

Three-dimensional structure of hadrons through hard exclusive processes

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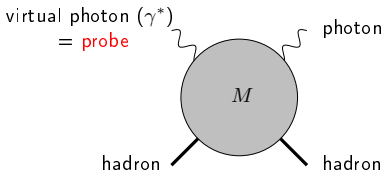
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Exclusive processes are theoretically challenging

How to deal with QCD?

example: Compton scattering



- Aim: describe M by separating:
 - quantities non-calculable perturbatively
 - some tools:
 - Discretization of QCD on a 4-d lattice: numerical simulations
 - AdS/CFT \Rightarrow AdS/QCD : $AdS_5 \times S^5 \leftrightarrow$ QCD
 - Polchinski, Strassler '01
 - for some issues related to Deep Inelastic Scattering (DIS):
 - B. Pire, L. Szymanowski, C. Roiesnel, S. W. Phys.Lett.B670 (2008) 84-90
 - for some issues related to Deep Virtual Compton Scattering (DVCS):
 - C. Marquet, C. Roiesnel, S. W. JHEP 1004:051 (2010) 1-26
 - perturbatively calculable quantities
 - We will here focus on theory and phenomenology of exclusive processes for which the dynamics is governed by QCD in the perturbative regime

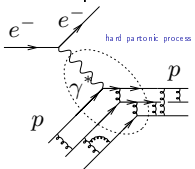
Exclusive processes are phenomenologically challenging

Key question of QCD:

how to obtain and understand the tri-dimensional structure of hadrons
in terms of quarks and gluons?

Can this be achieved using **hard** exclusive processes?

- The aim is to reduce the process to interactions involving a small number of *partons* (quarks, gluons), despite confinement
- This is possible if the considered process is driven by short distance phenomena ($d \ll 1 \text{ fm}$)
 $\implies \alpha_s \ll 1$: **Perturbative methods**
- One should hit strongly enough a hadron
Example: electromagnetic probe and form factor



τ electromagnetic interaction $\sim \tau$ parton life time after interaction
 $\ll \tau$ characteristic time of strong interaction

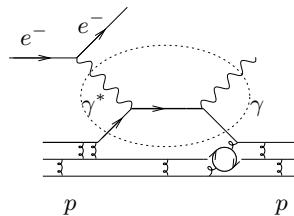
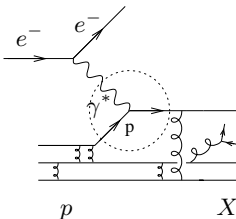
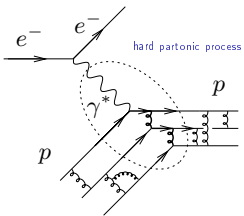
To get such situations in exclusive reactions is very challenging phenomenologically: **the cross sections are very small**

Introduction

Hard processes in QCD

Hard processes in QCD

- This is justified if the process is governed by a **hard scale**:
 - **virtuality of the electromagnetic probe**
 - in elastic scattering $e^\pm p \rightarrow e^\pm p$
 - in Deep Inelastic Scattering (DIS) $e^\pm p \rightarrow e^\pm X$
 - in Deep Virtual Compton Scattering (DVCS) $e^\pm p \rightarrow e^\pm p \gamma$
 - **Total center of mass energy** in $e^+e^- \rightarrow X$ annihilation
 - **t -channel momentum exchange** in meson photoproduction $\gamma p \rightarrow Mp$
- A precise treatment relies on **factorization theorems**
- The scattering amplitude is described by the **convolution** of the partonic amplitude with the non-perturbative hadronic content



Introduction

Counting rules and limitations

The partonic point of view... and its limitations

- Counting rules:

$$F_n(q^2) \simeq \frac{C}{(Q^2)^{n-1}} \quad n = \text{number of minimal constituents: } \begin{cases} \text{meson: } n = 2 \\ \text{baryon: } n = 3 \end{cases}$$

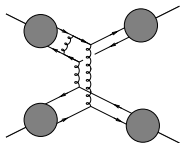
Brodsky, Farrar '73

- Large angle (i.e. $s \sim t \sim u$ large) elastic processes $h_a h_b \rightarrow h_a h_b$
e.g. : $\pi\pi \rightarrow \pi\pi$ or $pp \rightarrow pp$

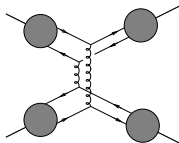
$$\frac{d\sigma}{dt} \sim \left(\frac{\alpha_S(p_\perp^2)}{s} \right)^{n-2} \quad n = \# \text{ of external fermionic lines } (n = 8 \text{ for } \pi\pi \rightarrow \pi\pi)$$

Brodsky, Lepage '81

Other contributions might be significant, even at large angle: e.g. $\pi\pi \rightarrow \pi\pi$



Brodsky Lepage mechanism: $\frac{d\sigma_{BL}}{dt} \sim \left(\frac{1}{s} \right)^6$



Landshoff '74 mechanism: $\frac{d\sigma_L}{dt} \sim \left(\frac{1}{s} \right)^5$

absent with at least one $\gamma^{(*)}$ (point-like coupling) 6 / 75

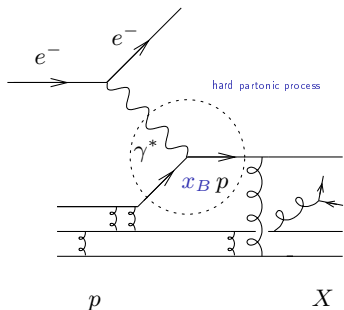
Introduction

DIS

Accessing the perturbative proton content using inclusive processes

no $1/Q$ suppression

example: DIS



$$s_{\gamma^* p} = (q_\gamma^* + p_p)^2 = 4 E_{\text{c.m.}}^2$$

$$Q^2 \equiv -q_{\gamma^*}^2 > 0$$

$$x_B = \frac{Q^2}{2 p_p \cdot q_\gamma^*} \simeq \frac{Q^2}{s_{\gamma^* p}}$$

- x_B = proton momentum fraction carried by the scattered quark
- $1/Q$ = transverse resolution of the photonic probe $\ll 1/\Lambda_{\text{QCD}}$

From inclusive to exclusive processes

Experimental effort

- Inclusive processes are not $1/Q$ suppressed (e.g. DIS);
Exclusive processes **are suppressed**
- Going from inclusive to exclusive processes is **difficult**
- High luminosity accelerators and high-performance detection facilities
HERA (H1, ZEUS), HERMES, JLab@6 GeV (Hall A, CLAS), BaBar, Belle, BEPC-II
(BES-III), LHC future: COMPASS-II, JLab@12 GeV, PANDA, LHeC, EIC, ILC
- What to do, and where?
 - Proton form factor: JLab@6 GeV future: PANDA (timelike proton form factor through $p\bar{p} \rightarrow e^+e^-$)
 - e^+e^- in $\gamma^*\gamma$ single-tagged channel: Transition form factor $\gamma^*\gamma \rightarrow \pi$, exotic hybrid meson production BaBar, Belle, BES,...
 - Deep Virtual Compton Scattering (GPD)
HERA (H1, ZEUS), HERMES, JLab@6 GeV
future: JLab@12GeV, COMPASS-II, EIC, LHeC
 - Non exotic and exotic hybrid meson electroproduction (GPD and DA), etc...
NMC (CERN), E665 (Fermilab), HERA (H1, ZEUS), COMPASS, HERMES, CLAS (JLab)
 - TDA (PANDA at GSI)
 - TMDs (BaBar, Belle, COMPASS, ...)
 - Diffractive processes, including ultraperipheral collisions
LHC (with or without fixed targets), ILC, LHeC

From inclusive to exclusive processes

Theoretical efforts

Very important theoretical developments during the last decade

- Key words:

DAs, GPDs, GDAs, TDAs ... TMDs

- Fundamental tools:

- At medium energies:

JLab, HERMES, COMPASS, BaBar, Belle, PANDA, EIC

collinear factorization

- At asymptotical energies:

HERA, Tevatron, LHC, LHeC, ILC (EIC and COMPASS at the boundary)

k_T -factorization

We will now explain and illustrate these concepts, and discuss issues and possible solutions...

The ultimate picture

6D

Wigner distributions
for hadrons

$$W(x, \vec{b}, k_T)$$

Experimentally
inaccessibleperturbative Regge
limit

uPDFs (gluons)

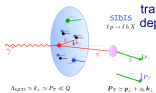
Unintegrated parton
distributions

3D

Semi-inclusive
processes

TMDs

$$f(x, k_T)$$

transverse momentum
dependent distribution $A_{\text{geom}} \sim k_z \sim p_T \ll Q$
 $P_T \approx p_{1+} + n_{1+} k_{1-}$

$$\int d^3 \vec{b}$$

$$\int d^2 k_T \int d b_z$$

$$b_T \leftrightarrow \Delta$$

$$f(x, b_T)$$

impact parameter
distributions

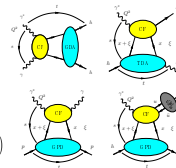
$$\int d^2 k_T \int \text{Fourier}(\vec{b})$$

$$\xi=0$$

GPDs

$$H(x, 0, t)$$

$$t = -\Delta^2$$

generalised parton
distributionsexclusive
processes

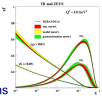
1D

inclusive and semi-
inclusive processes

PDFs

$$f(x)$$

parton distributions



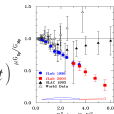
elastic processes



FFs

$$G_{E,M}(t)$$

form factors



GFFs

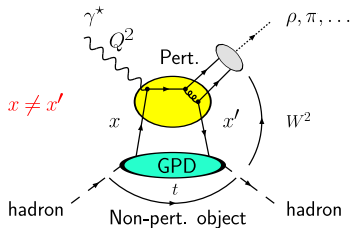
generalised form factors

lattices

Extensions from DVCS

- **Meson production:** γ replaced by ρ, π, \dots

$$\text{Amplitude} = \text{GPD (soft)} \otimes \text{CF (hard)} \otimes \text{Distribution Amplitude (soft)}$$

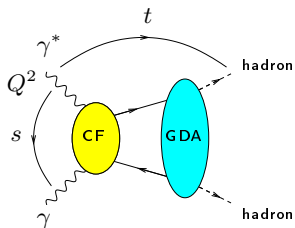


Collins, Frankfurt, Strikman '97; Radyushkin '97

proofs valid only for some restricted cases [backup]

- **Crossed process:** $s \ll -t$

$$\text{Amplitude} = \text{Coefficient Function (hard)} \otimes \text{Generalized Distribution Amplitude (soft)}$$



Diehl, Gousset, Pire, Teryaev '98

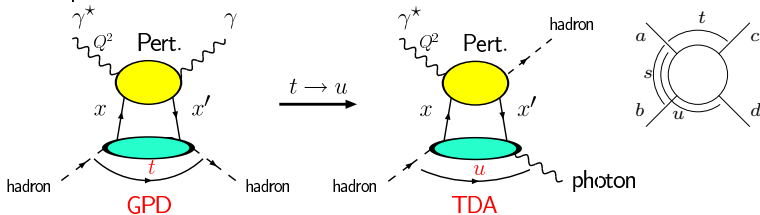
Extensions from DVCS

- Starting from usual DVCS, one allows: initial hadron \neq final hadron (in the same octuplet): transition GPDs

Even less diagonal:

baryonic number (initial state) \neq baryonic number (final state) \rightarrow TDA

Example:



Pire, Szymanowski '05

which can be further extended by replacing the outgoing γ by any hadronic state

$$\text{Amplitude} = \text{Transition Distribution Amplitude (soft)} \otimes \text{CF (hard)} \otimes \text{DA (soft)}$$

Lansberg, Pire, Szymanowski '06

Collinear factorization

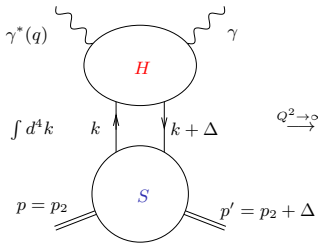
A bit more technical: DVCS and GPDs

The two steps for factorization, in a nutshell

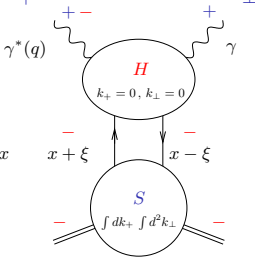
- momentum factorization: **light-cone vector dominance for $Q^2 \rightarrow \infty$**

$$p_1, p_2 : \text{the two light-cone directions} \quad \begin{cases} p_1 = \frac{\sqrt{s}}{2}(1, 0_\perp, 1) & p_1^+ = p_2^+ = 0 \\ p_2 = \frac{\sqrt{s}}{2}(1, 0_\perp, -1) & 2 p_1 \cdot p_2 = s \sim s_{\gamma^* p} \gtrsim Q^2 \end{cases}$$

$$\text{Sudakov decomposition: } k = \alpha p_1 + \beta p_2 + k_\perp$$



$$Q^2 \rightarrow \infty \quad \int d^4k \rightarrow \int dk^- \int dx$$



key point:
large (+) × (-) flux

⇒ short distance

(masses neglected)

$$\int d^4k S(k, k + \Delta) H(q, k, k + \Delta) = \int dk^- \int dk^+ d^2k_\perp S(k, k + \Delta) H(q, k^-, k^- + \Delta^-)$$

- Quantum numbers factorization (Fierz identity: spinors + color)

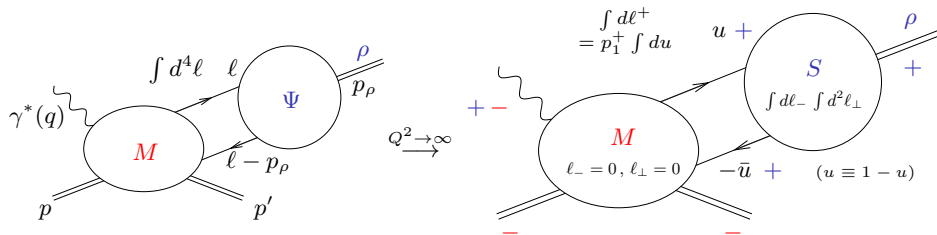
$$\Rightarrow \mathcal{M} = \text{GPD} \otimes \text{Hard part}$$

Collinear factorization

ρ -meson production: from the wave function to the DA

What is a ρ -meson in QCD?

