dynamics is dominated by gluons (dominance of spin 1 exchange in  $t$  channel)
- BFKL (and extensions: NLL, saturations effects, ...) is expected to dominate with respect to Born order at large relative rapidity.

Born order:



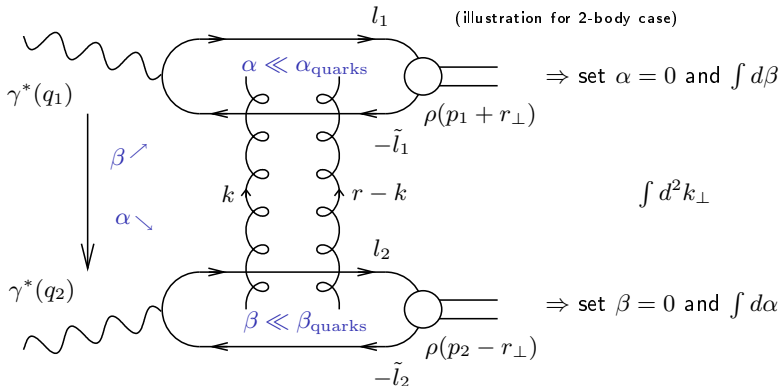
BFKL ladder:



## Impact factor for exclusive processes

 $k_T$  factorization $\gamma^* \gamma^* \rightarrow \rho\rho$  as an example

- Use Sudakov decomposition  $k = \alpha p_1 + \beta p_2 + k_\perp$  ( $p_1^2 = p_2^2 = 0$ ,  $2p_1 \cdot p_2 = s$ )
- write  $d^4k = \frac{s}{2} d\alpha d\beta d^2k_\perp$
- $t$ -channel gluons with non-sense polarizations ( $\epsilon_{NS}^{up} = \frac{2}{s} p_2$ ,  $\epsilon_{NS}^{down} = \frac{2}{s} p_1$ ) dominate at large  $s$



## Impact factor for exclusive processes

 $k_T$  factorization

impact representation

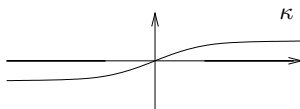
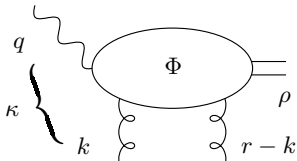
 $\underline{k} = \text{Eucl.} \leftrightarrow k_{\perp} = \text{Mink.}$ 

$$\mathcal{M} = is \int \frac{d^2 \underline{k}}{(2\pi)^2 \underline{k}^2 (\underline{r} - \underline{k})^2} \Phi^{\gamma^* (q_1) \rightarrow \rho(p_1^{\rho})}(\underline{k}, \underline{r} - \underline{k}) \Phi^{\gamma^* (q_2) \rightarrow \rho(p_2^{\rho})}(-\underline{k}, -\underline{r} + \underline{k})$$

The  $\gamma_{L,T}^*(q)g(k_1) \rightarrow \rho_{L,T}g(k_2)$  **impact factor** is normalized as

$$\Phi^{\gamma^* \rightarrow \rho}(\underline{k}^2) = e^{\gamma^* \mu} \frac{1}{2s} \int \frac{d\kappa}{2\pi} \text{Disc}_{\kappa} \mathcal{S}_{\mu}^{\gamma^* g \rightarrow \rho g}(\underline{k}^2),$$

with  $\kappa = (q+k)^2 = \beta s - Q^2 - \underline{k}^2$

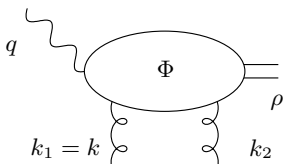


# Impact factor for exclusive processes

Gauge invariance within subleading twists

## Gauge invariance

- QCD gauge invariance (probes are colorless)  
 $\Rightarrow$  impact factor should vanish when  $\underline{k} \rightarrow 0$  or  $\underline{r} - \underline{k} \rightarrow 0$
- In the following we will restrict ourselves to the case  $t = t_{min}$ , i.e. to  $\underline{r} = 0$



