# Photoproduction of a $\pi\rho_T$ pair with a large invariant mass and Transversity Generalized Parton Distribution

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in collaboration with
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# Transversity of the nucleon using hard processes

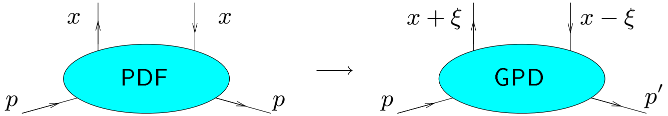
What is transversity?

- Transverse spin content of the proton:

$$
\begin{array}{ccc}
| \uparrow \rangle_{(x)} & \sim & | \rightarrow \rangle + | \leftarrow \rangle \\
| \downarrow \rangle_{(x)} & \sim & | \rightarrow \rangle - | \leftarrow \rangle \\
\text{spin along } x & & \text{helicity states}
\end{array}
$$

- Observable sensible to helicity flip thus give access to transversity $\Delta_T q(x)$. Very poorly known

- Transversity GPDs are completely unknown




- For massless (anti)particles, chirality = (-)helicity

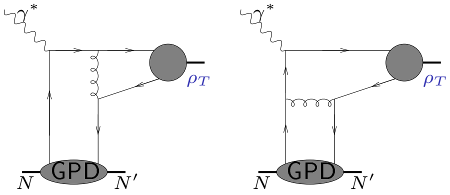
- Transversity is thus a chiral-odd quantity

- Since QCD and QED are chiral even, the chiral odd quantities which one want to measure should appear in pairs

# Transversity of the nucleon using hard processes: using a two body final state process?

How to get access to transversity?

- the dominant DA of $\rho_T$ is of twist 2 and chiral odd ($[\gamma^\mu, \gamma^\nu]$ coupling)
- unfortunately $\gamma^*\, N^\uparrow \to \rho_T\, N' = 0$
  - this is true at any order, because this would require a transfer of helicity of 2 from photon: impossible!
  - lowest order diagrammatic argument:




vanish: $\gamma^\alpha [\gamma^\mu, \gamma^\nu] \gamma_\alpha = 0$

## Transversity of the nucleon using hard processes: using a two body final state process?

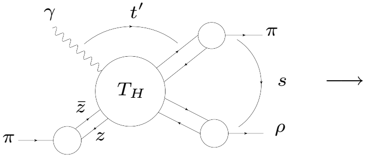
Can one circumvent this vanishing?

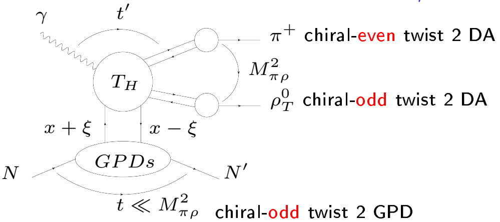
- this vanishing is true only a twist 2
- at twist 3 this process does not vanish
- however processes involving twist 3 DAs may face problems with factorization (end-point singularities)
- the problem of classification of twist 3 chiral-odd GPDs is still open (Pire, Szymanowski, S.W., in preparation, in the spirit of our Light-Cone Collinear Factorization framework: Anikin, Ivanov, Pire, Szymanowski, S. W. - see talk in Session *Small-x, diffraction and VM in DIS and hadron colliders*)

## Our process: $\gamma N \to \pi^+ \rho^0_T N'$

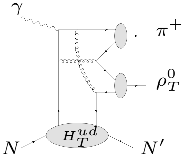
$\gamma N \to \pi^+ \rho^0_T N'$ gives access to transversity

- Factorization à la Brodsky Lepage of $\gamma + \pi \to \pi + \rho$ at large $s$ and fixed angle (i.e. fixed ratio $t'/s, u'/s$)
  $\Longrightarrow$ factorization of the amplitude for $\gamma + N \to \pi + \rho + N'$ at large $M^2_{\pi\rho}$



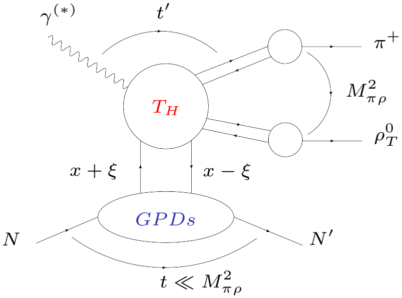




- a typical non-vanishing diagram:




- these processes with 3 body final state can give access to all GPDs.
  $M^2_{\pi\rho}$ plays the role of $\gamma^*$ in usual DVCS, and can be scanned.