It is described by its wave function Ψ which reduces in hard processes to its Distribution Amplitude



$$\int d^4l M(q, l, l - p_\rho) \Psi(l, l - p_\rho) = \int dl^+ M(q, l^+, l^+ - p_\rho^+) \int dl^- \int_{|\ell_\perp^2| < \mu_F^2} d^2l_\perp \Psi(l, l - p_\rho)$$

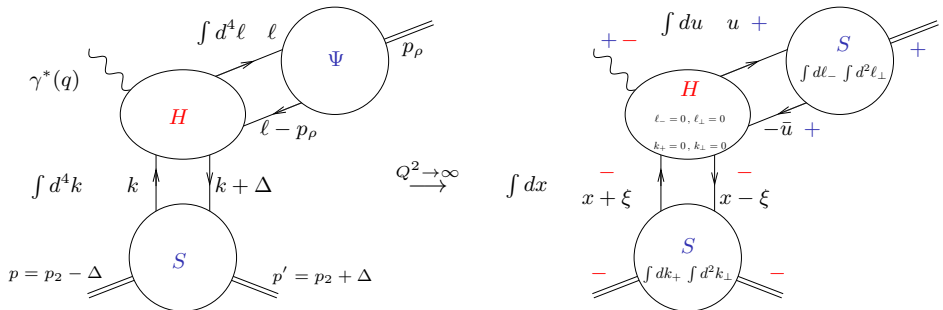
Hard part

DA $\Phi(u, \mu_F^2)$

(see Chernyak, Zhitnitsky '77; Brodsky, Lepage '79; Efremov, Radyushkin '80; ... in the case of form-factors studies)

Collinear factorization

Meson electroproduction: factorization with a GPD and a DA



$$\int d^4k d^4\ell$$

$$S(k, k + \Delta)$$

$$H(q, k, k + \Delta)$$

$$\Psi(\ell, \ell - p_\rho)$$

$$= \int dk^- d\ell^+ \int dk^+ \int_{|k_\perp^2| < \mu_{F_2}^2} d^2k_\perp S(k, k + \Delta) H(q; k^-, k^- + \Delta^-; \ell^+, \ell^+ - p_\rho^+) \int d\ell^- \int_{|\ell_\perp^2| < \mu_{F_1}^2} d^2\ell_\perp \Psi(\ell, \ell - p_\rho)$$

$$\text{GPD } F(x, \xi, t, \mu_{F_2}^2)$$

$$\text{Hard part } T(x/\xi, u, \mu_{F_1}^2, \mu_{F_2}^2, \mu_R^2)$$

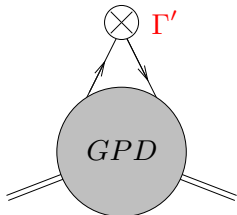
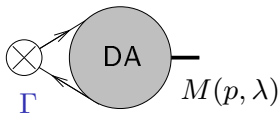
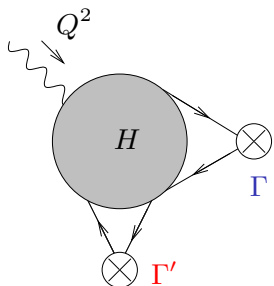
$$\text{DA } \Phi(u, \mu_{F_1}^2)$$

Collins, Frankfurt, Strikman '97; Radyushkin '97

Collinear factorization

Meson electroproduction: factorization with a GPD and a DA

The building blocks



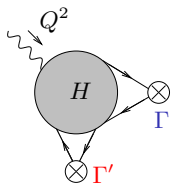
Γ, Γ' : Dirac matrices compatible
with quantum numbers: C, P, T , chirality

Similar structure for gluon exchange

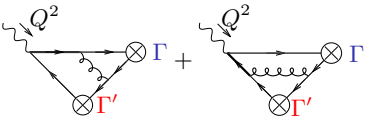
Collinear factorization

Meson electroproduction: factorization with a GPD and a DA

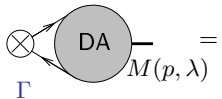
The building blocks



=



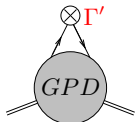
hand-bag diagrams



=

$$\langle M(p, \lambda) | \mathcal{O}(\Psi, \bar{\Psi} A) | 0 \rangle$$

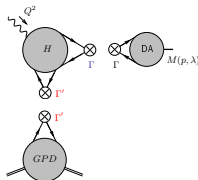
matrix element of a **non-local light-cone**
operator



=

$$\langle N(p') | \mathcal{O}'(\Psi, \bar{\Psi} A) | N(p) \rangle$$

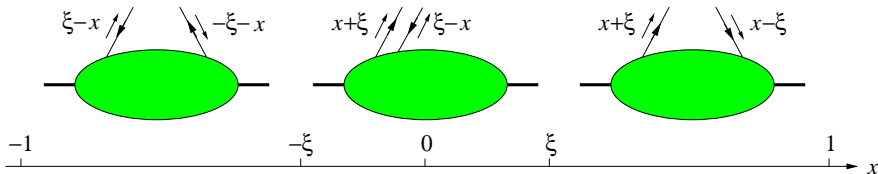
matrix element of a **non-local light-cone**
operator



Collinear factorization

Twist 2 GPDs

Physical interpretation for GPDs



Emission and reabsorption
of an antiquark
 \sim PDFs for antiquarks
DGLAP-II region

Emission of a quark and
emission of an antiquark
 \sim meson exchange
ERBL region

Emission and reabsorption
of a quark
 \sim PDFs for quarks
DGLAP-I region

Collinear factorization

Twist 2 GPDs

Classification of twist 2 GPDs

- For quarks, one should distinguish the exchanges

- without helicity flip (chiral-even Γ' matrices): 4 chiral-even GPDs:

$$H^q \xrightarrow{\xi=0, t=0} \text{PDF } q, E^q, \tilde{H}^q \xrightarrow{\xi=0, t=0} \text{polarized PDFs } \Delta q, \tilde{E}^q$$

$$\begin{aligned} F^q &= \frac{1}{2} \int \frac{dz^+}{2\pi} e^{ixP^-z^+} \langle p' | \bar{q}(-\frac{1}{2}z) \gamma^- q(\frac{1}{2}z) | p \rangle \Big|_{z^-=0, z_\perp=0} \\ &= \frac{1}{2P^-} \left[H^q(x, \xi, t) \bar{u}(p') \gamma^- u(p) + E^q(x, \xi, t) \bar{u}(p') \frac{i\sigma^{-\alpha} \Delta_\alpha}{2m} u(p) \right], \end{aligned}$$

$$\begin{aligned} \tilde{F}^q &= \frac{1}{2} \int \frac{dz^+}{2\pi} e^{ixP^-z^+} \langle p' | \bar{q}(-\frac{1}{2}z) \gamma^- \gamma_5 q(\frac{1}{2}z) | p \rangle \Big|_{z^-=0, z_\perp=0} \\ &= \frac{1}{2P^-} \left[\tilde{H}^q(x, \xi, t) \bar{u}(p') \gamma^- \gamma_5 u(p) + \tilde{E}^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^-}{2m} u(p) \right]. \end{aligned}$$

- with helicity flip (chiral-odd Γ' mat.): 4 chiral-odd GPDs:

$$H_T^q \xrightarrow{\xi=0, t=0} \text{quark transversity PDFs } \Delta_T q, E_T^q, \tilde{H}_T^q, \tilde{E}_T^q$$

$$\begin{aligned} &\frac{1}{2} \int \frac{dz^+}{2\pi} e^{ixP^-z^+} \langle p' | \bar{q}(-\frac{1}{2}z) i\sigma^{-i} q(\frac{1}{2}z) | p \rangle \Big|_{z^-=0, z_\perp=0} \\ &= \frac{1}{2P^-} \bar{u}(p') \left[H_T^q i\sigma^{-i} + \tilde{H}_T^q \frac{P^- \Delta^i - \Delta^- P^i}{m^2} + E_T^q \frac{\gamma^- \Delta^i - \Delta^- \gamma^i}{2m} + \tilde{E}_T^q \frac{\gamma^- P^i - P^- \gamma^i}{m} \right] \end{aligned}$$

Collinear factorization

Twist 2 GPDs

Classification of twist 2 GPDs

- analogously, for gluons:
 - 4 gluonic GPDs without helicity flip:

$$H^g \xrightarrow{\xi=0, t=0} \text{PDF } x g$$

$$E^g$$

$$\tilde{H}^g \xrightarrow{\xi=0, t=0} \text{polarized PDF } x \Delta g$$

$$\tilde{E}^g$$

- 4 gluonic GPDs with helicity flip:

$$H_T^g$$

$$E_T^g$$

$$\tilde{H}_T^g$$

$$\tilde{E}_T^g$$

(no forward limit reducing to gluons PDFs here: a change of 2 units of helicity cannot be compensated by a spin 1/2 target)

A few applications

Production of an exotic hybrid meson in hard processes

Quark model and meson spectroscopy

- spectroscopy: $\vec{J} = \vec{L} + \vec{S}$; neglecting any spin-orbital interaction
 $\Rightarrow S, L =$ additional quantum numbers to classify hadron states

$$\vec{J}^2 = J(J+1), \quad \vec{S}^2 = S(S+1), \quad \vec{L}^2 = L(L+1),$$

with $J = |L - S|, \dots, L + S$

- In the usual quark-model: meson = $q\bar{q}$ bound state with

$$C = (-)^{L+S} \quad \text{and} \quad P = (-)^{L+1}.$$

- Thus:

$$S = 0, \quad L = J, \quad J = 0, 1, 2, \dots : \quad J^{PC} = 0^{-+}(\pi, \eta), 1^{+-}(h_1, b_1), 2^{-+}, 3^{+-}, \dots$$

$$S = 1, \quad L = 0, \quad J = 1 : \quad J^{PC} = 1^{--}(\rho, \omega, \phi)$$

$$L = 1, \quad J = 0, 1, 2 : \quad J^{PC} = 0^{++}(f_0, a_0), 1^{++}(f_1, a_1), 2^{++}(f_2, a_2)$$

$$L = 2, \quad J = 1, 2, 3 : \quad J^{PC} = 1^{--}, 2^{--}, 3^{--}$$

...

- \Rightarrow the exotic mesons with $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, \dots$ are forbidden

A few applications

Production of an exotic hybrid meson in hard processes

Experimental candidates for light hybrid mesons (1)

three candidates:

- $\pi_1(1400)$
 - GAMS '88 (SPS, CERN): in $\pi^- p \rightarrow \eta \pi^0 n$ (through $\eta \pi^0 \rightarrow 4\gamma$ mode)
 $M = 1406 \pm 20 \text{ MeV}$ $\Gamma = 180 \pm 30 \text{ MeV}$
 - E852 '97 (BNL): $\pi^- p \rightarrow \eta \pi^- p$
 $M = 1370 \pm 16 \text{ MeV}$ $\Gamma = 385 \pm 40 \text{ MeV}$
 - VES '01 (Protvino) in $\pi^- Be \rightarrow \eta \pi^- Be$, $\pi^- Be \rightarrow \eta' \pi^- Be$,
 $\pi^- Be \rightarrow b_1 \pi^- Be$
 $M = 1316 \pm 12 \text{ MeV}$ $\Gamma = 287 \pm 25 \text{ MeV}$
 but resonance hypothesis ambiguous
 - Crystal Barrel (LEAR, CERN) '98 '99 in $\bar{p}n \rightarrow \pi^- \pi^0 \eta$ and $\bar{p}p \rightarrow 2\pi^0 \eta$
 (through $\pi\eta$ resonance)
 $M = 1400 \pm 20 \text{ MeV}$ $\Gamma = 310 \pm 50 \text{ MeV}$
 and $M = 1360 \pm 25 \text{ MeV}$ $\Gamma = 220 \pm 90 \text{ MeV}$

A few applications

Production of an exotic hybrid meson in hard processes

Experimental candidates for light hybrid mesons (2)

- $\pi_1(1600)$
 - **E852 (BNL)**: in peripheral $\pi^- p \rightarrow \pi^+ \pi^- \pi^- p$ (through $\rho\pi^-$ mode) '98 '02, $M = 1593 \pm 8$ MeV $\Gamma = 168 \pm 20$ MeV $\pi^- p \rightarrow \pi^+ \pi^- \pi^- \pi^0 \pi^0 p$ (in $b_1(1235)\pi^- \rightarrow (\omega\pi^0)\pi^- \rightarrow (\pi^+ \pi^- \pi^0)\pi^0 \pi^-$ '05 and $f_1(1285)\pi^-$ '04 modes), in peripheral $\pi^- p$ through $\eta'\pi^-$ '01
 $M = 1597 \pm 10$ MeV $\Gamma = 340 \pm 40$ MeV
 but **E852 (BNL)** '06: no exotic signal in $\pi^- p \rightarrow (3\pi)^- p$ for a larger sample of data!
 - **VES '00 (Protvino)**: in peripheral $\pi^- p$ through $\eta'\pi^-$ '93, '00, $\rho(\pi^+ \pi^-)\pi^-$ '00, $b_1(1235)\pi^- \rightarrow (\omega\pi^0)\pi^-$ '00
 - **Crystal Barrel (LEAR, CERN)** '03 $\bar{p}p \rightarrow b_1(1235)\pi\pi$
 - **COMPASS '10 (SPS, CERN)**: diffractive dissociation of π^- on Pb target through Primakov effect $\pi^- \gamma \rightarrow \pi^- \pi^- \pi^+$ (through $\rho\pi^-$ mode)
 $M = 1660 \pm 10$ MeV $\Gamma = 269 \pm 21$ MeV
- $\pi_1(2000)$: seen only at **E852 (BNL)** '04 '05 (through $f_1(1285)\pi^-$ and $b_1(1235)\pi^-$)

A few applications

Production of an exotic hybrid meson in hard processes

What about hard processes?