$$k_1 = \frac{\kappa + Q^2 + k^2}{s} p_2 + k_\perp$$

$$k_2 = \frac{\kappa + k^2}{s} p_2 + k_\perp,$$

$$k_1^2 = k_2^2 = -\underline{k}^2$$

This kinematics takes into account skewedness effects along  $p_2$

$t = t_{min} \Rightarrow$  restriction to the transitions

$$\begin{cases} 0 & \rightarrow & 0 & \text{(twist 2)} \\ (+ \text{ or } -) & \rightarrow & (+ \text{ or } -) & \text{(twist 3)} \end{cases}$$

- At twist 3 level (for  $\gamma_T^* \rightarrow \rho_T$  transition), gauge invariance is a non trivial statement which requires 2 and 3 body correlators

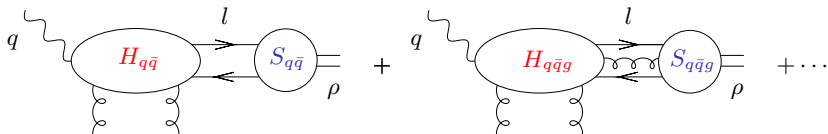


# Collinear factorization

## Light-Cone Collinear approach

- The impact factor can be written as

$$\Phi = \int d^4l \cdots \text{tr}[\underbrace{H(l \cdots)}_{\text{hard part}} \quad \underbrace{S(l \cdots)}_{\text{soft part}}]$$



- At the 2-body level:

$$S_{q\bar{q}}(l) = \int d^4z e^{-il \cdot z} \langle \rho(p) | \psi(0) \bar{\psi}(z) | 0 \rangle,$$

- $H$  and  $S$  are related by  $\int d^4l$  and by the **summation over spinor indices**

## Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

## 1 - Momentum factorization (1)

- Use Sudakov decomposition in the form ( $p = p_1$ ,  $n = 2p_2/s \Rightarrow p \cdot n = 1$ )

$$l_\mu = y p_\mu + l_\mu^\perp + (l \cdot p) n_\mu, \quad y = l \cdot n$$

$$\text{scaling:} \quad 1 \quad 1/Q \quad 1/Q^2$$

- decompose  $H(k)$  around the  $p$  direction:

$$H(l) = H(y p) + \left. \frac{\partial H(l)}{\partial l_\alpha} \right|_{l=y p} (l - y p)_\alpha + \dots \quad \text{with} \quad (l - y p)_\alpha \approx l_\alpha^\perp$$

- In Fourier space, the twist 3 term  $l_\alpha^\perp$  turns into a derivative of the soft term

$$\Rightarrow \text{one will deal with } \int d^4 z e^{-i l \cdot z} \langle \rho(p) | \psi(0) i \overleftrightarrow{\partial}_{\alpha^\perp} \bar{\psi}(z) | 0 \rangle$$

## Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

## 1 - Momentum factorization (2)

- write

$$d^4l \longrightarrow d^4l \delta(\mathbf{y} - l \cdot \mathbf{n}) d\mathbf{y}$$

- $\int d^4l \delta(\mathbf{y} - l \cdot \mathbf{n})$  is then absorbed in the soft term:

$$\begin{aligned} (\tilde{S}_{q\bar{q}}, \partial_{\perp} \tilde{S}_{q\bar{q}}) &\equiv \int d^4l \delta(\mathbf{y} - l \cdot \mathbf{n}) \int d^4z e^{-il \cdot z} \langle \rho(p) | \psi(0) (\mathbf{1}, i \overleftrightarrow{\partial}_{\perp}) \bar{\psi}(z) | 0 \rangle \\ (\delta(\mathbf{y} - l \cdot \mathbf{n}) = \int \frac{d\lambda}{2\pi} e^{-i\lambda(\mathbf{y} - l \cdot \mathbf{n})} \Rightarrow) &= \int \frac{d\lambda}{2\pi} e^{-i\lambda \mathbf{y}} \int d^4z \delta^{(4)}(z - \lambda \mathbf{n}) \langle \rho(p) | \psi(0) (\mathbf{1}, i \overleftrightarrow{\partial}_{\perp}) \bar{\psi}(z) | 0 \rangle \\ &= \int \frac{d\lambda}{2\pi} e^{-i\lambda \mathbf{y}} \langle \rho(p) | \psi(0) (\mathbf{1}, i \overleftrightarrow{\partial}_{\perp}) \bar{\psi}(\lambda \mathbf{n}) | 0 \rangle \end{aligned}$$

