A classification of chiral-odd pion generalized parton distributions beyond leading twist

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arXiv:1309.0083 [hep-ph]]

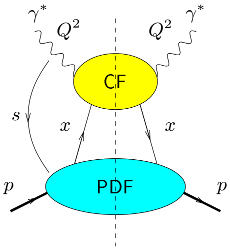
Extensions from DIS

- DIS: inclusive process → forward amplitude ($t = 0$) *(optical theorem)*

  (DIS: Deep Inelastic Scattering)

  ex: $e^{\pm}p \to e^{\pm}X$ at HERA

  $x \Rightarrow$ 1-dimensional structure



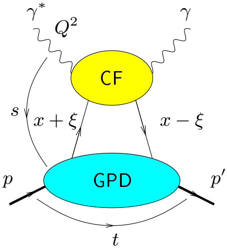
Structure Function

= Coefficient Function (hard) ⊗ Parton Distribution Function (soft)

- DVCS: exclusive process → non forward amplitude ($-t \ll s = W^2$)

  (DVCS: Deep Vitual Compton Scattering)

  Fourier transf.: $t \leftrightarrow$ impact parameter
  $(x, t) \Rightarrow$ 3-dimensional structure



Amplitude

= Coefficient Function (hard) ⊗ Generalized Parton Distribution (soft)

Müller et al. '91 - '94; Radyushkin '96; Ji '97

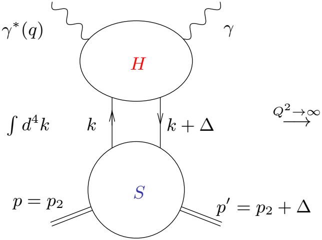
Collinear factorization
A bit more technical: DVCS and GPDs

The two steps for factorization, in a nutshell

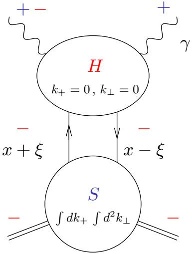
- momentum factorization: light-cone vector dominance for $Q^2 \to \infty$

$p_1,\, p_2$: the two light-cone directions
$$\begin{cases} p_1 = \frac{\sqrt{s}}{2}(1, 0_\perp, 1) & p_1^2 = p_2^2 = 0 \\ p_2 = \frac{\sqrt{s}}{2}(1, 0_\perp, -1) & 2\, p_1 \cdot p_2 = s \sim s_{\gamma^* p} \gtrsim Q^2 \end{cases}$$

Sudakov decomposition: $k \;=\; \underset{+}{\alpha\, p_1} \;+\; \underset{-}{\beta\, p_2} \;+\; \underset{\perp}{k_\perp}$



$$\xrightarrow{Q^2 \to \infty} \quad \int dk^- = p_2^- \int dx$$



key point:
large $(+) \times (-)$ flux

$\Rightarrow$ short distance

(masses neglected)

$$\int d^4k\; S(k,\, k+\Delta)\, H(q,\, k,\, k+\Delta) \;=\; \int dk^- \int dk^+ d^2k_\perp\, S(k,\, k+\Delta)\;\; H(q,\, k^-,\, k^- + \Delta^-)$$

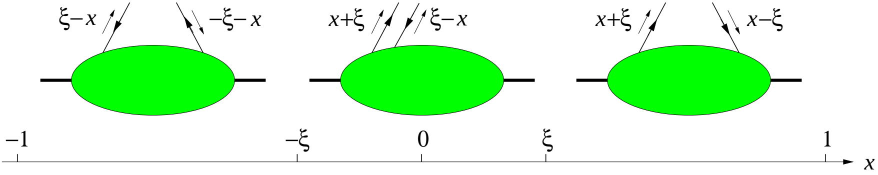
- Quantum numbers factorization (Fierz identity: spinors + color)

$$\Rightarrow \qquad \mathcal{M} = \text{GPD} \otimes \text{Hard part}$$

Müller et al. '91 - '94; Radyushkin '96; Ji '97

Collinear factorization
Twist 2 GPDs

Physical interpretation for GPDs



Emission and reabsoption of an antiquark
∼ PDFs for antiquarks
DGLAP-II region

Emission of a quark and emission of an antiquark
∼ meson exchange
ERBL region

Emission and reabsoption of a quark
∼ PDFs for quarks
DGLAP-I region

Collinear factorization
Twist 2 GPDs

Classification of twist 2 GPDs

- For quarks, one should distinguish the exchanges
  - without helicity flip (chiral-even Γ' matrices): 4 chiral-even GPDs:
    $H^q \xrightarrow{\xi=0,t=0}$ PDF q, E^q, $\tilde{H}^q \xrightarrow{\xi=0,t=0}$ polarized PDFs Δq, $\tilde{E}^q$

$$F^q = \frac{1}{2}\int \frac{dz^+}{2\pi}\, e^{ixP^- z^+} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, \gamma^- q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z^-=0,\, z_\perp=0}$$
$$= \frac{1}{2P^-}\left[H^q(x,\xi,t)\, \bar{u}(p')\gamma^- u(p) + E^q(x,\xi,t)\, \bar{u}(p')\frac{i\,\sigma^{-\alpha}\Delta_\alpha}{2m} u(p)\right],$$
$$\tilde{F}^q = \frac{1}{2}\int \frac{dz^+}{2\pi}\, e^{ixP^- z^+} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, \gamma^- \gamma_5\, q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z^-=0,\, z_\perp=0}$$
$$= \frac{1}{2P^-}\left[\tilde{H}^q(x,\xi,t)\, \bar{u}(p')\gamma^- \gamma_5 u(p) + \tilde{E}^q(x,\xi,t)\, \bar{u}(p')\frac{\gamma_5\, \Delta^-}{2m} u(p)\right].$$