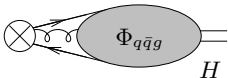
- Is there a hope to see such states in hard processes, with high counting rates, and to exhibit their light-cone wave-function?
- hybrid mesons = $q\bar{q}g$ states T. Barnes '77; R. L. Jaffe, K. Johnson, and Z. Ryzak, G. S. Bali
- popular belief: $H = q\bar{q}g \Rightarrow$ higher Fock-state component \Rightarrow twist-3 \Rightarrow hard electroproduction of H versus ρ suppressed as $1/Q$
- This is not true!! Electroproduction of hybrid is similar to electroproduction of usual ρ -meson: it is twist 2 dominated
I. V. Anikin, B. Pire, O. V. Teryaev, L. Szymanowski, S.W. '04

A few applications

Production of an exotic hybrid meson in hard processes

Distribution amplitude of exotic hybrid mesons at twist 2

- One may think that to produce $|q\bar{q}g\rangle$, the fields Ψ , $\bar{\Psi}$, A should appear explicitly in the non-local operator $\mathcal{O}(\Psi, \bar{\Psi} A)$



- If one tries to produce $H = 1^{-+}$ from a local operator, the dominant operator should be $\bar{\Psi}\gamma^\mu G_{\mu\nu}\Psi$ of twist = dimension - spin = 5 - 1 = 4
- It means that there should be a $1/Q^2$ suppression in the production amplitude of H versus the usual ρ -production (which is twist 2 dominated)
- But collinear approach describes hard exclusive processes in terms of non-local light-cone operators, among which are the twist 2 operator

$$\bar{\psi}(-z/2)\gamma_\mu[-z/2; z/2]\psi(z/2)$$

where $[-z/2; z/2]$ is a Wilson line, necessary to fulfill gauge invariance (i.e. a "color tube" between q and \bar{q}) which thus hides gluonic degrees of freedom: the needed gluon is there, at twist 2.

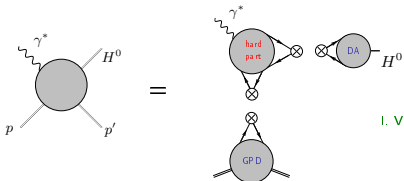
This does not require to introduce explicitly A !

A few applications

Production of an exotic hybrid meson in hard processes

Accessing the partonic structure of exotic hybrid mesons

- Electroproduction $\gamma^* p \rightarrow H^0 p$: JLab, COMPASS, EIC



$$\text{prediction: } \frac{d\sigma^H}{d\sigma^{\rho}} \approx 15\%$$

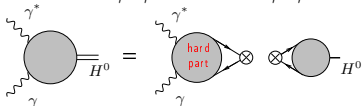
I. V. Anikin, B. Pire, O. V. Teryaev, L. Szymanowski, S.W.

Phys.Rev.D70 (2004) 011501

Phys.Rev.D71 (2005) 034021

Eur.Phys.J.C42 (2005) 163

- Channels $\gamma^* \gamma \rightarrow H$ and $\gamma^* \gamma \rightarrow \pi \eta$: BaBar, Belle, BES-III



$$\text{prediction: } \frac{|M^{\gamma^* \gamma \rightarrow H}|^2}{|M^{\gamma^* \gamma \rightarrow \pi^0}|^2} \approx 20\%$$

I. V. Anikin, B. Pire, O. V. Teryaev, L. Szymanowski, S.W.

Eur.Phys.J.C47 (2006)

[backup]

⇒ the partonic content of exotic hybrid meson is experimentally accessible

This is very complementary to spectroscopy studies, e.g. GLUEx (JLab@12Gev, Hall D) devoted to hybrid meson studies (with a photon source based on a diamond crystal)

A few applications

Spin transversity in the nucleon

What is transversity?

- Transverse spin content of the proton:

$$\begin{aligned} |\uparrow\rangle_{(x)} &\sim |\rightarrow\rangle + |\leftarrow\rangle \\ |\downarrow\rangle_{(x)} &\sim |\rightarrow\rangle - |\leftarrow\rangle \\ \text{spin along } x &\qquad \qquad \text{helicity state} \end{aligned}$$

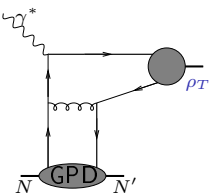
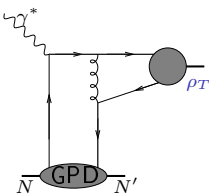
- An observable sensitive to helicity spin flip gives thus access to the transversity $\Delta_T q(x)$, which is very badly known (first data have recently been obtained by COMPASS)
- The transversity GPDs are completely unknown
- **Chirality:** $q_{\pm}(z) \equiv \frac{1}{2}(1 \pm \gamma^5)q(z)$ with $q(z) = q_+(z) + q_-(z)$
 Chiral-even: **chirality conserving**
 $\bar{q}_{\pm}(z)\gamma^{\mu}q_{\pm}(-z)$ and $\bar{q}_{\pm}(z)\gamma^{\mu}\gamma^5q_{\pm}(-z)$
 Chiral-odd: **chirality reversing**
 $\bar{q}_{\pm}(z) \cdot 1 \cdot q_{\mp}(-z)$, $\bar{q}_{\pm}(z) \cdot \gamma^5 \cdot q_{\mp}(-z)$ and $\bar{q}_{\pm}(z)[\gamma^{\mu}, \gamma^{\nu}]q_{\mp}(-z)$
- For a massless (anti)particle, chirality = (-)helicity
- **Transversity is thus a chiral-odd quantity**
- QCD and QED are chiral even $\Rightarrow \mathcal{A} \sim (\text{Ch.-odd})_1 \otimes (\text{Ch.-odd})_2$

A few applications

Spin transversity in the nucleon

How to get access to transversity?

- The dominant DA for ρ_T is of twist 2 and chiral-odd ($[\gamma^\mu, \gamma^\nu]$ coupling)
- Unfortunately $\gamma^* N^\uparrow \rightarrow \rho_T N' = 0$
 - this is true at any order in perturbation theory (i.e. corrections as powers of α_s), since this would require a transfer of 2 units of helicity from the proton: impossible!
Diehl, Gousset, Pire '99; Collins, Diehl '00
 - diagrammatic argument at Born order:



vanishes: $\gamma^\alpha [\gamma^\mu, \gamma^\nu] \gamma_\alpha = 0$

A few applications

Spin transversity in the nucleon

Can one circumvent this vanishing?

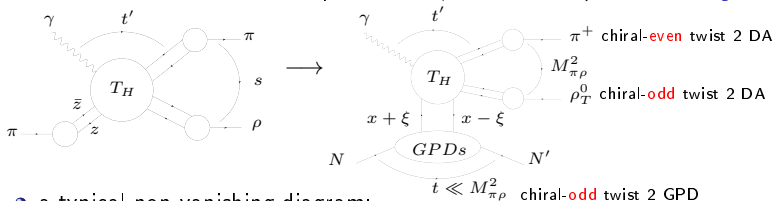
- This vanishing is true only a twist 2
- At twist 3 this process does not vanish
- However processes involving twist 3 DAs may face problems with factorization
(end-point singularities: see [back-up])
- Classification of twist 3 chiral-odd GPDs:
see later based on our **Light-Cone Collinear Factorization** framework
recently developed
(Pire, Szymanowski, S. W.)

A few applications

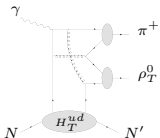
Spin transversity in the nucleon

$\gamma N \rightarrow \pi^+ \rho_T^0 N'$ gives access to transversity

- Factorization à la Brodsky Lepage of $\gamma + \pi \rightarrow \pi + \rho$ at large s and fixed angle (i.e. fixed ratio $t'/s, u'/s$)
 \implies factorization of the amplitude for $\gamma + N \rightarrow \pi + \rho + N'$ at large $M_{\pi\rho}^2$



- a typical non-vanishing diagram:



M. El Beiyad, P. Pire, M. Segond, L. Szymanowski, S.W. Phys.Lett. B688:154-167,2010

see also, at large s , with Pomeron exchange:

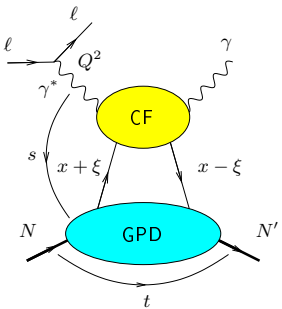
R. Ivanov, B. Pire, L. Szymanowski, O. Teryaev '02

R. Enberg, B. Pire, L. Szymanowski '06

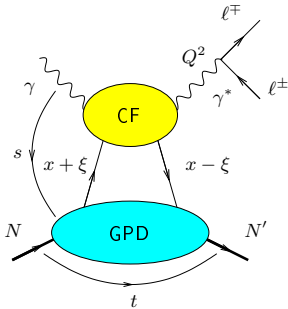
- These processes with 3 body final state can give access to all GPDs: $M_{\pi\rho}^2$ plays the role of the γ^* virtuality of usual DVCS (here in the time-like domain) JLab, COMPASS

Threshold effects for DVCS and TCS

DVCS and TCS



Deeply Virtual Compton Scattering
 $lN \rightarrow l'N'\gamma$



Timelike Compton Scattering
 $\gamma N \rightarrow l^+l^-N'$

- TCS versus DVCS:

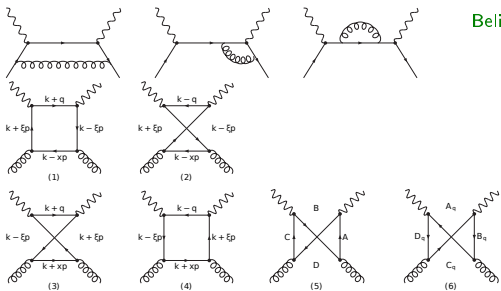
- **universality of the GPDs**
- another source for GPDs (special sensitivity on real part)
- spacelike-timelike crossing and understanding the structure of the NLO corrections

- Where to measure TCS? In **Ultra Peripheral Collisions**
LHC, JLab, COMPASS, AFTER

Threshold effects for DVCS and TCS

DVCS and TCS at NLO

One loop contributions to the coefficient function



Belitsky, Mueller, Niedermeier, Schafer,
 Phys.Lett.B474, 2000
 Pire, Szymanowski, Wagner
 Phys.Rev.D83, 2011

$$\mathcal{A}^{\mu\nu} = g_T^{\mu\nu} \int_{-1}^1 dx \left[\sum_q^{n_F} T^q(x) F^q(x) + T^g(x) F^g(x) \right]$$

(symmetric part of the factorised amplitude)

Threshold effects for DVCS and TCS

Resummations effects are expected

- The renormalized quark **coefficient functions** T^q is

$$T^q = C_0^q + C_1^q + C_{coll}^q \log \frac{|Q^2|}{\mu_F^2}$$

$$C_0^q = e_q^2 \left(\frac{1}{x - \xi + i\epsilon} - (x \rightarrow -x) \right)$$

$$C_1^q = \frac{e_q^2 \alpha_S C_F}{4\pi(x - \xi + i\epsilon)} \left[\log^2 \left(\frac{\xi - x}{2\xi} - i\epsilon \right) + \dots \right] - (x \rightarrow -x)$$

- Usual collinear approach: single-scale analysis w.r.t. Q^2

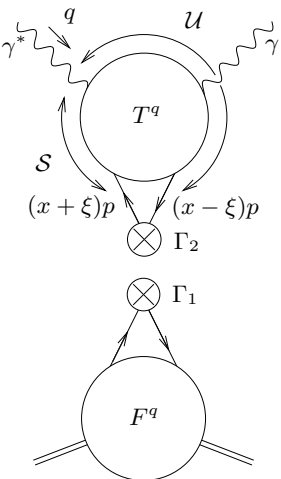
- Consider the invariants S and U :

$$S = \frac{x - \xi}{2\xi} Q^2 \ll Q^2 \quad \text{when } x \rightarrow \xi$$

$$U = -\frac{x + \xi}{2\xi} Q^2 \ll Q^2 \quad \text{when } x \rightarrow -\xi$$

⇒ **two scales problem; threshold singularities to be resummed**

analogous to the $\log(x - x_{Bj})$ resummation for DIS coefficient functions

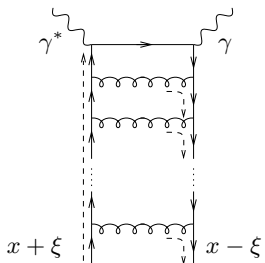


Threshold effects for DVCS and TCS

Resummation for Coefficient functions

Soft-collinear resummation effects for the coefficient function

- The resummation easier when using the axial gauge $p_1 \cdot A = 0$ ($p_\gamma \equiv p_1$)
- The dominant diagram are **ladder-like** [backup]



resummed formula (for DVCS), for $x \rightarrow \xi$:

$$(T^q)^{\text{res}} = \left(\frac{e_q^2}{x - \xi + i\epsilon} \left\{ \cosh \left[D \log \left(\frac{\xi - x}{2\xi} - i\epsilon \right) \right] - \frac{D^2}{2} \left[9 + 3 \frac{\xi - x}{x + \xi} \log \left(\frac{\xi - x}{2\xi} - i\epsilon \right) \right] \right\} + C_{\text{coll}}^q \log \frac{Q^2}{\mu_F^2} \right) - (x \rightarrow -x) \quad \text{with} \quad D = \sqrt{\frac{\alpha_s C_F}{2\pi}}$$

T. Altinoluk, B. Pire, L. Szymanowski, S. W.
JHEP 1210 (2012) 49; [arXiv:1206.3115]

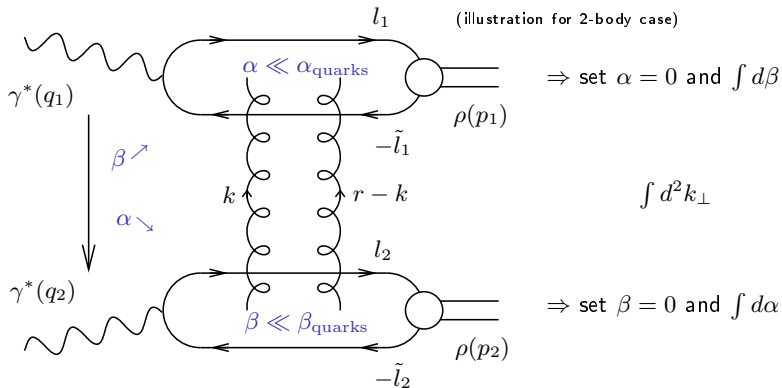
- Our analysis can be used for **the gluon coefficient function** [In progress].
- The measurement of the phenomenological impact of this procedure on the data analysis needs further analysis with the implementation of modeled generalized parton distributions [backup].