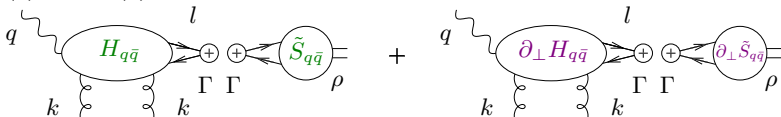
- $\int d\mathbf{y}$  performs the longitudinal momentum factorization

## Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (2-body case)

## 2 - Spinorial (and color) factorization

- Use Fierz decomposition of the Dirac (and color) matrices  $\psi(0) \bar{\psi}(z)$  and  $\psi(0) i \overleftrightarrow{\partial}_\perp \bar{\psi}(z)$ :



- $\Phi$  has now the simple factorized form:

$$\Phi = \int dx \left\{ \text{tr} [H_{q\bar{q}}(xp) \Gamma] S_{q\bar{q}}^\Gamma(x) + \text{tr} [\partial_\perp H_{q\bar{q}}(xp) \Gamma] \partial_\perp S_{q\bar{q}}^\Gamma(x) \right\}$$

$\Gamma = \gamma^\mu$  and  $\gamma^\mu \gamma^5$  matrices

$$S_{q\bar{q}}^\Gamma(x) = \int \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle \rho(p) | \bar{\psi}(\lambda n) \Gamma \psi(0) | 0 \rangle$$

$$\partial_\perp S_{q\bar{q}}^\Gamma(x) = \int \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle \rho(p) | \bar{\psi}(\lambda n) \Gamma i \overleftrightarrow{\partial}_\perp \psi(0) | 0 \rangle$$

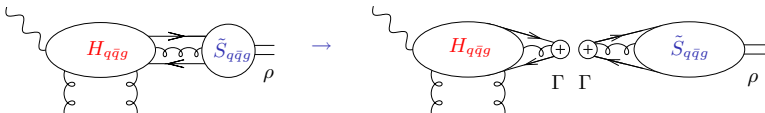
- choose axial gauge condition for gluons, i.e.  $n \cdot A = 0 \Rightarrow$  no Wilson line

## Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (3-body case)

## Factorization of 3-body contributions

- 3-body contributions start at **genuine twist 3**  
⇒ no need for **Taylor** expansion
- Momentum factorization goes in the same way as for 2-body case
- Spinorial (and color) factorization is similar:



## Collinear factorization

Parametrization of vacuum-to- $\rho$ -meson matrix elements (DAs): 2-body correlators2-body **non-local** correlators $\rho_L$ **twist 2**

- vector correlator

$$\langle \rho(p) | \bar{\psi}(z) \gamma_\mu \psi(0) | 0 \rangle \stackrel{\mathcal{F}}{=} m_\rho f_\rho \left[ \varphi_1(y) (e^* \cdot n) p_\mu + \varphi_3(y) e_\mu^{*T} \right]$$

 $\rho_T$ 
**kinematical twist 3 (WW)**  
**genuine twist 3**  
**genuine + kinematical twist 3**

- axial correlator

$$\langle \rho(p) | \bar{\psi}(z) \gamma_5 \gamma_\mu \psi(0) | 0 \rangle \stackrel{\mathcal{F}}{=} m_\rho f_\rho i \varphi_A(y) \varepsilon_{\mu\lambda\beta\delta} e_\lambda^{*T} p_\beta n_\delta$$