  - with helicity flip (chiral-odd Γ' mat.): 4 chiral-odd GPDs:
    $H_T^q \xrightarrow{\xi=0,t=0}$ quark transversity PDFs $\Delta_T q$, E_T^q, $\tilde{H}_T^q$, $\tilde{E}_T^q$

$$\frac{1}{2}\int \frac{dz^+}{2\pi}\, e^{ixP^- z^+} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, i\,\sigma^{-i}\, q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z^-=0,\, z_\perp=0}$$
$$= \frac{1}{2P^-}\bar{u}(p')\left[H_T^q\, i\sigma^{-i} + \tilde{H}_T^q\, \frac{P^-\Delta^i - \Delta^- P^i}{m^2} + E_T^q\, \frac{\gamma^-\Delta^i - \Delta^-\gamma^i}{2m} + \tilde{E}_T^q\, \frac{\gamma^- P^i - P^-\gamma^i}{m}\right]$$

Collinear factorization
Twist 2 GPDs

Classification of twist 2 GPDs

- analogously, for gluons:
  - 4 gluonic GPDs without helicity flip:

    $H^g \xrightarrow{\xi=0,t=0}$ PDF $x\,g$
    E^g
    $\tilde{H}^g \xrightarrow{\xi=0,t=0}$ polarized PDF $x\,\Delta g$
    $\tilde{E}^g$

  - 4 gluonic GPDs with helicity flip:

    H_T^g
    E_T^g
    $\tilde{H}_T^g$
    $\tilde{E}_T^g$
    (no forward limit reducing to gluons PDFs here: a change of 2 units of helicity cannot be compensated by a spin 1/2 target)

Spin transversity in the nucleon

What is transversity?

- Tranverse spin content of the proton:

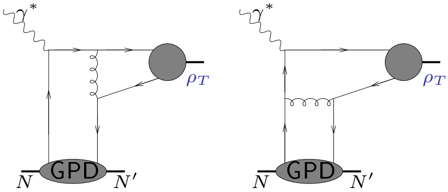
$$
\begin{array}{ccc}
|\uparrow\rangle_{(x)} & \sim & |\rightarrow\rangle + |\leftarrow\rangle \\
|\downarrow\rangle_{(x)} & \sim & |\rightarrow\rangle - |\leftarrow\rangle \\
\text{spin along } x & & \text{helicity state}
\end{array}
$$

- An observable sensitive to helicity spin flip gives thus access to the transversity $\Delta_T q(x)$, which is very badly known (first data have recently been obtained by COMPASS)
- The transversity GPDs are completely unknown
- Chirality: $q_\pm(z) \equiv \frac{1}{2}(1 \pm \gamma^5) q(z)$ with $q(z) = q_+(z) + q_-(z)$
  Chiral-even: chirality conserving
  $\bar{q}_\pm(z)\gamma^\mu q_\pm(-z)$ and $\bar{q}_\pm(z)\gamma^\mu\gamma^5 q_\pm(-z)$
  Chiral-odd: chirality reversing
  $\bar{q}_\pm(z) \cdot 1 \cdot q_\mp(-z)$, $\bar{q}_\pm(z) \cdot \gamma^5 \cdot q_\mp(-z)$ and $\bar{q}_\pm(z)[\gamma^\mu, \gamma^\nu] q_\mp(-z)$
- For a massless (anti)particle, chirality = (-)helicity
- Transversity is thus a chiral-odd quantity
- QCD and QED are chiral even $\Rightarrow \mathcal{A} \sim (\text{Ch.-odd})_1 \otimes (\text{Ch.-odd})_2$

Spin transversity in the nucleon

How to get access to transversity?

- The dominant DA for ρ_T is of twist 2 and chiral-odd ($[\gamma^\mu, \gamma^\nu]$ coupling)
- Unfortunately $\gamma^* \, N^\uparrow \to \rho_T \, N' = 0$
  - this is true at any order in perturbation theory (i.e. corrections as powers of α_s), since this would require a transfer of 2 units of helicity from the proton: impossible!
    Diehl, Gousset, Pire '99; Collins, Diehl '00
  - diagrammatic argument at Born order:



vanishes: $\gamma^\alpha [\gamma^\mu, \gamma^\nu] \gamma_\alpha = 0$

Spin transversity in the nucleon

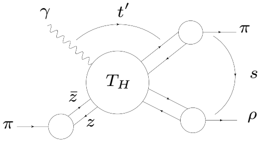
Can one circumvent this vanishing?

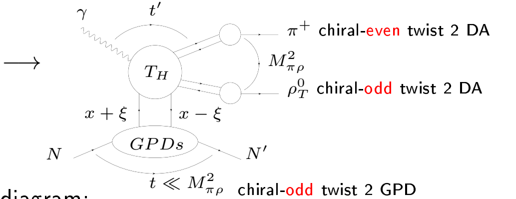
- This vanishing is true only at twist 2 in electroproduction:
  one may consider a final state with 3 particles
  (next slide)
- At twist 3 this process does not vanish but for consistency one needs to consider higher twist corrections *both* for the meson DAs and for the GPDs
  (next part of this talk: for simplicity we will consider the π^0 case)

Spin transversity in the nucleon

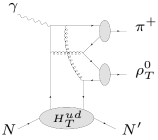
$\gamma N \to \pi^+ \rho^0_T N'$ gives access to transversity

- Factorization à la Brodsky Lepage of $\gamma + \pi \to \pi + \rho$ at large s and fixed angle (i.e. fixed ratio $t'/s, u'/s$)
  $\Longrightarrow$ factorization of the amplitude for $\gamma + N \to \pi + \rho + N'$ at large $M^2_{\pi\rho}$





- a typical non-vanishing diagram:



M. El Beiyad, P. Pire, M. Segond, L. Szymanowski, S.W Phys.Lett.B688:154-167,2010

see also, at large s, with $\mathbb{P}$omeron exchange:

R. Ivanov, B. Pire, L. Symanowski, O. Teryaev '02

R. Enberg, B. Pire, L. Symanowski '06

- These processes with 3 body final state can give access to all GPDs: $M^2_{\pi\rho}$ plays the role of the γ^* virtuality of usual DVCS (here in the time-like domain) JLab, COMPASS

Beyond leading twist

Light-Cone Collinear Factorization versus Covariant Collinear Factorization

- The Light-Cone Collinear Factorization, a self-consistent method, while non-covariant, is very efficient for practical computations
  Anikin, Ivanov, Pire, Szymanowski, S.W. '09
    - inspired by the inclusive case
      Ellis, Furmanski, Petronzio '83; Efremov, Teryaev '84
    - axial gauge
    - parametrization of matrix element along a light-like prefered direction $z = \lambda\, n$ $(n = 2\, p_2/s)$.
    - non-local correlators are defined along this prefered direction, with contributions arising from Taylor expansion up to needed term for a given twist order computation
    - their number is then reduced to a minimal set combining equations of motion and $n-$independency condition
- Another approach (Braun, Ball), based on non-local OPE and fully covariant but less convenient (at least at twist 3) when practically computing coefficient functions, can equivalently be used
- We have established the dictionnary between these two approaches
- This has been explicitly checked for the $\gamma^*_T \to \rho_T$ impact factor at twist 3

  Anikin, Ivanov, Pire, Szymanowski, S.W.

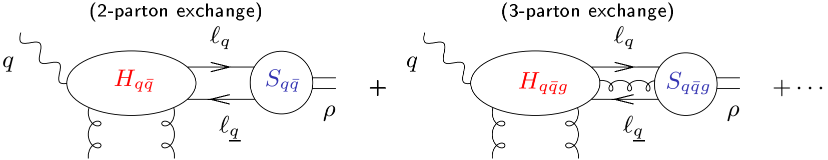
  Nucl.Phys.B 828 (2010) 1-68; Phys.Lett.B682 (2010) 413

Beyond leading twist : $\gamma^* \to \rho$ impact factor up to twist 3 as an example
Light-Cone Collinear Factorization

- The impact factor $\Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)}$ can be written as

$$\Phi^{\gamma^*(\lambda_\gamma)\to\rho(\lambda_\rho)} = \int d^4\ell \cdots \,\mathrm{tr}[H^{(\lambda_\gamma)}(\ell\cdots) \quad S^{(\lambda_\rho)}(\ell\cdots)]$$

hard part   soft part



- Soft parts:

$$S_{q\bar{q}}(\ell_q) = \int d^4z\, e^{-i\ell_q\cdot z}\langle\rho(p)|\psi(0)\,\bar{\psi}(z)|0\rangle$$

$$S_{q\bar{q}q}(\ell_q,\ell_g) = \int d^4z_1 \int d^4z_2\, e^{-i(\ell_q\cdot z_1+\ell_g\cdot z_2)}\langle\rho(p)|\psi(0)\,gA_\alpha^\perp(z_2)\bar{\psi}(z_1)|0\rangle$$

Beyond leading twist : $\gamma^* \to \rho$ impact factor up to twist 3 as an example

Light-Cone Collinear Factorization

Light-Cone Collinear Factorization

- **Sudakov expansion** in the basis $p \sim p_\rho$, n ($p^2 = n^2 = 0$ and $p \cdot n = 1$)

$$\ell_\mu \;=\; u\,p_\mu \;+\; \ell_\mu^\perp \;+\; (\ell\cdot p)\,n_\mu, \qquad u = \ell\cdot n$$

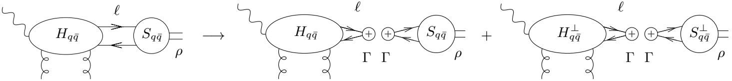
$$\qquad 1 \qquad\qquad 1/Q \qquad\qquad 1/Q^2$$

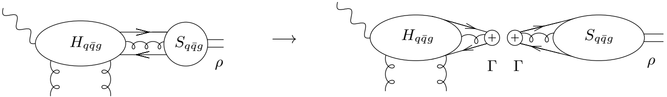
- **Taylor expansion** of the hard part $H(\ell)$ along the collinear direction p:

$$H(\ell) = H(up) \;+\; \left.\frac{\partial H(\ell)}{\partial \ell_\alpha}\right|_{\ell=up} (\ell - u\,p)_\alpha + \ldots \quad \text{with} \quad (\ell - u\,p)_\alpha \approx \ell_\alpha^\perp$$

- $l_\alpha^\perp \xrightarrow{Fourier}$ derivative of the soft term: $\int d^4z\; e^{-i\ell\cdot z}\langle\rho(p)|\psi(0)\,i\,\overleftrightarrow{\partial_{\alpha\perp}}\,\bar\psi(z)|0\rangle$

- Color + spinor factorization = Fierz transforms:





Beyond leading twist : $\gamma^* \to \rho$ impact factor up to twist 3 as an example

Light-Cone Collinear Factorization

2-body non-local correlators

ρ_L — twist 2

ρ_T — kinematical twist 3 (WW); genuine twist 3; genuine + kinematical twist 3

- vector correlator

$$\langle \rho(p)|\bar{\psi}(z)\gamma_\mu \psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho \left[\varphi_1(y)\,(e^*\cdot n)p_\mu + \varphi_3(y)\, e^{*T}_\mu\right]$$

- axial correlator

$$\langle \rho(p)|\bar{\psi}(z)\gamma_5\gamma_\mu \psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, i\,\varphi_A(y)\,\varepsilon_{\mu\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta\, n_\delta$$

- vector correlator with transverse derivative

$$\langle \rho(p)|\bar{\psi}(z)\gamma_\mu\, i\, \overleftrightarrow{\partial^\perp_\alpha}\, \psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, \varphi^T_1(y)\, p_\mu e^{*T}_\alpha$$