QCD at large s

k_T factorization

$\gamma^* \gamma^* \rightarrow \rho \rho$ as an example

- Use Sudakov decomposition $k = \alpha p_1 + \beta p_2 + k_\perp$ ($p_1^2 = p_2^2 = 0$, $2p_1 \cdot p_2 = s$)
- write
$$d^4k = \frac{s}{2} d\alpha d\beta d^2k_\perp$$
- t -channel gluons with non-sense polarizations ($\epsilon_{NS}^{up} = \frac{2}{s} p_2$, $\epsilon_{NS}^{down} = \frac{2}{s} p_1$) dominate at large s



QCD at large s

Phenomenological applications: Meson production at HERA

Diffractive meson production at HERA

HERA (DESY, Hamburg): first and single $e^\pm p$ collider (1992-2007)

- The "easy" case (from factorization point of view): J/Ψ production ($u \sim 1/2$: non-relativistic limit for bound state) combined with k_T -factorisation
Ryskin '93; Frankfurt, Koepf, Strikman '98; Ivanov, Kirschner, Schäfer, Szymanowski '00; Motyka, Enberg, Poludniowski '02
- Exclusive vector meson photoproduction at large t (= hard scale):
 $\gamma(q) + P \rightarrow \rho_{L,T}(p_1) + P$
based on k_T -factorization:
Forshaw, Ryskin '95; Bartels, Forshaw, Lotter, Wüsthoff '96; Forshaw, Motyka, Enberg, Poludniowski '03
 - H1, ZEUS data seems to favor BFKL
 - but end-point singularities for ρ_T are regularized with a quark mass:
 $m = m_\rho/2$
 - the spin density matrix is badly described
- Exclusive electroproduction of vector meson
 $\gamma_{L,T}^*(q) + P \rightarrow \rho_{L,T}(p_1) + P$ Goloskokov, Kroll '05
based on improved collinear factorization for the coupling with the meson
DA and collinear factorization for GPD coupling

QCD at large s

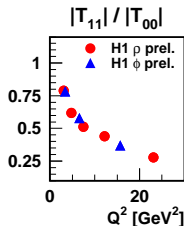
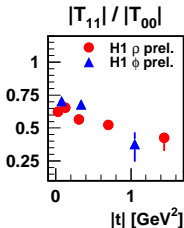
Phenomenological applications: Meson production at HERA

Polarization effects in $\gamma^* P \rightarrow \rho P$ at HERA

- Very precise experimental data on the spin density matrix (i.e. correlations between γ^* and ρ polarizations)
- for $t = t_{min}$ one can experimentally distinguish

$$\left\{ \begin{array}{l} \gamma_L^* \rightarrow \rho_L : \text{dominates ("twist 2": amplitude } |\mathcal{A}| \sim \frac{1}{Q}) \\ \gamma_T^* \rightarrow \rho_T : \text{visible ("twist 3": amplitude } |\mathcal{A}| \sim \frac{1}{Q^2}) \end{array} \right.$$

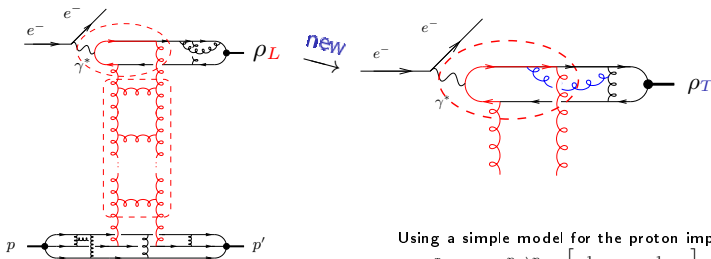
- How to calculate the $\gamma_T^* \rightarrow \rho_T$ transition from first principles?
- Can one avoid end-point singularities?



QCD at large s

Phenomenological applications: Meson production at HERA

Diffractive exclusive process $e^-p \rightarrow e^-p\rho_{L,T}$



first description combining beyond leading twist

- collinear factorisation
- k_T -factorisation

I. V. Anikin, D. Yu. Ivanov, B. Pire, L. Szymanowski, S.W.

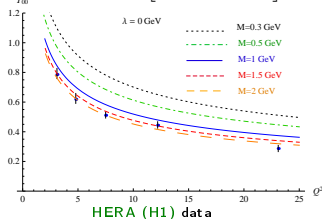
Phys.Lett. B682 (2010) 413-418

Nucl.Phys. B828 (2010) 1-68

HERA, EIC, LHeC, AFP@LHC

Using a simple model for the proton impact factor:

$$\Phi^{P \rightarrow p} \propto \left[\frac{1}{M^2} - \frac{1}{M^2 + k_T^2} \right]$$



I. V. Anikin, A. Besse, D. Yu. Ivanov, B. Pire,
L. Szymanowski, S.W.
Phys.Rev. D84 (2011) 054004

QCD at large s

Phenomenological applications: exclusive processes at Tevatron, RHIC, LHC, ILC

Exclusive $\gamma^{(*)}\gamma^{(*)}$ processes = gold place for testing QCD at large s

Proposals in order to test perturbative QCD in the large s limit

(t -structure of the hard Pomeron, saturation, Odderon...)

- $\gamma^{(*)}(q) + \gamma^{(*)}(q') \rightarrow J/\Psi J/\Psi$ Kwiecinski, Motyka '98
- $\gamma_{L,T}^*(q) + \gamma_{L,T}^*(q') \rightarrow \rho_L(p_1) + \rho_L(p_2)$ process in
 $e^+e^- \rightarrow e^+e^- \rho_L(p_1) + \rho_L(p_2)$ with double tagged lepton at ILC

Pire, Szymanowski, S. W. '04; Pire, Szymanowski, Enberg, S. W. '06; Ivanov, Papa '06;
 Segond, Szymanowski, S. W. '07

conclusion: feasible at ILC (high energy and high luminosity); BFKL NLL
 enhancement with respect to Born and DGLAP contributions [backup]

- What about the Odderon? C -parity of Odderon = -1
 consider $\gamma + \gamma \rightarrow \pi^+ \pi^- \pi^+ \pi^-$: $\pi^+ \pi^-$ pair has no fixed C -parity
 \Rightarrow Odderon and Pomeron can interfere
 \Rightarrow Odderon appears linearly in the charge asymmetry

Pire, Schwennsen, Szymanowski, S. W. '07

= example of possibilities offered by ultraperipheral exclusive processes at
 LHC [backup]

(p , \bar{p} or A as effective sources of photon)

but the distinction with pure QCD processes (with gluons instead of a photon) is tricky...

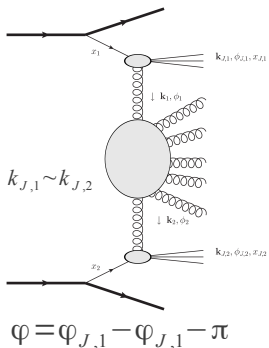
QCD at large s

Most recent signs of BFKL dynamics at LHC

Testing QCD in the perturbative Regge limit at LHC

Mueller-Navelet jets : the only observable for which a full NLO BFKL analysis is available

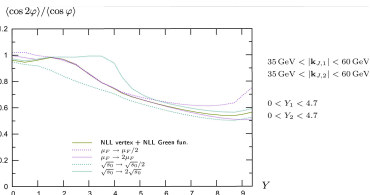
D. Colferai, F. Schwennsen, L. Szymanowski, S. W., JHEP 1012:026 (2010) 1-72 ;
B. Ducloué, L. Szymanowski, S. W., JHEP 1305 (2013) 096.



Surprisingly small decorrelation

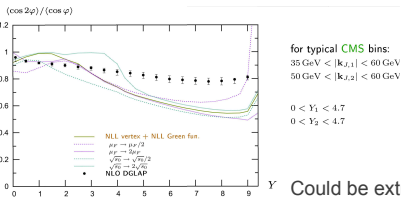
Predictions are stable with respect to

s_0, μ_F , PDFs, in the range $4.5 < Y < 8$



Symmetric configuration

See CMS data

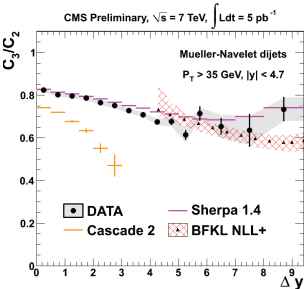
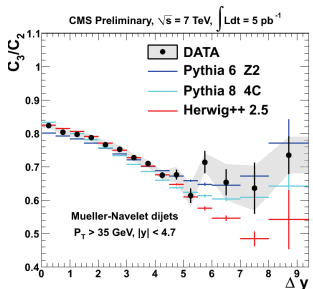
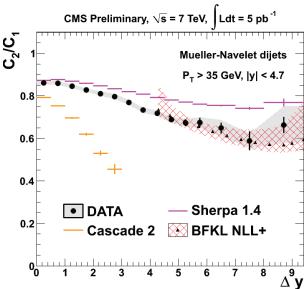
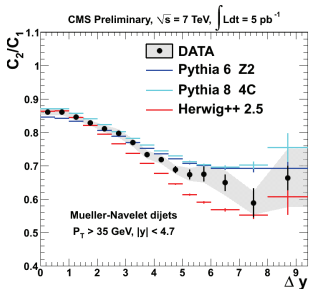


Asymmetric configuration

Could be extracted

QCD at large s

Most recent signs of BFKL dynamics at LHC

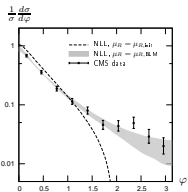
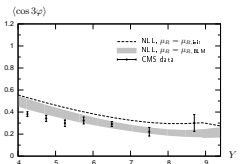
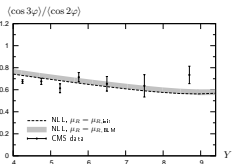
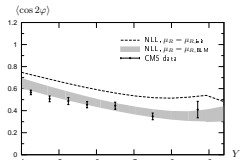
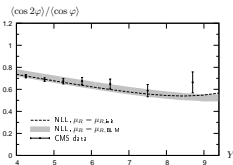
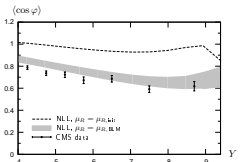


QCD at large s

Most recent signs of BFKL dynamics at LHC

With Brodsky-Lepage-Mackenzie renormalization scale fixing: no free-parameter!

B. Ducloué, L. Szymanowski, S. W. [arXiv:1309.3229]

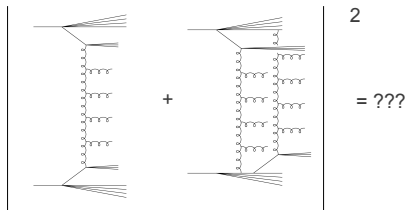


QCD at large s

MPI?

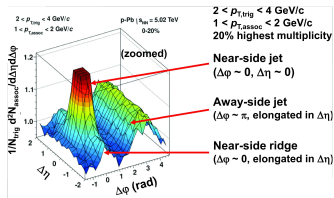
Testing QCD in the perturbative Regge limit at LHC

Mueller-Navelet jets : another mechanism ?



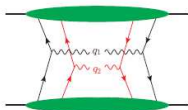
BFKL ladder

Color Glass Condensate ?
~ MPI at small x ?



Similar issues for the ridge effect in pp, pA

Multiparton interactions (MPI) : accessing to correlations between two partons inside a nucleon ?