# Master formula based on leading twist 2 factorization




$$\mathcal{A} = \frac{1}{\sqrt{2}} \int_{-1}^{1} dx \int_{0}^{1} dv \int_{0}^{1} dz \, \left(T^u(x,v,z) - T^d(x,v,z)\right)$$
$$\times \left(H_T^u(x,\xi,t) - H_T^d(x,\xi,t)\right) \Phi_\pi(z) \Phi_\rho(v) + \cdots$$

## Non pertubative matrix elements

One needs to encode the matrix elements of two kinds of chiral-odd operator:

- transversity GPDs (twist-2 level):

$$
\begin{aligned}
&\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p_2, \lambda_2 | \bar{\psi}_q \left(-\frac{1}{2}z^-\right) i\sigma^{+i} \psi \left(\frac{1}{2}z^-\right) | p_1, \lambda_1 \rangle \\
= \quad & \frac{1}{2P^+} \bar{u}(p_2, \lambda_2) \left[ H_T^q(x,\xi,t) i\sigma^{+i} + \tilde{H}_T^q(x,\xi,t) \frac{P^+\Delta^i - \Delta^+ P^i}{M_N^2} \right. \\
+ \quad & \left. E_T^q(x,\xi,t) \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2M_N} + \tilde{E}_T^q(x,\xi,t) \frac{\gamma^+ P^i - P^+\gamma^i}{M_N} \right] u(p_1, \lambda_1)
\end{aligned}
$$

  - for $\Delta_\perp = 0$ each above factors vanishes except for $H_T^q$ which thus dominates in the small $t$ domain
  - in the forward limit it is the only transversity GPD which survives: $H_T^q(x,0,0) = \Delta_T q(x)$ (quark transversity distribution)

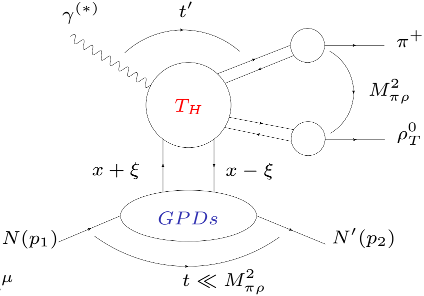
- transversity DAs (twist-2 level):

$$
\langle 0 | \bar{u}(0) \sigma^{\mu\nu} u(x) | \rho^0(p,s) \rangle = \frac{i}{\sqrt{2}} (\sigma^\mu_\rho p^\nu - \sigma^\nu_\rho p^\mu) f_\rho^\perp \int_0^1 du\ e^{-iup\cdot x}\ \phi_\perp(u)
$$

## Kinematics

- use a Sudakov basis :
  light-cone vectors $p$, $n$ with $2p \cdot n = s$

- assume the following kinematics:
  - $\Delta_\perp^\mu$ small
  - $M^2,\ m_\pi^2,\ m_\rho^2 \ll M_{\pi\rho}^2$




- initial state particle momenta:

$$q^\mu = n^\mu,\ p_1^\mu = (1+\xi)p^\mu + \frac{M^2}{s(1+\xi)} n^\mu$$

- final state particle momenta:

$$
\begin{aligned}
p_2^\mu &= (1-\xi)p^\mu + \frac{M^2+\vec{\Delta}_t^2}{s(1-\xi)} n^\mu + \Delta_\perp^\mu \\
p_\pi^\mu &= \alpha\, n^\mu + \frac{(\vec{p}_t - \vec{\Delta}_t/2)^2 + m_\pi^2}{\alpha s} p^\mu + p_\perp^\mu - \frac{\Delta_\perp^\mu}{2} \\
p_\rho^\mu &= \alpha_\rho\, n^\mu + \frac{(\vec{p}_t + \vec{\Delta}_t/2)^2 + m_\rho^2}{\alpha_\rho s} p^\mu - p_\perp^\mu - \frac{\Delta_\perp^\mu}{2}
\end{aligned}
$$