- axial correlator with transverse derivative

$$\langle \rho(p)|\bar{\psi}(z)\gamma_5\gamma_\mu\, i\, \overleftrightarrow{\partial^\perp_\alpha}\, \psi(0)|0\rangle \stackrel{\mathcal{F}}{=} m_\rho\, f_\rho\, i\,\varphi^T_A(y)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta\, n_\delta,$$

where y ($\bar{y} \equiv 1-y$) = momentum fraction along $p \equiv p_1$ of the quark (antiquark) and $\stackrel{\mathcal{F}}{=} \int_0^1 dy \exp\,[i\,y\,p\cdot z]$, with $z = \lambda n$

$\Rightarrow$ 5 2-body DAs

Beyond leading twist : $\gamma^* \to \rho$ impact factor up to twist 3 as an example

Light-Cone Collinear Factorization

3-body non-local correlators   **genuine twist 3**

- vector correlator

$$\langle \rho(p)|\bar{\psi}(z_1)\gamma_\mu g A^T_\alpha(z_2)\psi(0)|0\rangle \stackrel{\mathcal{F}_2}{=} m_\rho\, f_3^V\, B(y_1,y_2)\, p_\mu\, e^{*T}_\alpha,$$

- axial correlator

$$\langle \rho(p)|\bar{\psi}(z_1)\gamma_5\gamma_\mu g A^T_\alpha(z_2)\psi(0)|0\rangle \stackrel{\mathcal{F}_2}{=} m_\rho\, f_3^A\, i\, D(y_1,y_2)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e^{*T}_\lambda\, p_\beta\, n_\delta,$$

where y_1, $\bar{y}_2$, $y_2 - y_1$ = quark, antiquark, gluon momentum fraction

and $\stackrel{\mathcal{F}_2}{=} \int_0^1 dy_1 \int_0^1 dy_2\ \exp\left[i\, y_1\, p\cdot z_1 + i(y_2-y_1)\, p\cdot z_2\right]$, with $z_{1,2} = \lambda n$

$\Rightarrow$ 2 3-body DAs

Beyond leading twist : $\gamma^* \to \rho$ impact factor up to twist 3 as an example
Light-Cone Collinear Factorization

Minimal set of DAs

- Number of non-perturbative quantities: a priori 7 at twist 3
  (5 2-parton DA and 2 2-parton DA)
- Non-perturbative correlators cannot be obtained perturbatively!
- One should reduce their number to a minimal set before any use of a model or any measure on the QCD lattice
- Independence w.r.t the choice of the vector n defining
  - the light-cone direction z: $z = \lambda n$
  - the ρ_T polarization vector: $e_T \cdot n = 0$
  - the axial gauge: $n \cdot A = 0$
- We have proven that 3 independent Distribution Amplitudes are necessary:

$$\left\{ \begin{array}{lll} \text{QCD equations of motion} & \text{2 equations} & \text{(DAs from } \partial_\perp \text{ operators eliminated)} \\ \text{Arbitrariness in the choice of } n & \text{2 equations} & \end{array} \right.$$

$\varphi_1(y)$ $\leftarrow$ 2-body twist 2 correlator

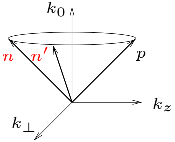
$B(y_1,\, y_2)$ $\leftarrow$ 3-body genuine twist 3 vector correlator

$D(y_1,\, y_2)$ $\leftarrow$ 3-body genuine twist 3 axial correlator

Beyond leading twist : $\gamma^* \to \rho$ impact factor up to twist 3 as an example

Light-Cone Collinear Factorization: $n-$independence

$n-$independence at the amplitude level

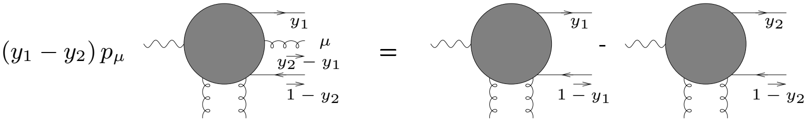


- ρ_T polarization: $e^{*T}_\mu = e^*_\mu - p_\mu \, e^* \cdot n$ keeping $\begin{cases} n \cdot p = 1 \\ n^2 = 0 \end{cases}$
- for the full factorized amplitude:

$$\mathcal{A} = H \otimes S \qquad \frac{d\mathcal{A}}{dn_\perp{}^\mu} = 0 \,,$$

- rewrite hard terms in one single form, of 2-body type: use Ward identities
  Example: hard 3-body $\longrightarrow$ hard 2-body

$$\mathrm{tr}\left[H_{3\rho}(y_1, y_2)\, p^\rho \, \not{p}\right] \; B(y_1,\, y_2) \;=\; \frac{1}{y_1 - y_2} \left(\mathrm{tr}\left[H_2(y_1)\, \not{p}\right] - \mathrm{tr}\left[H_2(y_2)\, \not{p}\right]\right) B(y_1,\, y_2)\,,$$



- thus, symbolically,

$$\frac{dS}{dn_\perp{}^\mu} = 0$$

Beyond leading twist : $\gamma^* \to \rho$ impact factor up to twist 3 as an example

Light-Cone Collinear Factorization: $n-$independence

$n-$independence from the operators

Variation of a Wilson line

- When implementing the two above generators, one should not forget the hidden Wilson line, entering the non-local operators!
- Wilson line $[y, x]_C$ between x and y along an arbitrary path C, defined as

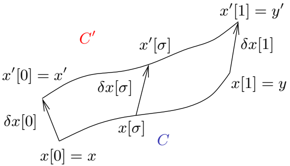
$$[y, x]_C \equiv P_C \exp ig \int_x^y dx_\mu \, A^\mu(x) \,.$$