Beyond leading twist : γ^* \rightarrow ρ impact factor up to twist 3

Light-Cone Collinear Factorization

3-body non-local correlators

genuine twist 3

- vector correlator

$$\langle \rho(p) | \bar{\psi}(z_1) \gamma_\mu g A_\alpha^T(z_2) \psi(0) | 0 \rangle \stackrel{\mathcal{F}_2}{\equiv} m_\rho f_3^V B(y_1, y_2) p_\mu e_{\alpha}^{*T},$$

- axial correlator

$$\langle \rho(p) | \bar{\psi}(z_1) \gamma_5 \gamma_\mu g A_\alpha^T(z_2) \psi(0) | 0 \rangle \stackrel{\mathcal{F}_2}{\equiv} m_\rho f_3^A i D(y_1, y_2) p_\mu \varepsilon_{\alpha\lambda\beta\delta} e_{\lambda}^{*T} p_{\beta} n_{\delta},$$

where $y_1, \bar{y}_2, y_2 - y_1 =$ quark, antiquark, gluon momentum fraction

$$\text{and } \stackrel{\mathcal{F}_2}{\equiv} \int_0^1 dy_1 \int_0^1 dy_2 \exp [i y_1 p \cdot z_1 + i (y_2 - y_1) p \cdot z_2], \text{ with } z_{1,2} = \lambda n$$

\Rightarrow 2 3-body DAs

Beyond leading twist : $\gamma^* \rightarrow \rho$ impact factor up to twist 3

Light-Cone Collinear Factorization

Minimal set of DAs

- Number of non-perturbative quantities: a priori 7 at twist 3
(5 2-parton DA and 2 2-parton DA)
- Non-perturbative correlators cannot be obtained perturbatively!
- One should reduce their number to a minimal set before any use of a model or any measure on the QCD lattice

- **independence w.r.t the choice of the vector n defining**

- the light-cone direction z : $z = \lambda n$
- the ρ_T polarization vector: $e_T \cdot n = 0$
- the axial gauge: $n \cdot A = 0$

$$\mathcal{A} = H \otimes S \quad \frac{d\mathcal{A}}{dn_{\perp}^{\mu}} = 0 \Rightarrow S \text{ are related}$$

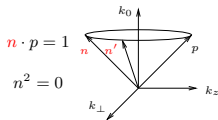
- We have proven that **3 independent Distribution Amplitudes** are necessary:

$$\begin{cases} \text{QCD equations of motion} & 2 \text{ equations} \\ \text{Arbitrariness in the choice of } n & 2 \text{ equations} \end{cases}$$

$\varphi_1(y)$ ← 2-body twist 2 correlator

$B(y_1, y_2)$ ← 3-body genuine twist 3 vector correlator

$D(y_1, y_2)$ ← 3-body genuine twist 3 axial correlator



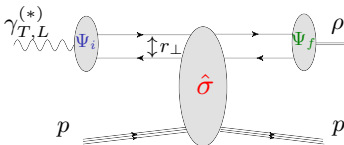
Beyond leading twist : $\gamma^* \rightarrow \rho$ impact factor up to twist 3

Dipole representation and saturation effects

The dipole picture at high energy

A key, inspiring and powerful paradigm for inclusive, diffractive, exclusive processes

in e-p, p-p, p-A, ...



Nikolaev, Zakharov '91

- Initial Ψ_i and final Ψ_f states wave functions of projectiles
- Primitive picture: proton = color dipole
scattering amplitude for two t -channel exchanged gluons:

$$\mathcal{N}(\underline{r}, \underline{k}) = \frac{4\pi\alpha_s}{N_c} \left(1 - e^{i\underline{k}\cdot\underline{r}}\right) \left(1 - e^{-i\underline{k}\cdot\underline{r}}\right)$$

- Real proton: $\mathcal{N} \rightarrow \hat{\sigma}_{\text{dipole-target}}$ = universal scattering amplitude
 - color transparency for small r_\perp : $\hat{\sigma}_{\text{dipole-target}} \sim r_\perp^2$
 - saturation for large $r_\perp \sim 1/Q_{\text{sat}}$: $T \lesssim 1$ Golec-Biernat Wusthoff '98
- Data for ρ production calls for models encoding saturation
Munier, Stasto, Mueller '04; Kowalski, Motyka, Watt '06
- The dipole representation is consistent with the twist 2 collinear factorization

Beyond leading twist : $\gamma^* \rightarrow \rho$ impact factor up to twist 3

Factorization in coordinate space: the 2-parton contribution

Light-Cone Collinear Factorization in the coordinate space

- Recall: impact factors $\Phi_{q\bar{q}}^{\gamma^* \rightarrow \rho} = -\frac{1}{4} \int d^4\ell \text{Tr}(H_{q\bar{q}}\Gamma)(\ell) S_{q\bar{q}}\Gamma(\ell)$
- Collinear approximation \Rightarrow expansion around $\ell_\perp = 0$:

$$\text{Tr}(H_{q\bar{q}}\Gamma)(\ell) = \int \frac{d^2 r_\perp}{2\pi} \tilde{H}_{q\bar{q}}^\Gamma(y, r_\perp) e^{-i\ell_\perp \cdot r_\perp} = \int \frac{d^2 r_\perp}{2\pi} \underbrace{\tilde{H}_{q\bar{q}}^\Gamma(y, r_\perp)}_{\text{factorizes out}} \overbrace{(1 - i\ell_\perp \cdot r_\perp + \dots)}^{\text{Gives the moments of } S_{q\bar{q}}\Gamma}$$

twist 2 and 3

- 2-parton impact factor **up to twist 3** (Wandzura-Wilczek (WW) approximation):

$$\Phi_{q\bar{q}}^{\gamma^* \rightarrow \rho} = -\frac{1}{4} m_\rho f_\rho \int dy \int \frac{d^2 r_\perp}{(2\pi)} \left\{ \tilde{H}_{q\bar{q}}^{\gamma, \mu}(y, \underline{r}) \left(\varphi_3(y) e_{\rho\mu}^* + i \varphi_1^T(y) p_{1\mu} (\underline{e}_\rho^* \cdot \underline{r}) \right) \right. \\ \left. + \tilde{H}_{q\bar{q}}^{\gamma_5, \mu}(y, \underline{r}) \left(i \varphi_A(y) \varepsilon_\mu e_{\rho}^* p_{1n} + \varphi_A^T(y) p_{1\mu} \varepsilon_{r_\perp} e_{\rho}^* p_{1n} \right) \right\}$$

- The **Fourier** transform of the hard part gives:

$$\Phi_{q\bar{q}}^{\gamma^* \rightarrow \rho} = \int dy \int d^2 \underline{r} \psi_{(q\bar{q})}^{\gamma^* \rightarrow \rho T} \times \mathcal{N}(\underline{r}, \underline{k}) + \text{Hard Terms} \times \overbrace{(2y\bar{y}\varphi_3(y) + (y - \bar{y})\varphi_1^T(y) + \varphi_A^T(y))}^{\text{Cancels due to EOM in WW approx.}}$$

\Rightarrow **dipole picture!**

Beyond leading twist : $\gamma^* \rightarrow \rho$ impact factor up to twist 3

Factorization in coordinate space: the 2-parton contribution

WW approximation: interpretation

- Scanning the ρ -meson wave function:

2-partons exchange overlap

$$\int d^2 \underline{r} \quad \Psi_{\lambda_\gamma, h}^{\gamma^*} \times \left(\underline{r} \cdot \partial_{\underline{z}} \left[\underline{z} \right] \phi_{\lambda_\rho, h}^{WW} + \dots \right) \Big|_{\underline{z}=0} \times \mathcal{N}(\underline{r}, \underline{k})$$

- Link with the ρ -meson wave function

$$\Psi_{\lambda_\rho, h}^{\rho qq} = \text{Spinor part} \times \varphi_{\lambda_\rho}^{(qq)}$$

$$\underbrace{\phi_{\lambda_\rho, h}^{WW}(y, \underline{r})}_{\sim \text{combination of DAs}} \propto (\underline{e}^{(\lambda_\rho)} \cdot \underline{r}) \frac{y \delta_{h, \lambda_\rho} + \bar{y} \delta_{h, -\lambda_\rho}}{y \bar{y}} \int^{|\ell_\perp| < \mu_F} d^2 \ell_\perp \ell_\perp^2 \varphi_{\lambda_\rho}^{(qq)}(y, \ell_\perp)$$

Beyond leading twist : $\gamma^* \rightarrow \rho$ impact factor up to twist 3

Factorization in coordinate space: the complete twist 3 contribution

- The 3-parton amplitude in transverse coordinate space at twist 3:

$$\Phi_{qqg}^{\gamma^* \rightarrow \rho} = -\frac{im_\rho f_\rho}{4} \int dy_1 dy_2 \int \frac{d^2 r_{1\perp}}{(2\pi)^2} \frac{d^2 r_{g\perp}}{(2\pi)^2} \left[\zeta_{3\rho}^V B(y_1, y_2) p_\mu e_{\rho\perp\alpha} \tilde{H}_{qqg}^{\alpha, \gamma^\mu}(y_1, y_2, r_{1\perp}, r_{g\perp}) \right. \\ \left. + \zeta_{3\rho}^A i D(y_1, y_2) p_\mu \varepsilon_{\alpha\beta\gamma\delta} p_n \tilde{H}_{qqg}^{\alpha, \gamma^\mu \gamma^5}(y_1, y_2, r_{1\perp}, r_{g\perp}) \right]$$

- 3-partons exchanged; however, **no quadrupole structure involved** (even at finite N_c , beyond the 't Hooft limit)

- 3-partons results:

$$\Phi_{qqg}^{\gamma_T^* \rightarrow \rho_T} \propto \int dy_1 \int dy_2 \int d^2 \underline{r} \psi_{(qqg)}^{\gamma_T^* \rightarrow \rho_T}(y_1, y_2, \underline{r}) \times \mathcal{N}(\underline{r}, \underline{k}) + \int dy_1 dy_2 \frac{2S(y_1, y_2)}{\bar{y}_1}$$

$$(S(y_1, y_2) = \zeta_\rho^V(\mu^2) B(y_1, y_2; \mu^2) + \zeta_\rho^A(\mu^2) D(y_1, y_2; \mu^2))$$

- Full twist 3 impact factor:

$$\Phi_{qT}^{\gamma_T^* \rightarrow \rho_T} = \Phi_{q\bar{q}}^{\gamma_T^* \rightarrow \rho_T} + \Phi_{q\bar{q}g}^{\gamma_T^* \rightarrow \rho_T} \propto \int dy_i \int d^2 \underline{r} \mathcal{N}(\underline{r}, \underline{k}) \left(\psi_{(q\bar{q})}^{\gamma_T^* \rightarrow \rho_T}(y, \underline{r}) + \psi_{(q\bar{q}g)}^{\gamma_T^* \rightarrow \rho_T}(y_1, y_2, \underline{r}) \right) \\ + \underbrace{\int \frac{dy}{y\bar{y}} \left(2y\bar{y}\varphi_3(y) + (y - \bar{y})\varphi_1^T(y) + \varphi_A^T(y) \right)}_{\text{Cancel due to EOM of QCD}} + \int dy_1 dy_2 \frac{2S(y_1, y_2)}{\bar{y}_1}$$

Cancel due to EOM of QCD

\Rightarrow dipole picture again!

Beyond leading twist : γ^* \rightarrow ρ impact factor up to twist 3

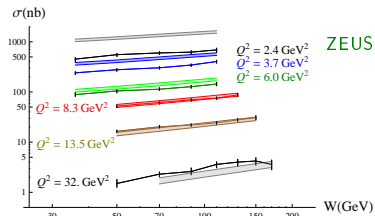
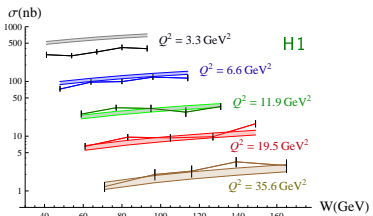
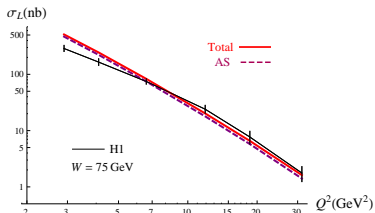
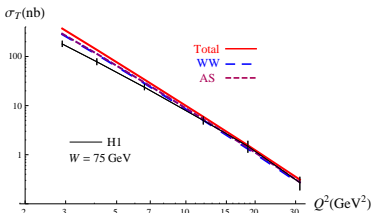
Comparison with data

Comparison with H1 and ZEUS data

A. Besse, L. Szymanowski, S.W.

[arXiv:1302.1766] to appear in JHEP

We use a model for the dipole cross-section $\hat{\sigma}$:
 running coupling **Balitsky Kovchegov** numerical solution (i.e. include saturation effects at Leading Order) **Albacete, Armesto, Milhano, Quiroga Arias, Salgado, 2011**



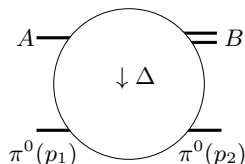
Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 5)

Light-Cone Collinear Factorization

Kinematics and factorization

Consider the process $A \pi^0 \rightarrow B \pi^0$
 (e.g. $\gamma^* \pi^0 \rightarrow \rho \pi^0 \pi^0$, i.e. $B = \rho \pi^0$).

$$P \equiv \frac{p_1 + p_2}{2} \quad \text{and} \quad \Delta \equiv p_2 - p_1.$$



- Sudakov basis provided by p and n ($p^2 = n^2 = 0, p \cdot n = 1$):

$$k = (k \cdot n) p + (k \cdot p) n + k_{\perp}.$$

- In particular $\Delta = -2\xi p + (\Delta \cdot p) n + \Delta_{\perp}$.
- Symmetric kinematics for p_1 and p_2 :

$$p_1 = (1 + \xi) p + \frac{m^2 - \frac{\Delta_{\perp}^2}{4}}{2(1 + \xi)} n - \frac{\Delta_{\perp}}{2},$$

$$p_2 = (1 - \xi) p + \frac{m^2 - \frac{\Delta_{\perp}^2}{4}}{2(1 - \xi)} n + \frac{\Delta_{\perp}}{2},$$

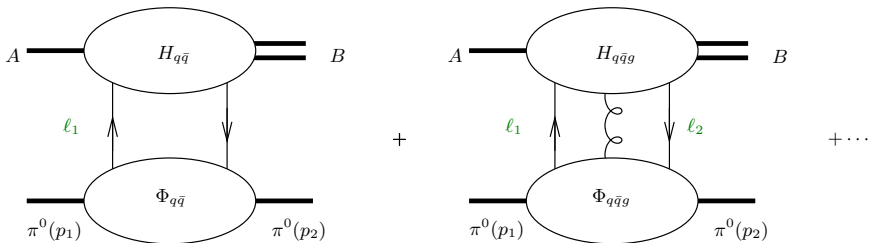
makes P longitudinal (no \perp component): $P = p + (P \cdot p) n = p + \frac{m^2 - \frac{\Delta_{\perp}^2}{4}}{1 - \xi^2} n$.