- vector correlator with transverse derivative

$$\langle \rho(p) | \bar{\psi}(z) \gamma_\mu i \overleftrightarrow{\partial}_\alpha^\perp \psi(0) | 0 \rangle \stackrel{\mathcal{F}}{=} m_\rho f_\rho \varphi_1^T(y) p_\mu e_\alpha^{*T}$$

- axial correlator with transverse derivative

$$\langle \rho(p) | \bar{\psi}(z) \gamma_5 \gamma_\mu i \overleftrightarrow{\partial}_\alpha^\perp \psi(0) | 0 \rangle \stackrel{\mathcal{F}}{=} m_\rho f_\rho i \varphi_A^T(y) p_\mu \varepsilon_{\alpha\lambda\beta\delta} e_\lambda^{*T} p_\beta n_\delta,$$

where  $y$  ( $\bar{y} \equiv 1 - y$ ) = momentum fraction along  $p \equiv p_1$  of the quark (antiquark) and

$$\stackrel{\mathcal{F}}{=} \int_0^1 dy \exp[i y p \cdot z], \text{ with } z = \lambda n$$

⇒ 5 2-body DAs

## Collinear factorization

Parametrization of vacuum-to- $\rho$ -meson matrix elements: 3-body correlators

## 3-body non-local correlators

genuine twist 3

- vector correlator

$$\langle \rho(p) | \bar{\psi}(z_1) \gamma_\mu g A_\alpha^T(z_2) \psi(0) | 0 \rangle \stackrel{\mathcal{F}_2}{=} m_\rho f_3^V B(y_1, y_2) p_\mu e_\alpha^{*T},$$

- axial correlator

$$\langle \rho(p) | \bar{\psi}(z_1) \gamma_5 \gamma_\mu g A_\alpha^T(z_2) \psi(0) | 0 \rangle \stackrel{\mathcal{F}_2}{=} m_\rho f_3^A i D(y_1, y_2) p_\mu \varepsilon_{\alpha\lambda\beta\delta} e_\lambda^{*T} p_\beta n_\delta,$$

where  $y_1, \bar{y}_2, y_2 - y_1 =$  quark, antiquark, gluon momentum fraction

$$\text{and } \stackrel{\mathcal{F}_2}{=} \int_0^1 dy_1 \int_0^1 dy_2 \exp[i y_1 p \cdot z_1 + i(y_2 - y_1) p \cdot z_2], \text{ with } z_{1,2} = \lambda n$$

$\Rightarrow$  2 3-body DAs

# Collinear factorization

## Equations of motion

### Equations of motion

twist 2  
kinematical twist 3 (WW)  
genuine twist 3  
genuine + kinematical twist 3

- Dirac equation leads to

$$\langle i(\not{D}(0)\psi(0))_\alpha \bar{\psi}_\beta(z) \rangle = 0 \quad (i \vec{D}_\mu = i \vec{\partial}_\mu + g A_\mu)$$

- Apply the Fierz decomposition to the above 2 and 3-body correlators

$$- \langle \psi(x) \bar{\psi}(z) \rangle = \frac{1}{4} \langle \bar{\psi}(z) \gamma_\mu \psi(x) \rangle \gamma_\mu + \frac{1}{4} \langle \bar{\psi}(z) \gamma_5 \gamma_\mu \psi(x) \rangle \gamma_\mu \gamma_5.$$

- $\Rightarrow$  2 Equations of motion:

$$\bar{y}_1 \varphi_3(y_1) + \bar{y}_1 \varphi_A(y_1) + \varphi_1^T(y_1) + \varphi_A^T(y_1) + \int dy_2 \left[ \zeta_3^V B(y_1, y_2) + \zeta_3^A D(y_1, y_2) \right] = 0 \quad \text{and} \quad (\bar{y}_1 \leftrightarrow y_1)$$