- Variation of a Wilson line from path C to path C'

$$\delta[y, x]_C =$$

$$-i\,g \int_0^1 [y, x[\sigma]]_C \; G_{\nu\gamma}(x[\sigma])\, \delta x^\gamma[\sigma] \, \frac{dx^\nu}{d\sigma}[\sigma] \; [x[\sigma], x]_C \, d\sigma$$

$$+\,i\,g\, A(y) \cdot \delta x[1] \; [y, x]_C - i\,g\,[y, x]_C \; A(x) \cdot \delta x[0] \,,$$



$$\left\{ \begin{array}{ccc} [0,1] & \to & C \\ \sigma & \mapsto & x[\sigma] \end{array} \right. \quad \text{with } x[0] = x \text{ and } x[1] = y \,.$$

Beyond leading twist : $\gamma^* \to \rho$ impact factor up to twist 3 as an example

Light-Cone Collinear Factorization

$n-$independence from the operators

Variation of a Wilson line

- consider now the Wilson line envolved in our non-local operators, like

$$\bar{\psi}(z)\,\Gamma\,[z,-z]\,\psi(-z) \quad \text{with } \Gamma \in \{\sigma^{\alpha\beta},\, \mathbb{1},\, i\gamma^5\}$$

- For simplicity, take a straight line from $-z$ to z: $x[\tau] = \tau\, z\,,\ \tau \in [-1,1]\,.$
- Consider an infinitesimal transformation $\delta\, z^\gamma$:

$$\frac{\partial}{\partial z^\gamma}\left[\bar{\psi}(z)\Gamma[z,\,-z]\psi(-z)\right] =$$
$$-\bar{\psi}(z)\Gamma[z,\,-z]\,\overrightarrow{D_\gamma}\,\psi(-z) + \bar{\psi}(z)\,\overleftarrow{D_\gamma}\,\Gamma[z,\,-z]\psi(-z)$$
$$-ig\int_{-1}^{1} dv\, v\, \bar{\psi}(z)[z,\,vz]z^\nu G_{\nu\gamma}(vz)\Gamma[vz,\,-z]\psi(-z)\,,$$

with $\overrightarrow{D_\alpha} = \overrightarrow{\partial_\alpha} - igA_\alpha(-z)$ and $\overleftarrow{D_\alpha} = \overleftarrow{\partial_\alpha} + igA_\alpha(z)\,.$ Balitsky, Braun '89

Beyond leading twist : $\gamma^* \to \rho$ impact factor up to twist 3 as an example

Light-Cone Collinear Factorization

$n-$independence from the operators

Application to matrix elements

$$\frac{\partial}{\partial z^\gamma}\left[\langle\rho(p)|\bar\psi(z)\Gamma[z,\,-z]\psi(-z)|0\rangle\right] =$$
$$-\langle\rho(p)|\bar\psi(z)\Gamma[z,\,-z]\overrightarrow{D_\gamma}\,\psi(-z)+\bar\psi(z)\overleftarrow{D_\gamma}\,\Gamma[z,\,-z]\psi(-z)|0\rangle$$
$$-ig\int\limits_{-1}^{1}dv\,v\,\langle\rho(p)|\bar\psi(z)[z,\,vz]z^\nu G_{\nu\gamma}(vz)\Gamma[vz,\,-z]\psi(-z)|0\rangle\,. \qquad (1)$$

- Use light-like gauge: $n\cdot A=0$
- Thus

$$z^\nu G_{\nu\gamma}=z^\nu\partial_\nu A_\gamma$$

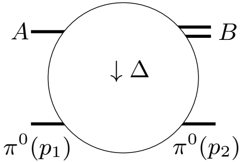
- Only the $\gamma_\perp$ index contributes non-trivially
- Thus (1) only involves matrix elements with the $\perp$ components of the field A_γ introduced before
- One finally gets a set of two integral equations between DAs

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 6)

Light-Cone Collinear Factorization

Kinematics and factorization

Consider the process $A\,\pi^0 \to B\,\pi^0$
(e.g. $\gamma^*\,\pi^0 \to \rho\,\pi^0\,\pi^0$, i.e. $B = \rho\,\pi^0$).



$$P \equiv \frac{p_1 + p_2}{2} \quad \text{and} \quad \Delta \equiv p_2 - p_1 \,.$$

- Sudakov basis provided by p and n ($p^2 = n^2 = 0\,, p \cdot n = 1$):

$$k = (k \cdot n)\, p + (k \cdot p)\, n + k_\perp \,.$$

- In particular $\Delta = -2\xi\, p + (\Delta \cdot p)\, n + \Delta_\perp \,.$

- Symmetric kinematics for p_1 and p_2:

$$p_1 = (1+\xi)\, p + \frac{m^2 - \frac{\Delta_\perp^2}{4}}{2(1+\xi)} n - \frac{\Delta_\perp}{2} \,,$$

$$p_2 = (1-\xi)\, p + \frac{m^2 - \frac{\Delta_\perp^2}{4}}{2(1-\xi)} n + \frac{\Delta_\perp}{2} \,,$$

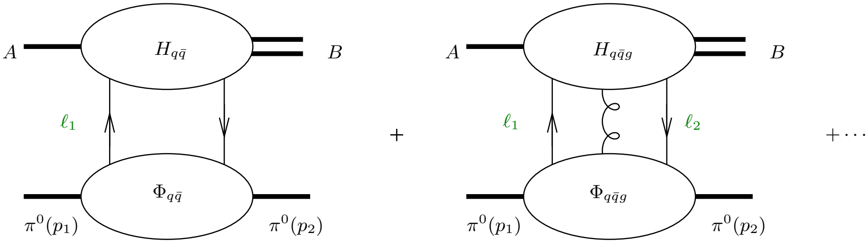
makes P longitudinal (no $\perp$ component): $P = p + (P \cdot p)\, n = p + \frac{m^2 - \frac{\Delta_\perp^2}{4}}{1 - \xi^2} n \,.$

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 6)