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 5)

Light-Cone Collinear Factorization

Light-Cone Collinear Factorization

- The p, \perp, n basis is natural for the twist expansion
- To implement T -invariance, the basis P, \perp, n is more suitable
- We only consider 2- and 3-parton correlators

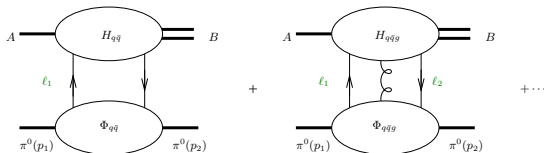


Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 5)

Light-Cone Collinear Factorization

Light-Cone Collinear Factorization

- Loop integrations:



- Taylor expansion of the hard part w.r.t. loop momenta ℓ_i

$$H(\ell_i) = H(y_i p) + \frac{\partial H(\ell_i)}{\partial \ell_\alpha} \Big|_{\ell_i = y_i p} (\ell_i - y_i p)_\alpha + \dots$$

with $(\ell_i - y_i p)_\alpha = \ell_{i\alpha}^\perp + (\ell \cdot p) n_\alpha$

- Using $\int d^4 \ell_i = \int d^4 \ell_i \int dy_i \delta(y_i - \ell_i \cdot n)$ we integrate according to

$$\int d^4 \ell_i = \int dy_i \times \int d(\ell_i \cdot n) \delta(y_i - \ell_i \cdot n) \times \int d^2 \ell_{i\perp} \times \int d(\ell_i \cdot p)$$

\hookrightarrow fact.

\hookrightarrow trivial

\hookrightarrow soft-part

\hookrightarrow integration by res

- We can always close on the $\ell_i^2 = 0$ pole \Rightarrow this fixes the derivatives along n
- Fourier transf. w.r.t. $\ell_i^\perp \Rightarrow$ non-local operators with ∂_\perp (e.g. $\bar{\psi} \partial^\perp \psi$)
 \Rightarrow non-perturbative correlators $\Phi^\perp(l)$

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 5)

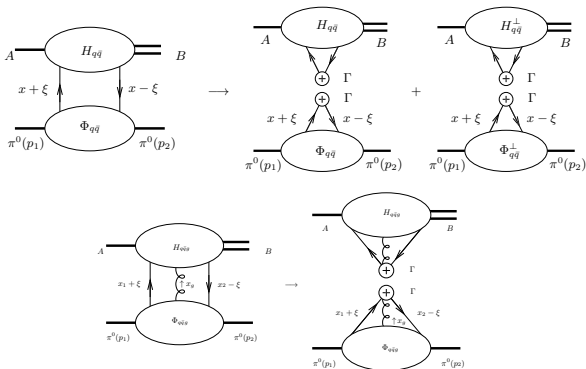
Light-Cone Collinear Factorization

Light-Cone Collinear Factorization

- For consistency, we stop at order 1: the A field and the derivative should appear in a QCD gauge invariant way, through the covariant derivative

$$D_\mu = \partial_\mu - igA_\mu(z).$$

- Here: number of gluons $\leq 1 \implies$ number of (transverse) derivatives ≤ 1
- Color + spinor factorization = Fierz transforms



Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 5)

Light-Cone Collinear Factorization

Parametrization of the non-local correlators

2-parton (with no derivative) **non-local** correlators

Based on C, P, T , this leads to the following set of 4 real GPDs:

$$\langle \pi^0(p_2) | \bar{\psi}(z) \begin{bmatrix} \sigma^{\alpha\beta} \\ \mathbb{1} \\ i\gamma^5 \end{bmatrix} \psi(-z) | \pi^0(p_2) \rangle = \int_{-1}^1 dx e^{i(x-\xi)P \cdot z + i(x+\xi)P \cdot z} \times$$

$$\left[\begin{array}{ccc} -\frac{i}{m_\pi} (P^\alpha \Delta_\perp^\beta - P^\beta \Delta_\perp^\alpha) H_T & + i m_\pi (P^\alpha n^\beta - P^\beta n^\alpha) H_{T3} & - i m_\pi (\Delta_\perp^\alpha n^\beta - \Delta_\perp^\beta n^\alpha) H_{T4} \\ & m_\pi H_S & \\ & 0 & \end{array} \right]$$

twist 2 & 4 twist 3 twist 4

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 5)

Light-Cone Collinear Factorization

Parametrization of the non-local correlators

2-parton (with derivative) and 3-parton non-local correlators: $\sigma^{\alpha\beta}$ structure

Based on C, P, T , this leads to the following set of 12 real GPDs:

$$\langle \pi^0(p_2) | \bar{\psi}(z) \sigma^{\alpha\beta} \left\{ \begin{array}{c} i \overleftrightarrow{\partial}_\perp^\gamma \\ g A^\gamma(y) \end{array} \right\} \psi(-z) | \pi^0(p_1) \rangle = \left\{ \begin{array}{c} \int_{-1}^1 dx e^{i(x-\xi)P \cdot z + i(x+\xi)P \cdot z} \\ \int d^3[x_{1,2,g}] e^{iP \cdot z(x_1+\xi) - iP \cdot y x_g + iP \cdot z(x_2-\xi)} \end{array} \right\}$$

$$\times \left[i m_\pi \left(P^\alpha g_\perp^{\beta\gamma} - P^\beta g_\perp^{\alpha\gamma} \right) \left\{ \begin{array}{c} T_1^T \\ T_1 \end{array} \right\} + \frac{i}{m_\pi} \left(P^\alpha \Delta_\perp^\beta - P^\beta \Delta_\perp^\alpha \right) \Delta_\perp^\gamma \left\{ \begin{array}{c} T_2^T \\ T_2 \end{array} \right\} \right] \text{ (twist 3 \& 5)}$$

$$+ i m_\pi \left(\Delta_\perp^\alpha g_\perp^{\beta\gamma} - \Delta_\perp^\beta g_\perp^{\alpha\gamma} \right) \left\{ \begin{array}{c} T_3^T \\ T_3 \end{array} \right\} + i m_\pi \left(P^\alpha n^\beta - P^\beta n^\alpha \right) \Delta_\perp^\gamma \left\{ \begin{array}{c} T_4^T \\ T_4 \end{array} \right\} \text{ (twist 4)}$$

$$+ i m_\pi^3 \left(n^\alpha g_\perp^{\beta\gamma} - n^\beta g_\perp^{\alpha\gamma} \right) \left\{ \begin{array}{c} T_5^T \\ T_5 \end{array} \right\} + i m_\pi \left(n^\alpha \Delta_\perp^\beta - n^\beta \Delta_\perp^\alpha \right) \Delta_\perp^\gamma \left\{ \begin{array}{c} T_6^T \\ T_6 \end{array} \right\} \Big], \text{ (twist 5)}$$

$$\int d^3[x_{1,2,g}] \equiv \int_{-1+\xi}^{1+\xi} dx_g \int_{-1}^1 dx_1 \int_{-1}^1 dx_2 \delta(x_g - x_2 + x_1), \quad \text{and} \quad \overleftrightarrow{\partial}_\perp^\gamma \equiv \frac{1}{2} (\overrightarrow{\partial}_\perp^\gamma - \overleftarrow{\partial}_\perp^\gamma).$$

$$T_i^T \equiv T_i^T(x, \xi, t) \quad \text{and} \quad T_i \equiv T_i(x_1, x_2, \xi, t) \quad (i = 1, \dots, 6).$$

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 5)

Light-Cone Collinear Factorization

Parametrization of the non-local correlators

2-parton (with derivative) and 3-parton non-local correlators: $\mathbb{1}$ and $i\gamma^5$ structures

Based on C , P , T , this leads to the following set of 4 real GPDs:

$$\langle \pi^0(p_2) | \bar{\psi}(z) \mathbb{1} \left\{ \begin{array}{c} i \overleftrightarrow{\partial}_\perp^\gamma \\ g A^\gamma(y) \end{array} \right\} \psi(-z) | \pi^0(p_1) \rangle = \left\{ \begin{array}{c} \int_{-1}^1 dx e^{i(x-\xi)P \cdot z + i(x+\xi)P \cdot z} \\ \int d^3[x_{1,2,g}] e^{iP \cdot z(x_1+\xi) - iP \cdot y x_g + iP \cdot z(x_2-\xi)} \end{array} \right\} \\ \times m_\pi \Delta_\perp^\gamma \left\{ \begin{array}{c} H_S^{T4} \\ T_S \end{array} \right\}. \quad (\text{twist } 4)$$

$$\langle \pi^0(p_2) | \bar{\psi}(z) i\gamma^5 \left\{ \begin{array}{c} i \overleftrightarrow{\partial}_\perp^\gamma \\ g A^\gamma(y) \end{array} \right\} \psi(-z) | \pi^0(p_1) \rangle = \left\{ \begin{array}{c} \int_{-1}^1 dx e^{i(x-\xi)P \cdot z + i(x+\xi)P \cdot z} \\ \int d^3[x_{1,2,g}] e^{iP \cdot z(x_1+\xi) - iP \cdot y x_g + iP \cdot z(x_2-\xi)} \end{array} \right\} \\ \times m_\pi \epsilon^{\gamma n P \Delta_\perp} \left\{ \begin{array}{c} H_P^T \\ T_P \end{array} \right\}. \quad (\text{twist } 4)$$

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 5)

Light-Cone Collinear Factorization

Minimal set of GPDs

- Number of GPDs: a priori 20 up to twist 5
- Two constraints:
 - QCD equations of motion (EOM)
 - Arbitrariness of p and n

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 5)

Light-Cone Collinear Factorization

Minimal set of GPDs: QCD equations of motion

Dirac equation in a covariant form (no inclusion of mass effects):

$$(i\not{D}\psi)_\alpha = 0 \quad \text{and} \quad (i\not{D}\bar{\psi})_\beta = 0$$

i.e. at correlator level:

$$\langle \pi^0(p_2) | (i\not{D}\psi)_\alpha(-z) \bar{\psi}_\beta(z) | \pi^0(p_1) \rangle = 0$$

and

$$\langle \pi^0(p_2) | \psi_\alpha(-z) (i\not{D}\bar{\psi})_\beta(z) | \pi^0(p_1) \rangle = 0.$$

\implies relations between various correlators.

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 5)

Light-Cone Collinear Factorization

Minimal set of GPDs: Arbitrariness of p and n

- P is fixed by the kinematics
- neither p nor n are fixed
- constraint: $n \cdot p = n \cdot P = 1$
- start from an initial choice for p and n , denoted as $p^{(0)}$ and $n^{(0)}$
- expand

$$n = \alpha n^{(0)} - \frac{n_{\perp}^2}{2\alpha} p^{(0)} + n_{\perp}, \quad (1)$$

$$p = \beta p^{(0)} - \frac{p_{\perp}^2}{2\beta} n^{(0)} + p_{\perp}. \quad (2)$$

- Use global Lorentz invariance \implies consider (1) only
- The two generators of (1) are:
 - scaling of $n^{(0)}$ (i.e. α)
 - the two translations in \perp space (i.e. n_{\perp})

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 5)

Light-Cone Collinear Factorization

Minimal set of GPDs: Arbitrariness of p and n

Variation of a Wilson line

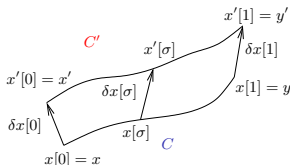
- When implementing the two above generators, one should not forget the hidden Wilson line, entering the non-local operators!
- Wilson line $[y, x]_C$ between x and y along an arbitrary path C , defined as

$$[y, x]_C \equiv P_C \exp ig \int_x^y dx_\mu A^\mu(x).$$

- Variation of a Wilson line from path C to path C'

$$\begin{aligned} \delta[y, x]_C = & \\ -ig \int_0^1 [y, x[\sigma]]_C G_{\nu\gamma}(x[\sigma]) \delta x^\gamma[\sigma] \frac{dx^\nu}{d\sigma}[\sigma] [x[\sigma], x]_C d\sigma & \\ + ig A(y) \cdot \delta x[1] [y, x]_C - ig [y, x]_C A(x) \cdot \delta x[0], & \end{aligned}$$

$$\begin{cases} [0, 1] & \rightarrow & C \\ \sigma & \mapsto & x[\sigma] \end{cases} \quad \text{with } x[0] = x \text{ and } x[1] = y.$$



Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 5)

Light-Cone Collinear Factorization

Minimal set of GPDs: Arbitrariness of p and n

Variation of a Wilson line

- consider now the Wilson line evolved in our non-local operators, like

$$\bar{\psi}(z) \Gamma [z, -z] \psi(-z) \quad \text{with } \Gamma \in \{\sigma^{\alpha\beta}, \mathbb{1}, i\gamma^5\}$$

- For simplicity, take a straight line from $-z$ to z : $x[\tau] = \tau z$, $\tau \in [-1, 1]$.
- Consider the two above mentioned generators:

- Dilation:** $\delta z^\gamma = z^\gamma$

- Translation:** $\delta z^\gamma = \delta z_\perp^\gamma$

$$\begin{aligned} \implies & \frac{\partial}{\partial z^\gamma} \left[\bar{\psi}(z) \Gamma [z, -z] \psi(-z) \right] = \\ & -\bar{\psi}(z) \Gamma [z, -z] \overrightarrow{D}_\gamma \psi(-z) + \bar{\psi}(z) \overleftarrow{D}_\gamma \Gamma [z, -z] \psi(-z) \\ & -ig \int_{-1}^1 dv v \bar{\psi}(z) [z, vz] z^\nu G_{\nu\gamma}(vz) \Gamma [vz, -z] \psi(-z), \end{aligned}$$

with

- $\overrightarrow{D}_\alpha = \partial_\alpha - ig A_\alpha(-z)$ and $\overleftarrow{D}_\alpha = \partial_\alpha + ig A_\alpha(z)$,

- $\frac{\partial}{\partial z^\gamma}$ acts either along the n direction or along the \perp direction

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 5)