## Pertinent physical parameters

- Total center-of-mass energy squared of the $\gamma$-N system

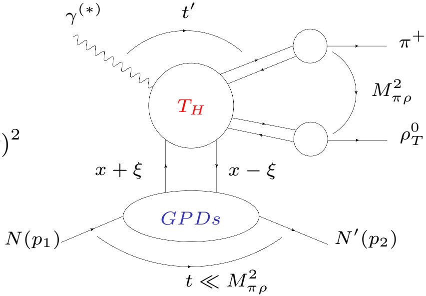
$$S_{\gamma N} = (q + p_1)^2$$

- Hard scale: invariant squared mass of the $(\pi^+, \rho^0)$ system

$$M^2_{\pi\rho} = (p_\pi + p_\rho)^2 \simeq -u' = -(p_\rho - q)^2$$
$$\simeq -t' = -(p_\pi - q)^2 \simeq -p_\perp^2$$

- Transferred squared momentum:

$$t = (p_2 - p_1)^2 \qquad \text{small} \quad t$$




- Skewedness: $\xi = \frac{\tau}{2-\tau}$

with $\tau = \frac{M^2_{\pi\rho}}{S_{\gamma N} - M^2_{\pi\rho}}$ (generalized Bjorken variable for Drell Yan)

# Computation of the hard part

Typical Feynman diagrams (62 in total)

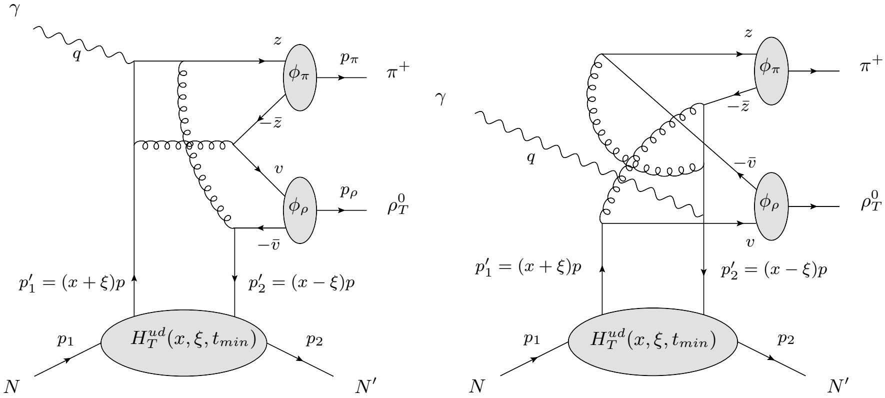
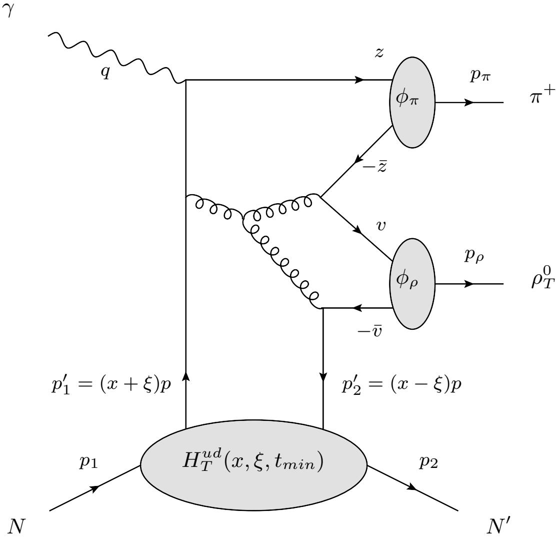