- In WW approximation: genuine twist 3 = 0 i.e.  $B = D = 0$

$$\begin{cases} \varphi_A^T(y) = \frac{1}{2} [(y - \bar{y}) \varphi_A^{WW}(y) - \varphi_3^{WW}(y)] \\ \varphi_1^T(y) = \frac{1}{2} [(y - \bar{y}) \varphi_3^{WW}(y) - \varphi_A^{WW}(y)] \end{cases}$$



# Collinear factorization

$n$ -independence

## A minimal set of DAs

- The non-perturbative correlators cannot be obtained from perturbative QCD (!)
- one should reduce them to a minimal set before using any model
- this can be achieved by using an additional condition:  
**independence of the full amplitude with respect to the light-cone vector  $n$**   
 $n$  enters 3 places:
  - light-cone direction of  $z$ :  $z = \lambda n$
  - definition of  $\rho_T$  polarization:  $e_T \cdot n = 0$
  - axial gauge:  $n \cdot A = 0$

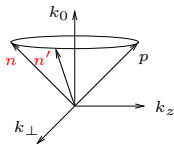
⇒ we prove that **3 independent Distribution Amplitudes** are needed:  
 $7 - 2$  (=nb of equations of motion) -  $2$  (=nb of eq. from  $n$ -ind. cond.)

$\phi_1(y)$  ← **2 body twist 2 correlator**  
 $B(y_1, y_2)$  ← **3 body genuine twist 3 vector correlator**  
 $D(y_1, y_2)$  ← **3 body genuine twist 3 axial correlator**

# Collinear factorization

$n$ -independence

$n$ -independence in practice

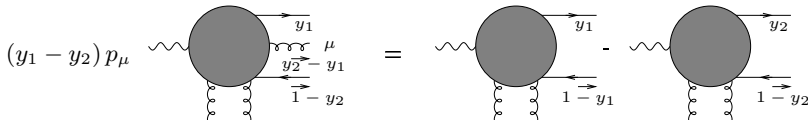


- $\rho_T$  polarization:  $e_{\mu}^{*T} = e_{\mu}^* - p_{\mu} e^* \cdot n$  keeping  $n \cdot p = 1$
- for the full factorized amplitude:

$$\mathcal{A} = H \otimes S \quad \frac{d\mathcal{A}}{dn_{\perp}^{\mu}} = 0,$$

- rewrite hard terms in one single form, of 2-body type: use Ward identities  
Example: hard 3-body  $\rightarrow$  hard 2-body

$$\text{tr} [H_{3\rho}(y_1, y_2) p^{\rho} \not{p}] B(y_1, y_2) = \frac{1}{y_1 - y_2} (\text{tr} [H_2(y_1) \not{p}] - \text{tr} [H_2(y_2) \not{p}]) B(y_1, y_2),$$



- thus, symbolically,

$$\frac{dS}{dn_{\perp}^{\mu}} = 0$$

## Collinear factorization

 $n$ -independenceConstraints from  $n$ -independence

twist 2

kinematical twist 3 (WW)

genuine twist 3

genuine + kinematical twist 3

- vector correlators

$$\frac{d}{dy_1} \varphi_1^T(y_1) = -\varphi_1(y_1) + \varphi_3(y_1)$$

$$-\zeta_3^V \int_0^1 \frac{dy_2}{y_2 - y_1} (B(y_1, y_2) + B(y_2, y_1))$$

- axial correlators

$$\frac{d}{dy_1} \varphi_A^T(y_1) = \varphi_A(y_1) - \zeta_3^A \int_0^1 \frac{dy_2}{y_2 - y_1} (D(y_1, y_2) + D(y_2, y_1))$$

# Collinear factorization

A set of independent non-perturbative correlators

## Solution

twist 2  
kinematical twist 3 (WW)  
genuine twist 3  
genuine + kinematical twist 3

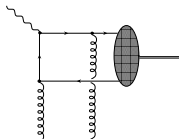
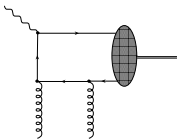
- the set of 4 equations (2 EOM + 2  $n$ -independence relations) can be solved analytically
- 7  $\rightarrow$  3 independent DAs