Light-Cone Collinear Factorization

Minimal set of GPDs: Arbitrariness of p and n

Application to matrix elements

$$\begin{aligned} & \frac{\partial}{\partial z^\gamma} \left[\langle \pi^0(p_2) | \bar{\psi}(z) \Gamma[z, -z] \psi(-z) | \pi^0(p_1) \rangle \right] = \\ & - \langle \pi^0(p_2) | \bar{\psi}(z) \Gamma[z, -z] \overrightarrow{D}_\gamma \psi(-z) + \bar{\psi}(z) \overleftarrow{D}_\gamma \Gamma[z, -z] \psi(-z) | \pi^0(p_1) \rangle \\ & - ig \int_{-1}^1 dv v \langle \pi^0(p_2) | \bar{\psi}(z) [z, vz] z^\nu G_{\nu\gamma}(vz) \Gamma[vz, -z] \psi(-z) | \pi^0(p_1) \rangle. \quad (3) \end{aligned}$$

- Use light-like gauge: $n \cdot A = 0$
- Thus

$$z^\nu G_{\nu\gamma} = z^\nu \partial_\nu A_\gamma$$

- Only the γ_\perp index contributes non-trivially
- Thus (3) only involves matrix elements with the \perp components of the field A_γ introduced before
- One finally gets a set of integral equations between GPDs

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 5)

Light-Cone Collinear Factorization

Minimal set of GPDs: results

B. Pire, L. Szymanowski, S. W.

[arXiv:1309.3229]

- Twist 5 case: **20 GPDs**
 - 8 EOM
 - 8 n -independence constraints

⇒ the **20** GPDs can be expressed in terms of **8** GPDs (T_i ($i = 1, \dots, 6$), T_P , T_S ,) satisfying **4** sum rules (note: **20-8-8=8-4**).
- Twist 4 case: **16 GPDs**
 - 8 EOM
 - 6 n -independence constraints

⇒ the **16** GPDs can be expressed in terms of **6** GPDs (T_i ($i = 1, \dots, 4$), T_P , T_S ,) satisfying **4** sum rules (note: **16-8-6=6-4**).
- Twist 3 case: **7 GPDs**
 - 5 EOM
 - 2 n -independence constraints

⇒ the **7** GPDs can be expressed in terms of **2** GPDs (T_1 and T_2) satisfying **2** sum rules (note: **7-5-2=2-2**).
- The **vanishing Wandzura-Wilczek** limit:
 - one assumes that the 3-parton correlators vanish
 - ⇒ **all GPDs vanish**
 - ⇒ amplitude of any process involving the chiral-odd π^0 GPDs = 0!

Conclusion

- Since a decade, there have been much progress in the understanding of **hard** exclusive processes
 - at medium energies, there is now a conceptual framework starting from first principle, allowing to describe a huge number of processes
 - at high energy, the impact representation is a powerful tool for describing exclusive processes in diffractive experiments; they are and will be essential for studying QCD in the hard Regge limit (Pomeron, Odderon, saturation...)
- Still, some problems remain:
 - proofs of factorization have been obtained only for very few processes (ex.: $\gamma^* p \rightarrow \gamma p$, $\gamma_L^* p \rightarrow \rho_L p$)
 - for some other processes factorization is highly plausible, but not fully demonstrated at any order (ex.: processes involving GDAs and TDAs)
 - some processes explicitly show sign of breaking of factorization (ex.: $\gamma_T^* p \rightarrow \rho_T p$ which has end-point singularities at Leading Order)
 - models and results from the lattice or from AdS/QCD for the non-perturbative correlators entering GPDs, DAs, GDAs, TDAs are needed, even at a qualitative level!
 - QCD evolution, NLO corrections, choice of renormalization/factorization scale, power corrections, threshold resummations will be very relevant to interpret and describe the forthcoming data
- Constructing a consistent framework including GPDs (skewness) and TMDs/uPDFs (k_T -dependency) with realistic experimental observables is an (almost) open problem (GTMDs)
- Links between theoretical and experimental communities are very fruitful!

A few applications

Production of an exotic hybrid meson in hard processes

Distribution amplitude and quantum numbers: C -parity

- Define the H DA as (for long. pol.)

$$\langle H(p, 0) | \bar{\psi}(-z/2) \gamma_\mu [-z/2; z/2] \psi(z/2) | 0 \rangle \Big|_{\substack{z^2=0 \\ z_+=0 \\ z_\perp=0}} = i f_H M_H e_\mu^{(0)} \int_0^1 dy e^{i(\bar{y}-y)p \cdot z/2} \phi_L^H(y)$$

- Expansion in terms of local operators

$$\langle H(p, \lambda) | \bar{\psi}(-z/2) \gamma_\mu [-z/2; z/2] \psi(z/2) | 0 \rangle = \sum_n \frac{1}{n!} z_{\mu_1} \dots z_{\mu_n} \langle H(p, \lambda) | \bar{\psi}(0) \gamma_\mu \overleftrightarrow{D}_{\mu_1} \dots \overleftrightarrow{D}_{\mu_n} \psi(0) | 0 \rangle,$$

- C -parity: $\begin{cases} H \text{ selects the odd-terms: } C_H = (-) \\ \rho \text{ selects even-terms: } C_\rho = (-) \end{cases}$

$$\langle H(p, \lambda) | \bar{\psi}(-z/2) \gamma_\mu [-z/2; z/2] \psi(z/2) | 0 \rangle = \sum_{n \text{ odd}} \frac{1}{n!} z_{\mu_1} \dots z_{\mu_n} \langle H(p, \lambda) | \bar{\psi}(0) \gamma_\mu \overleftrightarrow{D}_{\mu_1} \dots \overleftrightarrow{D}_{\mu_n} \psi(0) | 0 \rangle$$

- Special case $n = 1$: $\mathcal{R}_{\mu\nu} = S_{(\mu\nu)} \bar{\psi}(0) \gamma_\mu \overleftrightarrow{D}_\nu \psi(0)$

$S_{(\mu\nu)}$ = symmetrization operator: $S_{(\mu\nu)} T_{\mu\nu} = \frac{1}{2}(T_{\mu\nu} + T_{\nu\mu})$

A few applications

Electroproduction of an exotic hybrid meson

Non perturbative input for the hybrid DA

- We need to fix f_H (the analogue of f_ρ)
- This is a non-perturbative input
- Lattice does not yet give information
- The operator $\mathcal{R}_{\mu\nu}$ is related to quark energy-momentum tensor $\Theta_{\mu\nu}$:

$$\mathcal{R}_{\mu\nu} = -i \Theta_{\mu\nu}$$

- Rely on QCD sum rules: resonance for $M \approx 1.4$ GeV
I. I. Balitsky, D. Diakonov, and A. V. Yung

$$f_H \approx 50 \text{ MeV}$$

$$f_\rho = 216 \text{ MeV}$$

- Note: f_H evolves according to the γ_{QQ} anomalous dimension

$$f_H(Q^2) = f_H \left(\frac{\alpha_S(Q^2)}{\alpha_S(M_H^2)} \right)^{K_1} \quad K_1 = \frac{2\gamma_{QQ}(1)}{\beta_0},$$

A few applications

Electroproduction of an exotic hybrid meson

Counting rates for H versus ρ electroproduction: order of magnitude

- Ratio:

$$\frac{d\sigma^H(Q^2, x_B, t)}{d\sigma^\rho(Q^2, x_B, t)} = \left| \frac{f_H (e_u \mathcal{H}_{uu}^- - e_d \mathcal{H}_{dd}^-) \mathcal{V}^{(H,-)}}{f_\rho (e_u \mathcal{H}_{uu}^+ - e_d \mathcal{H}_{dd}^+) \mathcal{V}^{(\rho,+)}} \right|^2$$

- Rough estimate:

- neglect \bar{q} i.e. $x \in [0, 1]$

$\Rightarrow Im\mathcal{A}_H$ and $Im\mathcal{A}_\rho$ are equal up to the factor \mathcal{V}^M

- Neglect the effect of $Re\mathcal{A}$

$$\frac{d\sigma^H(Q^2, x_B, t)}{d\sigma^\rho(Q^2, x_B, t)} \approx \left(\frac{5f_H}{3f_\rho} \right)^2 \approx 0.15$$

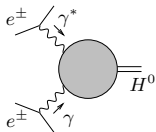
- More precise study based on *Double Distributions* to model GPDs + effects of varying μ_R : order of magnitude unchanged
- The range around 1400 MeV is dominated by the $a_2(1329)(2^{++})$ resonance
 - possible interference between H and a_2
 - identification through the $\pi\eta$ GDA, main decay mode for the $\pi_1(1400)$ candidate, through angular asymmetry in θ_π in the $\pi\eta$ cms

A few applications

Electroproduction of an exotic hybrid meson

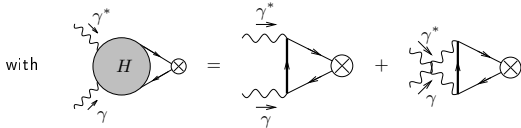
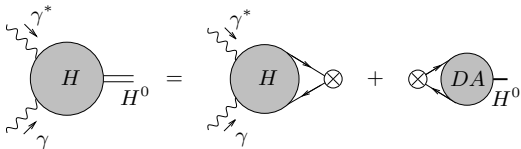
Hybrid meson production in e^+e^- colliders

- Hybrid can be copiously produced in $\gamma^*\gamma$, i.e. at e^+e^- colliders with one tagged out-going electron



BaBar, Belle

- This can be described in a hard factorization framework:



A few applications

Electroproduction of an exotic hybrid meson

Counting rates for H^0 versus π^0

- Factorization gives:

$$\mathcal{A}^{\gamma\gamma^* \rightarrow H^0}(\gamma\gamma^* \rightarrow H_L) = (\epsilon_\gamma \cdot \epsilon_\gamma^*) \frac{(e_u^2 - e_d^2) f_H}{2\sqrt{2}} \int_0^1 dz \Phi^H(z) \left(\frac{1}{z} - \frac{1}{\bar{z}} \right)$$

- Ratio H^0 versus π^0 :

$$\frac{d\sigma^H}{d\sigma^{\pi^0}} = \left| \frac{f_H \int_0^1 dz \Phi^H(z) \left(\frac{1}{z} - \frac{1}{\bar{z}} \right)}{f_\pi \int_0^1 dz \Phi^\pi(z) \left(\frac{1}{z} + \frac{1}{\bar{z}} \right)} \right|^2$$

- This gives, with *asymptotical* DAs (i.e. limit $Q^2 \rightarrow \infty$):

$$\frac{d\sigma^H}{d\sigma^{\pi^0}} \approx 38\%$$

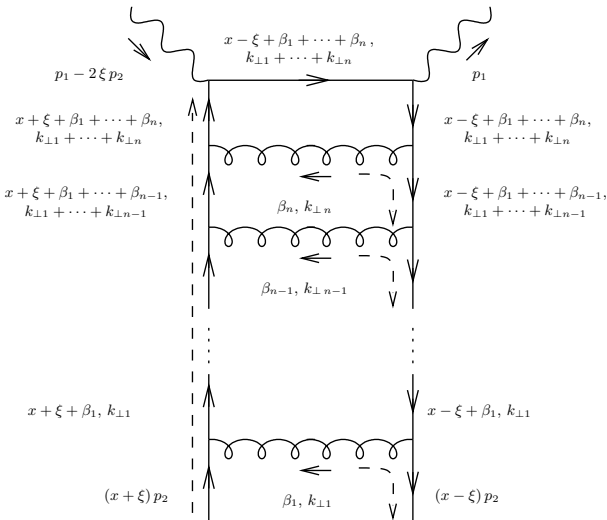
still larger than 20% at $Q^2 \approx 1 \text{ GeV}^2$ (including kinematical twist-3 effects à la [Wandzura-Wilczek](#) for the H^0 DA) and similarly

$$\frac{d\sigma^H}{d\sigma^\eta} \approx 46\%$$

Threshold effects for DVCS and TCS

Resummation for Coefficient functions (1)

Computation of the n -loop ladder-like diagram



- All gluons are assumed to be on mass shell.
- Strong ordering in \underline{k}_i , α_i and β_i .
- The dominant momentum flows along p_2 are indicated

Threshold effects for DVCS and TCS

Resummation for Coefficient functions

Computation of the n -loop ladder-like diagram (2)

- Strong ordering is given as :

$$|\underline{k}_n| \gg |\underline{k}_{n-1}| \gg \dots \gg |\underline{k}_1| \quad , \quad 1 \gg |\alpha_n| \gg |\alpha_{n-1}| \gg \dots \gg |\alpha_1|$$

$$x \sim \xi \gg |\beta_1| \sim |x - \xi| \gg |x - \xi + \beta_1| \sim |\beta_2| \gg \dots \gg |x - \xi + \beta_1 + \beta_2 - \dots + \beta_{n-1}| \sim |\beta_n|$$

- eikonal coupling on the left
- coupling on the right goes beyond eikonal
- Integral for n -loop:

$$I_n = \left(\frac{s}{2}\right)^n \int d\alpha_1 d\beta_1 d_2 \underline{k}_1 \dots \int d\alpha_n d\beta_n d_2 \underline{k}_n (\text{Num})_n \frac{1}{L_1^2} \dots \frac{1}{L_n^2} \frac{1}{S^2} \frac{1}{R_1^2} \dots \frac{1}{R_n^2} \frac{1}{k_1^2} \dots \frac{1}{k_n^2}$$

- Numerator:

$$(\text{Num})_2 = -4s \underbrace{\frac{-2k_1^2(x+\xi)}{\beta_1} \left[1 + \frac{2(x-\xi)}{\beta_1}\right]}_{\text{gluon 1}} \underbrace{\frac{-2k_2^2(x+\xi)}{\beta_2} \left[1 + \frac{2(\beta_1+x-\xi)}{\beta_2}\right]}_{\text{gluon 2}} \dots \underbrace{\frac{-2k_n^2(x+\xi)}{\beta_n} \left[1 + \frac{2(\beta_{n-1}+\dots+\beta_1+x-\xi)}{\beta_n}\right]}_{\text{gluon n}}$$