diagram with photon $u$-quark coupling

diagram with photon $d$-quark coupling




representative diagram with a 3 gluon vertex

## Tensorial structure of the amplitude

$$\begin{aligned}\mathcal{A}_{H_T^q} &= (N^{\perp}_{\lambda_1\lambda_2}\cdot\epsilon_{\rho\,\pm})(p_\perp\cdot\epsilon_{\gamma\perp})A + (N^{\perp}_{\lambda_1\lambda_2}\cdot\epsilon_{\gamma\perp})(p_\perp\cdot\epsilon_{\rho\,\pm})\,B\\ &+ (N^{\perp}_{\lambda_1\lambda_2}\cdot p_\perp)(\epsilon_{\gamma\perp}\cdot\epsilon_{\rho\,\pm})\,C\\ &- (N^{\perp}_{\lambda_1\lambda_2}\cdot p_\perp)(p_\perp\cdot\epsilon_{\gamma\perp})(p_\perp\cdot\epsilon_{\rho\,\pm})D\end{aligned}$$

with

- $A$, $B$, $C$, $D$ scalar functions of $S_{\gamma N}$, $-u'$ and $M^2_{\pi\rho}$
- $\epsilon^{\mu}_{\gamma\perp}$ the transverse polarization of the on-shell photon
- $N^{\perp\mu}_{\lambda_1\lambda_2} = \frac{2i}{p\cdot n} g^{\mu\nu}_{\perp}\bar{u}(p_2,\lambda_2)\not{n}\gamma_\nu\gamma^5 u(p_1,\lambda_1)$

Rich spin structure of $\mathcal{A}_{H_T^q}$ : access to the spin density matrix of $\rho^0_T$, polarization asymmetries, ...

# A model based on Double Distribution

## Realistic Parametrization of $H_T^q$

- GPDs can be represented in terms of Double Distribution (Radyushkin)
  based on Schwinger representation of a toy model for GPDs which has the structure of a triangle diagram in scalar $\phi^3$ theory

$$H_T^q(x,\xi,t=0) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \; \delta(\beta+\xi\alpha-x)\; f_T^q(\beta,\alpha)$$

- ansatz for these Double Distribution (Radyushkin):
  - $f_T^q(\beta,\alpha) = \Pi(\beta,\alpha)\; \Delta_T q(\beta)$
  - $\Delta_T q(x)$ : chiral-odd PDF (Anselmino *et al.*)
  - $\Pi(\beta,\alpha) = \frac{3}{4}\frac{(1-\beta)^2-\alpha^2}{(1-\beta)^3}$ : profile function ($f_T^q(\beta,0) = \Delta_T q(\beta)$)

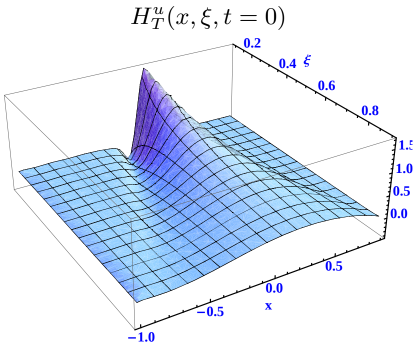
- ansatz for the $t$-dependence:

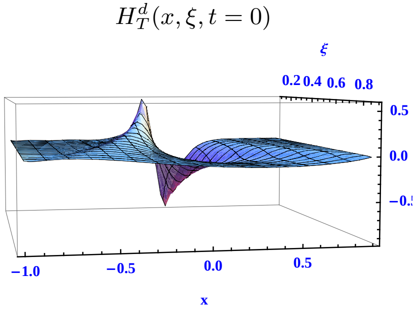
$$H_T^q(x,\xi,t) = H_T^q(x,\xi,t=0) \times F_H(t)$$

with $F_H(t) = \frac{C^2}{(t-C)^2}$ a standard dipole form factor ($C = .71$ GeV)

# Plots of our model for transversity GPD

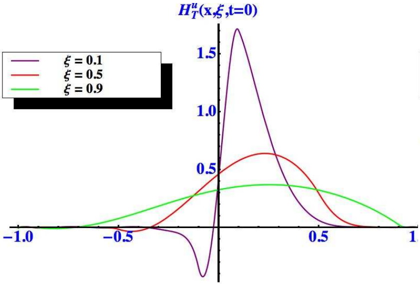
$x$ and $\xi$-dependence of $H_T^q(x, \xi, t = 0)$