# Computation and results

## Computation of the hard part

### 2-body diagrams

- without derivative



twist 2  $(\gamma_L^* \rightarrow \rho_L)$

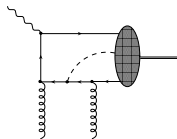
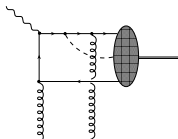
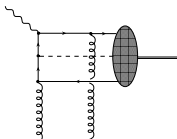
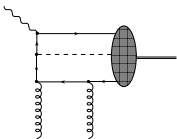
twist 3  $(\gamma_T^* \rightarrow \rho_T)$

- practical trick for computing  $\partial_\perp H$  : use the Ward identity

$$\frac{\partial}{p_\mu} \rightarrow p = \rightarrow p \quad \bullet \quad \rightarrow p$$

$\gamma^\mu$

where  $\rightarrow p = \frac{1}{m - \not{p} - i\epsilon}$

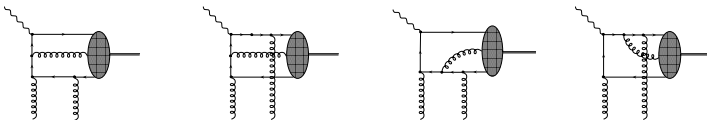


# Computation and results

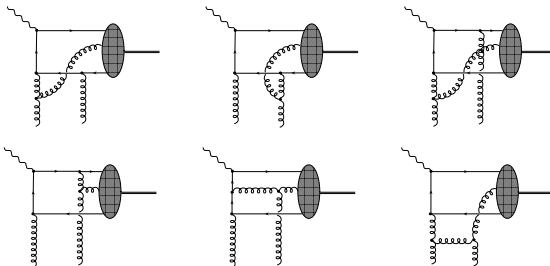
## Computation of the hard part

### 3-body diagrams

- “abelian” type



- “non-abelian” type



# Computation and results

Recall:  $\gamma_L^* \rightarrow \rho_L$  impact factor

$\gamma_L^* \rightarrow \rho_L$  impact factor

$$\Phi^{\gamma_L^* \rightarrow \rho_L}(\underline{k}^2) = \frac{2 e g^2 f_\rho}{Q} \frac{\delta^{ab}}{2 N_c} \int dy \varphi_1(y) \frac{\underline{k}^2}{y \bar{y} Q^2 + \underline{k}^2}$$

pure twist 2 scaling (from  $\rho$ -factorization point of view)

## Computation and results

Results:  $\gamma_T^* \rightarrow \rho_T$  impact factor $\gamma_T^* \rightarrow \rho_T$  impact factor:

Spin Non-Flip/Flip separation appears

$$\Phi^{\gamma_T^* \rightarrow \rho_T}(\underline{k}^2) = \Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T}(\underline{k}^2) T_{n.f.} + \Phi_{f.}^{\gamma_T^* \rightarrow \rho_T}(\underline{k}^2) T_f.$$

where

$$T_{n.f.} = -(e_\gamma \cdot e^*) \quad \text{and} \quad T_f = \frac{(e_\gamma \cdot k)(e^* k)}{\underline{k}^2} + \frac{(e_\gamma \cdot e^*)}{2}$$

non-flip transitions  $\begin{cases} + \rightarrow + \\ - \rightarrow - \end{cases}$ flip transitions  $\begin{cases} + \rightarrow - \\ - \rightarrow + \end{cases}$