- Propagators:

$$L_1^2 = \alpha_1(x+\xi)s, \quad R_1^2 = -\underline{k}_1^2 + \alpha_1(\beta_1+x-\xi)s,$$

$$L_2^2 = \alpha_2(x+\xi)s, \quad R_2^2 = -\underline{k}_2^2 + \alpha_2(\beta_1+\beta_2+x-\xi)s,$$

$$\vdots$$

$$L_n^2 = \alpha_n(x+\xi)s, \quad R_n^2 = -\underline{k}_n^2 + \alpha_n(\beta_1+\dots+\beta_n+x-\xi)s,$$

Threshold effects for DVCS and TCS

Resummation for Coefficient functions

Computation of the n -loop ladder-like diagram (3)

$$I_n = -4 \frac{(2\pi i)^n}{x - \xi} \int_0^{\xi-x} d\beta_1 \cdots \int_0^{\xi-x-\beta_1-\cdots-\beta_{n-1}} d\beta_n \frac{1}{\beta_1 + x - \xi} \cdots \frac{1}{\beta_1 + \cdots + \beta_n + x - \xi} \\ \times \int_0^\infty d_N \underline{k}_n \cdots \int_{\underline{k}_2}^\infty d_N \underline{k}_1 \frac{1}{\underline{k}_1^2} \cdots \frac{1}{\underline{k}_{n-1}^2} \frac{1}{\underline{k}_n^2 - (\beta_1 + \cdots + \beta_n + x - \xi)s}$$

integration over \underline{k}_i and β_i leads to our final result :

$$I_n^{\text{fin.}} = -4 \frac{(2\pi i)^n}{x - \xi + i\epsilon} \frac{1}{(2n)!} \log^{2n} \left[\frac{\xi - x}{2\xi} - i\epsilon \right]$$

Resummation :

remember that $K_n = -\frac{1}{4} e_q^2 \left(-i C_F \alpha_s \frac{1}{(2\pi)^2} \right)^n I_n$

$$\left(\sum_{n=0}^{\infty} K_n \right) - (x \rightarrow -x) = \frac{e_q^2}{x - \xi + i\epsilon} \cosh \left[D \log \left(\frac{\xi - x}{2\xi} - i\epsilon \right) \right] - (x \rightarrow -x)$$

where $D = \sqrt{\frac{\alpha_s C_F}{2\pi}}$

Threshold effects for DVCS and TCS

Resummed formula

Inclusion of our resummed formula into the NLO coefficient function

The inclusion procedure is not unique and it is natural to propose two choices:

- modifying only the Born term and the \log^2 part of the C_1^q and keeping the rest of the terms untouched :

$$(T^q)^{\text{res1}} = \left(\frac{e_q^2}{x-\xi+i\epsilon} \left\{ \cosh \left[D \log \left(\frac{\xi-x}{2\xi} - i\epsilon \right) \right] - \frac{D^2}{2} \left[9 + 3 \frac{\xi-x}{x+\xi} \log \left(\frac{\xi-x}{2\xi} - i\epsilon \right) \right] \right\} \right. \\ \left. + C_{coll}^q \log \frac{Q^2}{\mu_F^2} \right) - (x \rightarrow -x)$$

- the resummation effects are accounted for in a multiplicative way for C_0^q and C_1^q :

$$(T^q)^{\text{res2}} = \left(\frac{e_q^2}{x-\xi+i\epsilon} \cosh \left[D \log \left(\frac{\xi-x}{2\xi} - i\epsilon \right) \right] \left[1 - \frac{D^2}{2} \left\{ 9 + 3 \frac{\xi-x}{x+\xi} \log \left(\frac{\xi-x}{2\xi} - i\epsilon \right) \right\} \right] \right) \\ \left. + C_{coll}^q \log \frac{Q^2}{\mu_F^2} \right) - (x \rightarrow -x)$$

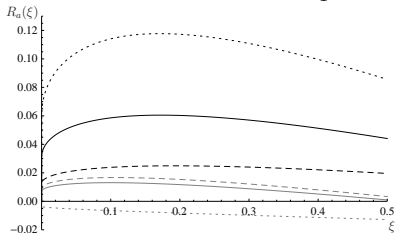
These resummed formulas differ through logarithmic contributions which are beyond the precision of our study.

Threshold effects for DVCS and TCS

Phenomenological implications

- We use a Double Distribution based model
 - S. V. Goloskokov and P. Kroll, *Eur. Phys. J. C* **50**, 829 (2007)
- Blind integral in the whole x -range: amplitude = NLO result $\pm 1\%$
- To respect the domain of applicability of our resummation procedure:
 - restrict the use of our formula to $\xi - a\gamma < |x| < \xi + a\gamma$
 - width $a\gamma$ defined through $|D \log(\gamma/(2\xi))| = 1$
 - theoretical uncertainty evaluated by varying a
 - a more precise treatment is beyond the leading logarithmic approximation

$$R_a(\xi) = \frac{[\int_{\xi-a\gamma}^{\xi+a\gamma} + \int_{-\xi-a\gamma}^{-\xi+a\gamma}] dx (C^{\text{res}} - C_0 - C_1) H(x, \xi, 0)}{|\int_{-1}^1 dx (C_0 + C_1) H(x, \xi, 0)|}$$



$Re[R_a(\xi)]$: black upper curves

$Im[R_a(\xi)]$: grey lower curves

$a = 1$ (solid)

$a = 1/2$ (dotted)

$a = 2$ (dashed)

Problems

ρ -electroproduction: Selection rules and factorization status

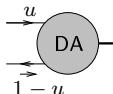
- chirality = helicity for a particle, chirality = -helicity for an antiparticle
- for massless quarks: QED and QCD vertices = chiral even (no chirality flip during the interaction)
 - \Rightarrow the total helicity of a $q\bar{q}$ produced by a γ^* should be 0
 - \Rightarrow helicity of the $\gamma^* = L_z^{q\bar{q}}$ (z projection of the $q\bar{q}$ angular momentum)
- in the pure collinear limit (i.e. twist 2), $L_z^{q\bar{q}}=0 \Rightarrow \gamma_L^*$
- at $t = 0$, no source of orbital momentum from the proton coupling \Rightarrow helicity of the meson = helicity of the photon
- in the collinear factorization approach, $t \neq 0$ change nothing from the hard side \Rightarrow the above selection rule remains true
- thus: 2 transitions possible (s -channel helicity conservation (SCHC)):
 - $\gamma_L^* \rightarrow \rho_L$ transition: QCD factorization holds at $t=2$ at any order in perturbation (i.e. LL, NLL, etc...)

Collins, Frankfurt, Strikman '97 Radyushkin '97

- $\gamma_T^* \rightarrow \rho_T$ transition: QCD factorization has problems at $t=3$

Mankiewicz-Piller '00

$$\int_0^1 \frac{du}{u} \text{ or } \int_0^1 \frac{du}{1-u} \text{ diverge (end-point singularity)}$$



Problems

ρ -electroproduction: Selection rules and factorization status

Improved collinear approximation: a solution?

- keep a transverse ℓ_{\perp} dependency in the q, \bar{q} momenta, used to regulate end-point singularities
- soft and collinear gluon exchange between the valence quark are responsible for large double-logarithmic effects which are conjectured to exponentiate
- this is made easier when using the impact parameter space b_{\perp} conjugated to $\ell_{\perp} \Rightarrow$ **Sudakov** factor

$$\exp[-S(u, b, Q)]$$

- S diverges when $b_{\perp} \sim O(1/\Lambda_{QCD})$ (large transverse separation, i.e. **small transverse momenta**) or $u \sim O(\Lambda_{QCD}/Q)$ **Botts, Serman '89**
 \Rightarrow regularization of end-point singularities for $\pi \rightarrow \pi\gamma^*$ and $\gamma\gamma^*\pi^0$ form factors, based on the factorization approach **Li, Serman '92**
- it has been proposed to combine this perturbative resummation tail effect with an ad-hoc non-perturbative gaussian ansatz for the DAs

$$\exp[-a^2 |k_{\perp}^2|/(u\bar{u})]$$

which gives back the usual asymptotic DA $6u\bar{u}$ when integrating over k_{\perp}
 \Rightarrow practical tools for meson electroproduction phenomenology

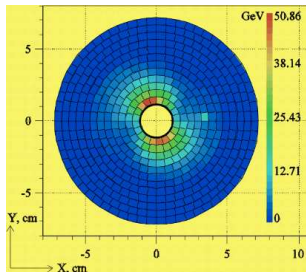
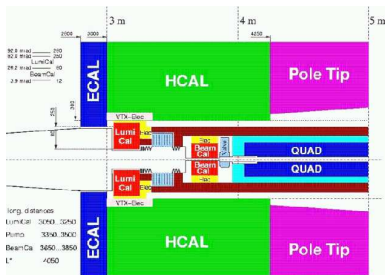
Goloskokov, Kroll '05

QCD at large s

Phenomenological applications: exclusive test of Pomeron

An example of realistic exclusive test of Pomeron: $\gamma^{(*)}\gamma^{(*)} \rightarrow \rho\rho$
 as a subprocess of $e^-e^+ \rightarrow e^-e^+\rho_L^0\rho_L^0$

- ILC should provide $\left\{ \begin{array}{l} \text{very large } \sqrt{s} \text{ (} = 500 \text{ GeV)} \\ \text{very large luminosity (} \simeq 125 \text{ fb}^{-1}/\text{year)} \end{array} \right.$
- detectors are planned to cover the **very forward** region, close from the beampipe (directions of out-going e^+ and e^- at large s)

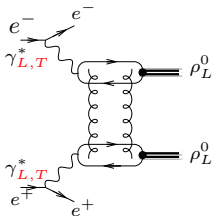
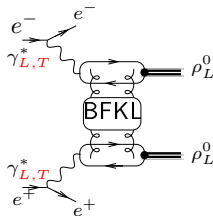
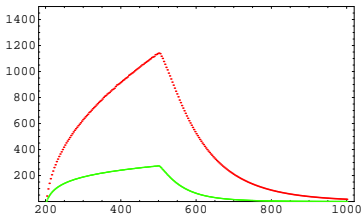


good efficiency of tagging for outgoing e^\pm for $E_e > 100$ GeV and $\theta > 4$ mrad (illustration for LDC concept)

- could be equivalently done at LHC based on the AFP project

QCD at large s

Phenomenological applications: exclusive test of Pomeron

QCD effects in the Regge limit on $\gamma^{(*)}\gamma^{(*)} \rightarrow \rho\rho$  $\simeq 4 \cdot 10^3$ events/year $\simeq 2 \cdot 10^4$ events/year $\frac{d\sigma^{tmin}}{dt} (fb/GeV^2)$  $\sqrt{s_{e+e-}} [GeV]$

proof of feasibility:

B. Pire, L. Szymanowski and S. W.
Eur.Phys.J.C44 (2005) 545

proof of visible BFKL enhancement:

R. Enberg, B. Pire, L. Szymanowski and S. W.
Eur.Phys.J.C45 (2006) 759comprehensive study of γ^* polarization effects
and event rates:M. Segond, L. Szymanowski and S. W.
Eur. Phys. J. C 52 (2007) 93

NLO BFKL study:

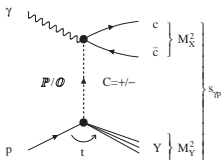
Ivanov, Papa '06 '07; Caporale, Papa, Vera '08

Finding the hard Odderon

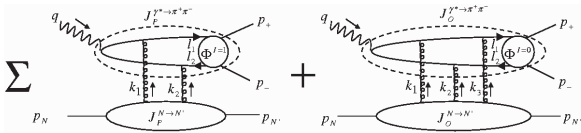
- colorless gluonic exchange
 - $C = +1$: Pomeron, in pQCD described by **BFKL** equation
 - $C = -1$: Odderon, in pQCD described by **BJKP** equation
- best but still weak evidence for \mathbb{O} : pp and $p\bar{p}$ data at **ISR**
- no evidence for perturbative \mathbb{O}

Finding the hard Odderon

- exchange much weaker than $\mathbb{P} \Rightarrow$ two strategies in QCD
 - consider **processes**, where \mathbb{P} vanishes due to C -parity conservation:
 - exclusive $\eta, \eta_c, f_2, a_2, \dots$ in ep ; $\gamma\gamma \rightarrow \eta_c\eta_c \sim |\mathcal{M}_0|^2$ Braunewell, Ewerz '04
 - exclusive $J/\Psi, \Upsilon$ in pp (\mathbb{P} fusion, not $\mathbb{P}\mathbb{P}$) Bzdak, Motyka, Szymanowski, Cudell '07
 - consider **observables** sensitive to the **interference** between \mathbb{P} and \mathbb{O} (open charm in ep ; $\pi^+\pi^-$ in ep) $\sim \text{Re} \mathcal{M}_{\mathbb{P}} \mathcal{M}_{\mathbb{O}}^* \Rightarrow$ observable **linear** in \mathcal{M}_0



Brodsky, Rathsman, Merino '99



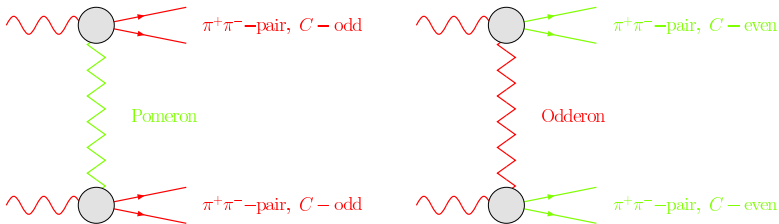
Ivanov, Nikolaev, Ginzburg '01 in photo-production

Hägler, Pire, Szymanowski, Teryaev '02 in electro-production

Finding the hard Odderon

P – O interference in double UPC

P – O interference in $\gamma\gamma \rightarrow \pi^+ \pi^- \pi^+ \pi^-$



Hard scale = t

B. Pire, F. Schwennsen, L. Szymanowski, S. W.
 Phys.Rev.D78:094009 (2008)

pb at LHC: pile-up!