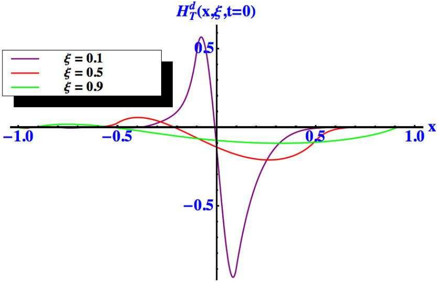


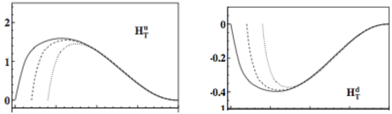
# Plots of our model for transversity GPD

$x$-dependence of $H_T^q(x, \xi, t=0)$ for fixed values of $\xi$




Same order of magnitude but significant differences with other parametrizations (Pincetti *et al.*) and lattice calculations (Göckeler *et al.*)




$$
\begin{aligned}
A^u_{T10}(t \sim 0) &\simeq 0.4(0.9)\\
A^d_{T10}(t \sim 0) &\simeq -0.1(-0.2)
\end{aligned}
$$

# Unpolarized differential cross section

## Differential Cross Section and Physical Cuts

$$\left.\frac{d\sigma}{dt\, du'\, dM^2_{\pi\rho}}\right|_{t=t_{min}} = \frac{|\mathcal{M}|^2}{32 S^2_{\gamma N} M^2_{\pi\rho} (2\pi)^3}$$

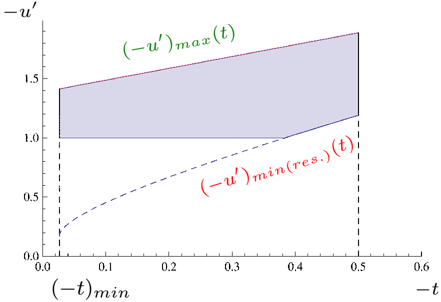
- Validity of the factorization of the partonic amplitude :
  $-t', -u' > \Lambda^2 \gg \Lambda^2_{QCD}$ with $\Lambda \sim 1$ GeV
- Suppress final states interactions (to justify factorization):

  $M^2_{\pi N'},\ M^2_{\rho N'} > M^2_R$ with $M^2_R = 2$ GeV$^2$

  - Cuts over $-u'$ and $M^2_{\pi N'}$

    $\Rightarrow (-u')_{min(res.)}(t, S_{\gamma N}, M^2_{\pi\rho})$.
  - Cuts over $-t'$ and $M^2_{\rho N'}$

    $\Rightarrow (-u')_{max}(t, S_{\gamma N}, M^2_{\pi\rho})$.



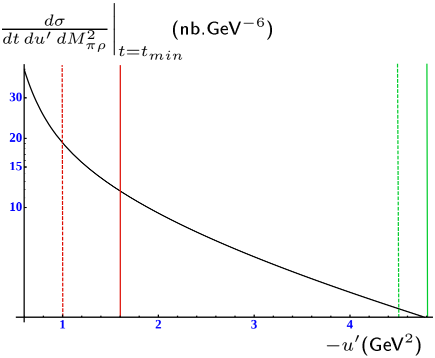

$S_{\gamma N} = 20$ GeV$^2$, $M^2_{\pi\rho} = 3$ GeV$^2$

# Predictions

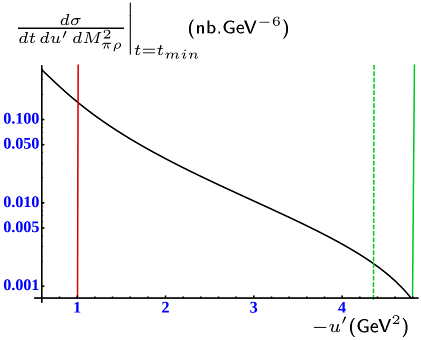
Differential cross section for $M^2_{\pi\rho} = 6$ GeV$^2$

windows: dotted: $t = t_{min}$
solid: $t = -.5$ GeV$^2$

$S_{\gamma N} = 20\ GeV^2$




$S_{\gamma N} = 200\ GeV^2$




$$\left.\frac{d\sigma}{dtdu'dM^2_{\pi\rho}}\right|_{t=t_{min}} \propto 10\ \mathrm{nb}.\mathrm{GeV}^{-6}$$