## Computation and results

Results:  $\gamma_T^* \rightarrow \rho_T$  impact factorpure twist 3 scaling (from  $\rho$ -factorization point of view)

$$\begin{aligned} & \Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T}(\underline{k}^2) \\ &= -\frac{e g^2 m_\rho f_\rho}{2\sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \left\{ -2 \int dy_1 \frac{(\underline{k}^2 + 2 Q^2 y_1 (1 - y_1)) \underline{k}^2}{y_1 (1 - y_1) (\underline{k}^2 + Q^2 y_1 (1 - y_1))^2} \left[ (2y_1 - 1) \varphi_1^T(y_1) + \varphi_A^T(y_1) \right] \right. \\ &+ 2 \int dy_1 dy_2 \left[ \zeta_3^V B(y_1, y_2) - \zeta_3^A D(y_1, y_2) \right] \frac{y_1 (1 - y_1) \underline{k}^2}{\underline{k}^2 + Q^2 y_1 (1 - y_1)} \left[ \frac{(2 - N_c/C_F) Q^2}{\underline{k}^2 (y_1 - y_2 + 1) + Q^2 y_1 (1 - y_2)} \right. \\ &- \left. \frac{N_c}{C_F} \frac{Q^2}{y_2 \underline{k}^2 + Q^2 y_1 (y_2 - y_1)} \right] - 2 \int dy_1 dy_2 \left[ \zeta_3^V B(y_1, y_2) + \zeta_3^A D(y_1, y_2) \right] \left[ \frac{2 + N_c/C_F}{1 - y_1} \right. \\ &+ \frac{y_1 Q^2}{\underline{k}^2 + Q^2 y_1 (1 - y_1)} \left( \frac{(2 - N_c/C_F) y_1 \underline{k}^2}{\underline{k}^2 (y_1 - y_2 + 1) + Q^2 y_1 (1 - y_2)} - 2 \right) \\ &\left. \left. + \frac{N_c}{C_F} \frac{(y_1 - y_2) (1 - y_2)}{1 - y_1} \frac{Q^2}{\underline{k}^2 (1 - y_1) + Q^2 (y_2 - y_1) (1 - y_2)} \right] \right\} \end{aligned}$$

and

$$\begin{aligned} & \Phi_{f.}^{\gamma_T^* \rightarrow \rho_T}(\underline{k}^2) = -\frac{e g^2 m_\rho f_\rho}{2\sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \left\{ 4 \int dy_1 \frac{\underline{k}^2 Q^2}{(\underline{k}^2 + Q^2 y_1 (1 - y_1))^2} \left[ \varphi_A^T(y_1) - (2y_1 - 1) \varphi_1^T(y_1) \right] \right. \\ &- 4 \int dy_1 dy_2 \frac{y_1 \underline{k}^2}{\underline{k}^2 + Q^2 y_1 (1 - y_1)} \left[ \zeta_3^A D(y_1, y_2) (-y_1 + y_2 - 1) + \zeta_3^V B(y_1, y_2) (y_1 + y_2 - 1) \right] \\ &\left. \times \left[ \frac{(2 - N_c/C_F) Q^2}{\underline{k}^2 (y_1 - y_2 + 1) + Q^2 y_1 (1 - y_2)} - \frac{N_c}{C_F} \frac{Q^2}{y_2 \underline{k}^2 + Q^2 y_1 (y_2 - y_1)} \right] \right\} \end{aligned}$$

# Computation and results

Results:  $\gamma_T^* \rightarrow \rho_T$  impact factor

## WW limit

- WW limit: keep only **twist 2** + **kinematical twist 3** terms (i.e  $B = D = 0$ )
- The only remaining contributions come from the two-body correlators
- non-flip transition

$$\Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T}(\underline{k}^2) = \frac{-e m_\rho f_\rho}{2\sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \int_0^1 dy \left\{ \frac{(y - \bar{y}) \varphi_1^{TWW}(y) + 2 y \bar{y} \varphi_3^{WW}(y) + \varphi_A^{TWW}(y)}{y \bar{y}} - \frac{2 \underline{k}^2 (\underline{k}^2 + 2 Q^2 y \bar{y}) ((y - \bar{y}) \varphi_1^{TWW}(y) + \varphi_A^{TWW}(y))}{y \bar{y} (\underline{k}^2 + Q^2 y (1 - y))^2} \right\}$$

which simplifies, using equation of motion:

$$\int dy [(y - \bar{y}) \varphi_1^{TWW}(y) + 2 y \bar{y} \varphi_3^{WW}(y) + \varphi_A^{TWW}(y)] = 0$$

$$\Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T}(\underline{k}^2) = \frac{e m_\rho f_\rho}{\sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \int_0^1 dy \frac{2 \underline{k}^2 (\underline{k}^2 + 2 Q^2 y \bar{y})}{y \bar{y} (\underline{k}^2 + Q^2 y \bar{y})^2} [(2y - 1) \varphi_1^{TWW}(y) + \varphi_A^{TWW}(y)].$$