Light-Cone Collinear Factorization

Light-Cone Collinear Factorization

- The p, $\perp$, n basis is natural for the twist expansion
- To implement $T-$invariance, the basis P, $\perp$, n is more suitable
- We only consider 2- and 3-parton correlators

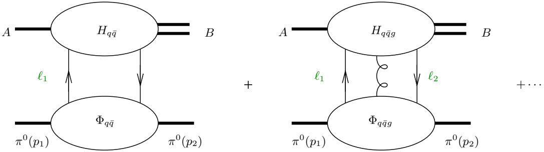


Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 6)

Light-Cone Collinear Factorization

Light-Cone Collinear Factorization

- Loop integrations:



- Taylor expansion of the hard part w.r.t. loop momenta ℓ_i

$$H(\ell_i) = H(y_i\, p) + \left.\frac{\partial H(\ell_i)}{\partial \ell_\alpha}\right|_{\ell_i = y_i p} (\ell_i - y\, p)_\alpha + \dots .$$

with $(\ell_i - y_i\, p)_\alpha = \ell^\perp_{i\alpha} + (\ell \cdot p)\, n_\alpha$

- Using $\int d^4\ell_i = \int d^4\ell_i \int dy_i\, \delta(y_i - \ell_i \cdot n)$ we integrate according to

$$\int d^4\ell_i = \int dy_i \times \int d(\ell_i \cdot n)\, \delta(y_i - \ell_i \cdot n) \times \int d^2\ell_{i\perp} \times \int d(\ell_i \cdot p)$$

$\hookrightarrow$ fact. $\quad\hookrightarrow$ trivial $\quad\hookrightarrow$ soft-part

- Fourier transf. w.r.t. :
  - $\ell_i^\perp \Rightarrow$ non-local op. with $\partial_\perp$ (e.g. $\bar{\psi}\, \partial^\perp \psi$) $\Rightarrow$ correlators $\Phi^\perp(l)$
  - $(\ell \cdot p) n_\alpha \Rightarrow$ non-local op. with $\partial_n^\gamma \equiv (\partial \cdot p) n^\gamma$ (e.g. $\bar{\psi}\, \partial_n^\gamma \psi$) $\Rightarrow$ correl. $\Phi^n(l)$

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 6)

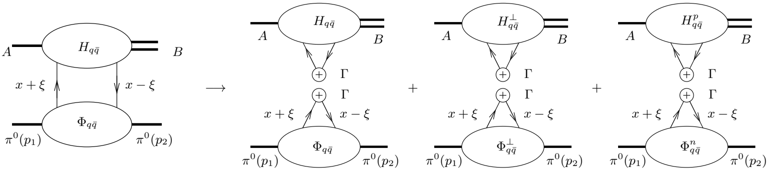
Light-Cone Collinear Factorization

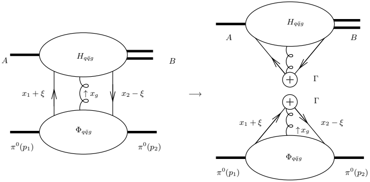
Light-Cone Collinear Factorization

- For consistency, we stop at order 1: the A field and the derivative should appear in a QCD gauge invariant way, through the covariant derivative

$$D_\mu = \partial_\mu - igA_\mu(z)\,.$$

- Here: number of gluons $\leq 1 \Longrightarrow$ number of derivatives ≤ 1
- Color + spinor factorization = Fierz transforms





Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 6)

Light-Cone Collinear Factorization

Parametrization of the non-local correlators

2-parton (with no derivative) non-local correlators

Based on C, P, T, this leads to the following set of 4 real GPDs:

$$\langle \pi^0(p_2)|\bar{\psi}(z) \begin{bmatrix} \sigma^{\alpha\beta} \\ \mathbb{1} \\ i\gamma^5 \end{bmatrix} \psi(-z)|\pi^0(p_2)\rangle = \int_{-1}^{1} dx\, e^{i(x-\xi)P\cdot z + i(x+\xi)P\cdot z} \times$$

$$\begin{bmatrix} -\frac{i}{m_\pi}\left(P^\alpha \Delta_\perp^\beta - P^\beta \Delta_\perp^\alpha\right) H_T + i\, m_\pi \left(P^\alpha n^\beta - P^\beta n^\alpha\right) H_{T3} - i\, m_\pi \left(\Delta_\perp^\alpha n^\beta - \Delta_\perp^\beta n^\alpha\right) H_{T4} \\ m_\pi\, H_S \\ 0 \end{bmatrix}$$

twist 2 & 4 — twist 3 — twist 4

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 6)

Light-Cone Collinear Factorization

Parametrization of the non-local correlators

2-parton (with $\perp$ derivative) and 3-parton non-local correlators: $\sigma^{\alpha\beta}$ structure

Based on C, P, T, this leads to the following set of 12 real GPDs:

$$\langle\pi^0(p_2)|\bar{\psi}(z)\,\sigma^{\alpha\beta}\left\{\begin{matrix} i\,\overleftrightarrow{\partial}^{\gamma}_{\perp} \\ g\,A^{\gamma}(y)\end{matrix}\right\}\psi(-z)|\pi^0(p_1)\rangle = \left\{\begin{matrix}\int_{-1}^{1} dx\, e^{i(x-\xi)P\cdot z+i(x+\xi)P\cdot z} \\ \int d^3[x_{1,\,2,\,g}]\, e^{iP\cdot z(x_1+\xi)-iP\cdot y\, x_g+iP\cdot z\,(x_2-\xi)}\end{matrix}\right\}$$

$$\times\Big[i\,m_\pi\left(P^\alpha g_\perp^{\beta\gamma}-P^\beta g_\perp^{\alpha\gamma}\right)\left\{\begin{matrix}T_1^T\\T_1\end{matrix}\right\}+\frac{i}{m_\pi}\left(P^\alpha\Delta_\perp^\beta-P^\beta\Delta_\perp^\alpha\right)\Delta_\perp^\gamma\left\{\begin{matrix}T_2^T\\T_2\end{matrix}\right\}\text{(twist 3 \& 5)}$$