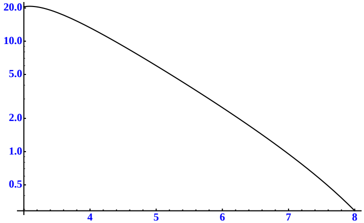
$$\left.\frac{d\sigma}{dtdu'dM^2_{\pi\rho}}\right|_{t=t_{min}} \propto 0.01\ \mathrm{nb}.\mathrm{GeV}^{-6}$$

# Predictions

$M^2_{\pi\rho}$-dependence of the differential cross section $\frac{d\sigma}{dM^2_{\pi\rho}}$

$$\frac{d\sigma}{dM^2_{\pi\rho}} = \int_{-0.5}^{t_{min}} dt \int_{-u'_{min}}^{-u'_{max}} d(-u')\; F_H^2(t) \times \left.\frac{d\sigma}{dt\,du'\,dM^2_{\pi\rho}}\right|_{t=t_{min}}$$

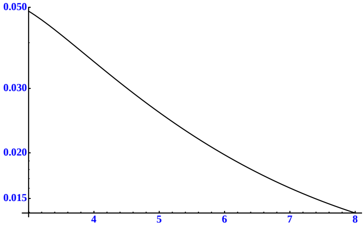
$\frac{d\sigma}{dM^2_{\pi\rho}}$ (nb.GeV$^{-2}$)




$M^2_{\pi\rho}$(GeV$^2$)

$S_{\gamma N} = 20\ GeV^2$

$\frac{d\sigma}{dM^2_{\pi\rho}}$ (nb.GeV$^{-2}$)




$M^2_{\pi\rho}$(GeV$^2$)

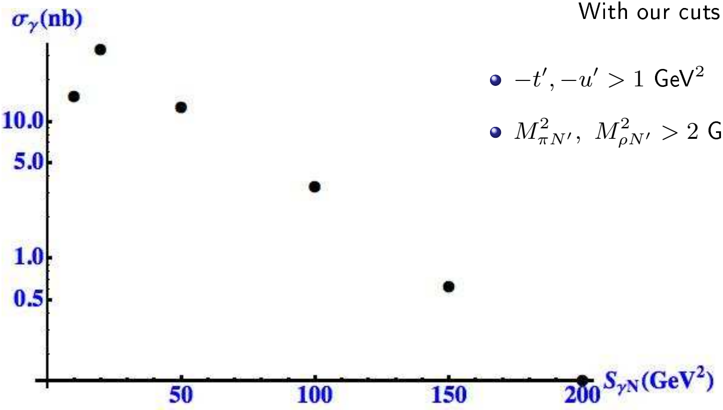
$S_{\gamma N} = 200\ GeV^2$

Total cross sections for photoproduction:

$\sigma(S_{\gamma N} = 20\ \mathrm{GeV}^2) \simeq$ **33 nb** $\quad \sigma(S_{\gamma N} = 200\ \mathrm{GeV}^2) \simeq$ **0.1 nb**

# Predictions

$S_{\gamma N}$-dependence of the differential cross section $\sigma$




With our cuts

- $-t', -u' > 1$ GeV$^2$
- $M^2_{\pi N'},\ M^2_{\rho N'} > 2$ GeV$^2$

# Muoproduction at Compass (CERN)

Very sizable rates

- denote $\Gamma_T^\mu(Q^2,\ \nu)$ the quasi real (transverse) photon flux ($E_\mu = 160$ GeV).
- Total cross section for the muoproduction $\mu N \to \mu \pi^+ \rho_T^0 N'$

$$\sigma_\mu = \int_{0.02}^{1} dQ^2 \int_{16}^{144} d\nu\ \Gamma_T^\mu(Q^2,\nu)\ \sigma_{\gamma^* N \to \pi^+ \rho_T^0 N'}(Q^2,\nu) \simeq 0.25 \text{ pb}$$