- flip transition:

$$\Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T}(\underline{k}^2) = -\frac{e m_\rho f_\rho}{\sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \int_0^1 \frac{2 \underline{k}^2 Q^2}{(\underline{k}^2 + Q^2 y \bar{y})^2} [(1 - 2y) \varphi_1^{TWW}(y) + \varphi_A^{TWW}(y)].$$

# Computation and results

Discussion: [gauge invariance](#)

- The obtained results are gauge invariant:

$$\Phi^{\gamma_T^* \rightarrow \rho_T} \rightarrow 0 \quad \text{when} \quad \underline{k} \rightarrow 0$$

- this is straightforward in the WW limit
- at the full twist 3 order:
  - the  $C_F$  part of the abelian 3-body contribution cancels the 2-body contribution **after using the equation of motion**
  - the  $N_c$  part of the abelian 3-body contribution cancels the 3-body non-abelian contribution
  - thus  $\gamma_T^* \rightarrow \rho_T$  impact factor is **gauge-invariant only provided the 2 and 3-body contributions have been taken into account in a consistent way**

# Computation and results

Discussion: **consistence with factorization**

- **Our results are free of end-point singularities**, in both **WW** approximation and full twist-3 order calculation:
  - the flip contribution obviously does not have any end-point singularity because of the  $\underline{k}^2$  which regulates them
  - the potential end-point singularity for the non-flip contribution is spurious since  $\varphi_A^T(y)$ ,  $\varphi_1^T(y)$  vanishes at  $y = 0, 1$  as well as  $B(y_1, y_2)$  and  $D(y_1, y_2)$ .

# Conclusions

1

- We have performed a full up to twist 3 computation of the  $\gamma^* \rightarrow \rho$  impact factor, in the  $t = t_{min}$  limit.
- Our result respects gauge invariance. This is achieved only after including 2 and 3 body correlators.
- It is free of end-point singularities  
(this should be contrasted with standard collinear treatment, at moderate  $s$ , where  $k_T$ -factorization is NOT applicable: see [Mankiewicz-Piller](#)).
- Phenomenological applications will be done in the near future.
- In this talk we relied on the [Light-Cone Collinear approach](#)  
([Ellis + Furmanski + Petronzio](#); [Efremov + Teryaev](#); [Anikin + Teryaev](#)), which is non-covariant, but [very efficient for practical computations](#).
- This [Light-Cone Collinear approach is systematic](#), and can be extended to any process, including higher twist effects (but does not preclude potential end-point singularities)

## Conclusions

2

- Comparison with a fully **covariant approach** by **Ball+Braun et al**:  
The dictionary between the two approaches within a full twist 3 treatment is now established:

$$B(y_1, y_2) = -\frac{V(y_1, 1 - y_2, y_2 - y_1)}{y_2 - y_1},$$

$$D(y_1, y_2) = -\frac{A(y_1, 1 - y_2, y_2 - y_1)}{y_2 - y_1}$$

$$\varphi_1(y) = f_\rho m_\rho \phi_{\parallel}(y)$$

$$\varphi_3(y) = f_\rho m_\rho g^{(v)}(y),$$

$$\varphi_A(y) = -\frac{1}{4} f_\rho m_\rho \frac{\partial g^{(a)}(y)}{\partial y}$$

- We also performed calculations of the same impact factor within the **covariant approach** by **Ball+Braun et al**: calculations proceed in quite different way : eg. no  $\varphi_{1,A}^T$ -DAs but **Wilson** line effects are important !!  
We got a full agreement with our approach