$$+i\,m_\pi\left(\Delta_\perp^\alpha g_\perp^{\beta\gamma}-\Delta_\perp^\beta g_\perp^{\alpha\gamma}\right)\left\{\begin{matrix}T_3^T\\T_3\end{matrix}\right\}+i\,m_\pi\left(P^\alpha n^\beta-P^\beta n^\alpha\right)\Delta_\perp^\gamma\left\{\begin{matrix}T_4^T\\T_4\end{matrix}\right\}\quad\text{(twist 4)}$$

$$+i\,m_\pi^3\left(n^\alpha g_\perp^{\beta\gamma}-n^\beta g_\perp^{\alpha\gamma}\right)\left\{\begin{matrix}T_5^T\\T_5\end{matrix}\right\}+i m_\pi\left(n^\alpha\Delta_\perp^\beta-n^\beta\Delta_\perp^\alpha\right)\Delta_\perp^\gamma\left\{\begin{matrix}T_6^T\\T_6\end{matrix}\right\}\Big],\quad\text{(twist 5)}$$

$$\int d^3[x_{1,\,2,\,g}]\equiv\int_{-1+\xi}^{1+\xi}dx_g\int_{-1}^{1}dx_1\int_{-1}^{1}dx_2\;\delta(x_g-x_2+x_1)\,,\quad\text{and}\quad\overleftrightarrow{\partial}^{\gamma}_{\perp}\equiv\frac{1}{2}(\overrightarrow{\partial}^{\gamma}_{\perp}-\overleftarrow{\partial}^{\gamma}_{\perp})\,.$$

$$T_i^T\equiv T_i^T(x,\xi,t)\quad\text{and}\quad T_i\equiv T_i(x_1,x_2,\xi,t)\;(i=1,\cdots 6)\,.$$

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 6)

Light-Cone Collinear Factorization

Parametrization of the non-local correlators

2-parton (with $\perp$ derivative) and 3-parton non-local correlators: $\mathbb{1}$ and $i\gamma^5$ structures

Based on C, P, T, this leads to the following set of 4 real GPDs:

$$\langle \pi^0(p_2)|\bar{\psi}(z)\,\mathbb{1}\left\{ \begin{array}{c} i\,\overleftrightarrow{\partial}^{\gamma}_{\perp} \\ g\,A^{\gamma}(y) \end{array} \right\}\psi(-z)|\pi^0(p_1)\rangle = \left\{ \begin{array}{l} \int_{-1}^{1} dx\, e^{i(x-\xi)P\cdot z + i(x+\xi)P\cdot z} \\ \int d^3[x_{1,\,2,\,g}]\, e^{iP\cdot z(x_1+\xi) - iP\cdot y\, x_g + iP\cdot z\,(x_2-\xi)} \end{array} \right\}$$

$$\times \quad m_\pi\, \Delta^{\gamma}_{\perp} \left\{ \begin{array}{c} H_S^{T4} \\ T_S \end{array} \right\}. \qquad \text{(twist 4)}$$

$$\langle \pi^0(p_2)|\bar{\psi}(z)\,i\gamma^5\left\{ \begin{array}{c} i\,\overleftrightarrow{\partial}^{\gamma}_{\perp} \\ g\,A^{\gamma}(y) \end{array} \right\}\psi(-z)|\pi^0(p_1)\rangle = \left\{ \begin{array}{l} \int_{-1}^{1} dx\, e^{i(x-\xi)P\cdot z + i(x+\xi)P\cdot z} \\ \int d^3[x_{1,\,2,\,g}]\, e^{iP\cdot z(x_1+\xi) - iP\cdot y\, x_g + iP\cdot z\,(x_2-\xi)} \end{array} \right\}$$

$$\times \quad m_\pi\, \epsilon^{\gamma\, n\, P\, \Delta_\perp} \left\{ \begin{array}{c} H_P^{T} \\ T_P \end{array} \right\}. \qquad \text{(twist 4)}$$

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 6)

Light-Cone Collinear Factorization

Parametrization of the non-local correlators

2-parton (with long. derivative) and 3-parton non-local correlators: $\sigma^{\alpha\beta}$ structure

Based on C, P, T, this leads to the following set of 6 real GPDs:
$(\partial_n^\gamma \equiv (\partial \cdot p) n^\gamma \quad \text{and} \quad A_n^\gamma \equiv (A \cdot p) n^\gamma)$

$$\langle \pi^0(p_2)|\bar\psi(z)\, \sigma^{\alpha\beta} \left\{ \begin{matrix} i \overleftrightarrow{\partial_n^\gamma} \\ g\, A_n^\gamma(y) \end{matrix} \right\} \psi(-z)|\pi^0(p_1)\rangle = \left\{ \begin{matrix} \int_{-1}^{1} dx\, e^{i(x-\xi)P\cdot z + i(x+\xi)P\cdot z} \\ \int d^3[x_{1,\,2,\,g}]\; e^{iP\cdot z(x_1+\xi) - iP\cdot y\, x_g + iP\cdot z\,(x_2-\xi)} \end{matrix} \right\}$$

$$\times \Bigg[i m_\pi \left(P^\alpha \Delta_\perp^\beta - P^\beta \Delta_\perp^\alpha \right) n^\gamma \left\{ \begin{matrix} M_1^- \\ M_1 \end{matrix} \right\} \quad \text{(twist 4 \& 6)}$$

$$+ i\, m_\pi^3 \left(P^\alpha n^\beta - P^\beta n^\alpha \right) n^\gamma \left\{ \begin{matrix} M_2^- \\ M_2 \end{matrix} \right\} \quad \text{(twist 5)}$$

$$+ i\, m_\pi^3 \left(n^\alpha \Delta_\perp^\beta - n^\beta \Delta_\perp^\alpha \right) n^\gamma \left\{ \begin{matrix} M_3^- \\ M_3 \end{matrix} \right\} \Bigg], \quad \text{(twist 6)}$$

$$\int d^3[x_{1,\,2,\,g}] \equiv \int_{-1+\xi}^{1+\xi} dx_g \int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2\; \delta(x_g - x_2 + x_1), \quad \text{and} \quad \overleftrightarrow{\partial_n^\gamma} \equiv \frac{1}{2}(\overrightarrow{\partial_n^\gamma} - \overleftarrow{\partial_n^\gamma}),$$

$$M_i^- \equiv M_i^-(x,\xi,t) \quad \text{and} \quad M_i \equiv M_i(x_1, x_2, \xi, t)\; (i = 1, \cdots 3).$$