- Experimental rate: For a muon beam luminosity of 2.5 $10^{32}$ cm$^{-2}$.s$^{-1}$,

$$\mathbf{R \simeq 6\ 10^{-2}\ Hz}$$

Very sizable

# Rate estimates at JLab

Very high rates

- CLAS12 Hall B:

  with a photon (7 - 10.5 GeV) flux $N_\gamma \sim 5\ 10^7$ photons/s

  Experimental rate: $R \sim$ **0.1 Hz**

- Hall D (12 GeV)
  - photon (8 - 9 GeV) flux $N_\gamma \sim 10^8$ photons/s
  - number of protons per surface unit $N_p \sim 1.27$ b$^{-1}$ (target : liquid hydrogen (30 cm))

  Experimental rate: $R = \sigma \times N_\gamma \times N_p \sim$ **5 Hz**

# Conclusion

- Photoproduction of a $\pi\rho^0_T$ pair with a large hard scale $M^2_{\pi\rho}$ sensitive to the transversity GPDs even for unpolarized target and at twist-2 level
- Parametrization of the dominant chiral-odd GPD $H^q_T$ based on double distribution
- Promising way to get informations on the generalized chiral-odd quark content of the nucleon:
  large enough rates to extract transversity GPDs, at COMPASS and JLab@12 GeV
- Possibility to access to :
  - spin density matrix of $\rho^0_T$
  - spin asymmetries
  - chiral-even GPDs $H$ and $\tilde{H}$ with $\rho^0_L$

    El Beiyad, Pire, Szymanowski, S.W. in preparation
- Such processes with 3 body final state are also promising for non transversity GPD measurement, on top of the now standard DVCS based studies

BACKUP

## Transverse polarization of $\rho_T^0$

$$
\begin{aligned}
\epsilon_\pm^\mu(p_\rho) &= \left(\frac{\vec{p}_\rho\cdot\vec{\epsilon}_\pm}{m_\rho}\ ,\ \vec{\epsilon}_\pm + \frac{\vec{p}_\rho\cdot\vec{\epsilon}_\pm}{m_\rho(E_\rho+m_\rho)}\vec{p}_\rho\right)\\
&\Rightarrow\ 2\bar{\alpha}\frac{\vec{p}_t\cdot\vec{\epsilon}_\pm}{\bar{\alpha}^2 s+\vec{p}_t^{\,2}}\,(p^\mu+n^\mu)+(0,\vec{\epsilon}_\pm)\\
&\Rightarrow\ 2\bar{\alpha}\frac{\vec{p}_t\cdot\vec{\epsilon}_\pm}{\bar{\alpha}^2 s+\vec{p}_t^{\,2}}\left[1-\frac{\vec{p}_t^{\,2}}{\bar{\alpha}^2 s}\right]p^\mu+2\frac{\vec{p}_t\cdot\vec{\epsilon}_\pm}{\bar{\alpha}^2 s+\vec{p}_t^{\,2}}p_T^\mu+(0,\vec{\epsilon}_\pm)
\end{aligned}
$$

## Transversity PDFs

$$
\begin{aligned}
\Delta_T u(x) &= 7.5 * 0.5 * (1-x)^5 (x * u(x) + x * \Delta u(x)) \\
\Delta_T \bar{u}(x) &= 7.5 * 0.5 * (1-x)^5 (x * \bar{u}(x) + x * \Delta \bar{u}(x)) \\
\Delta_T d(x) &= 7.5 * (-0.6) * (1-x)^5 (x * d(x) + x * \Delta d(x)) \\
\Delta_T \bar{d}(x) &= 7.5 * (-0.6) * (1-x)^5 (x * \bar{d}(x) + x * \Delta \bar{d}(x))
\end{aligned}
$$

## Polarized PDFs

$$
\begin{aligned}
\Delta u(x) &= \sqrt{x}\; u(x) \\
\Delta \bar{u}(x) &= -0.3\; x^{0.4}\; \bar{u}(x) \\
\Delta d(x) &= -0.7\; \sqrt{x}\; d(x) \\
\Delta \bar{d}(x) &= -0.3\; x^{0.4}\; \bar{d}(x)
\end{aligned}
$$