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 6)

Light-Cone Collinear Factorization

Parametrization of the non-local correlators

2-parton (with long. derivative) and 3-parton non-local correlators: $\mathbb{1}$ and $i\gamma^5$ structures

Based on C, P, T, this leads to the following set of 2 real GPDs:

$$\langle\pi^0(p_2)|\bar{\psi}(z)\,\mathbb{1}\left\{\begin{array}{c} i\ \overleftrightarrow{\partial}_n^{\gamma} \\ g\,A_n^{\gamma}(y)\end{array}\right\}\psi(-z)|\pi^0(p_1)\rangle = \left\{\begin{array}{l}\int_{-1}^{1} dx\, e^{i(x-\xi)P\cdot z+i(x+\xi)P\cdot z} \\ \int d^3[x_{1,\,2,\,g}]\; e^{iP\cdot z(x_1+\xi)-iP\cdot y\,x_g+iP\cdot z\,(x_2-\xi)}\end{array}\right.$$

$$\times\quad m_\pi^3\,n^{\gamma}\left\{\begin{array}{c} H_S^- \\ M_S\end{array}\right\}. \qquad \text{(twist 5)}$$

For the $i\gamma^5$ structure, we cannot define correlators with the needed parity :

$$\langle\pi^0(p_2)|\bar{\psi}(z)\,i\gamma^5\left\{\begin{array}{c} i\ \overleftrightarrow{\partial}_n^{\gamma} \\ g\,A_n^{\gamma}(y)\end{array}\right\}\psi(-z)|\pi^0(p_1)\rangle = 0\,.$$

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 6)

Light-Cone Collinear Factorization

Minimal set of GPDs

- Number of GPDs: a priori 28 up to twist 5
- Two constraints:
  - QCD equations of motion (EOM)
  - Arbitrariness of p and n

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 6)

Light-Cone Collinear Factorization

Minimal set of GPDs: QCD equations of motion

Dirac equation in a covariant form (no inclusion of mass effects):

$$(i\not{D}\psi)_\alpha = 0 \quad \text{and} \quad (i\not{D}\bar{\psi})_\beta = 0$$

i.e. at correlator level:

$$\langle \pi^0(p_2)|\,(i\not{D}\psi)_\alpha(-z)\,\bar{\psi}_\beta(z)\,|\pi^0(p_1)\rangle = 0$$

and

$$\langle \pi^0(p_2)|\,\psi_\alpha(-z)\,(i\not{D}\bar{\psi})_\beta(z)\,|\pi^0(p_1)\rangle = 0\,.$$

$\Longrightarrow$ relations between various correlators
$\Longrightarrow$ 8 equations between GPDs.

Beyond leading twist : Chiral-odd pion GPDs beyond leading twist (up to 6)

Light-Cone Collinear Factorization

Minimal set of GPDs: $n-$independence

The implementation of $n-$independence is much more difficult here, in comparison with the case of the DAs (because of ξ). Under progress...

Conclusion

- The transversity GPDs are difficult extract
- In order to extract the quark transversity GPDs:
  - At twist 2 one may think of rather involved processes w.r.t. to usual DVCS or vector meson electroproduction, with 3 instead of 2 particles in the final state
  - Another possibility is to consider vector meson electroproduction beyond leading twist
  - This requires to classify the corresponding DAs and GPDs
- For simplicity, we considered the π^0
  - In the light-cone collinear factorization framework, we introduced the relevant matrix element for:
    - 2-partons non-local correlators, with and without transverse and longitudinal derivatives
    - 3-partons non-local correlators
  - Their detailled parametrization is fixed by C, P, T
  - This leads to the introduction of 28 real GPDs
  - Their symmetry properties have been obtained
  - Their reduction to a minimal set requires the use of
    - QCD equations of motions
    - Implementation of the $n-$independence constraint
  - The complete reduction to a minimal set is under process
  - The next stage is to perform the same analysis for the nucleons and to use it for phenomenology

SCHOOL: "Correlations between partons in nucleons"

ORSAY, LPT, June 30th - July 4th

https://indico.in2p3.fr/conferenceDisplay.py?ovw=True&confId=9917

- Long lectures :
  - Marco Stratmann, BNL (USA)
    Partons Distribution Functions and the LHC (6h)
  - Markus Diehl, DESY (Germany)
    Multi Parton Interactions (6h)
  - Cédric Lorcé, IPNO (France) and IFPA Liège (Belgium)
    Nucleon structure (4h)
  - Raju Venugopalan, BNL and Stony Brook University (USA)
    Color Glass Condensate (4h)
  - Leif Lönnblad, Lund Observatory (Sweden)
    Introduction to event generators physics (3h)
  - Abhay Deshpande, Stony Brook University (USA)
    The questions of Hadronic physics (3h)
- Short lectures :
  - Paolo Bartalini, CERN and Central China Normal University (China)
    CMS and ATLAS signals for MPI processes (1.5h)
  - Sarah Porteboeuf-Houssais, LPC Clermont Ferrand (France)
    ALICE signals for MPI processes (1.5h)
  - David Kosower, IPhT (France)
    Introduction to multi-gluons processes (1.5h)