Mueller Navelet jets at LHC: The first complete NLL BFKL study

Samuel Wallon

Université Pierre et Marie Curie
and
Laboratoire de Physique Théorique
CNRS / Université Paris Sud
Orsay

Parallel session "Nuclear matter under extreme conditions and nuclear medium"

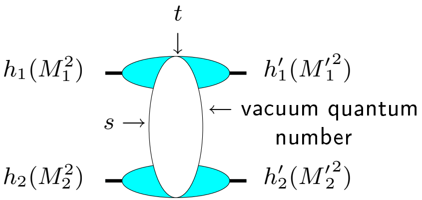
Palaiseau, April 18th 2012

in collaboration with
B. Ducloué (LPT), D. Colferai (Firenze), F. Schwennsen (DESY),
L. Szymanowski (SINS, Varsaw)

D. Colferai; F. Schwennsen, L. Szymanowski, S.W.
JHEP 1012:026 (2010) 1-72 [arXiv:1002.1365 [hep-ph]]

Motivations

- One of the important longstanding theoretical questions raised by QCD is its behaviour in the perturbative Regge limit $s \gg -t$
- Based on theoretical grounds, one should identify and test suitable observables in order to test this peculiar dynamics



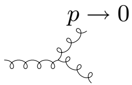
hard scales: $M_1^2,\, M_2^2 \gg \Lambda^2_{QCD}$ or $M'^2_1,\, M'^2_2 \gg \Lambda^2_{QCD}$ or $t \gg \Lambda^2_{QCD}$
where the $t-$channel exchanged state is the so-called hard Pomeron

How to test QCD in the perturbative Regge limit?

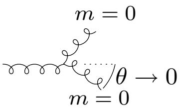
What kind of observable?

- perturbation theory should be applicable:
  selecting external or internal probes with transverse sizes $\ll 1/\Lambda_{QCD}$ (*hard* γ^*, *heavy* meson (J/Ψ, Υ), *energetic* forward jets) or by choosing large t in order to provide the hard scale.

- governed by the *"soft"* perturbative dynamics of QCD



and *not* by its *collinear* dynamics



$\Longrightarrow$ select semi-hard processes with $s \gg p_{T\,i}^2 \gg \Lambda_{QCD}^2$ where $p_{T\,i}^2$ are typical transverse scale, all of the same order.

How to test QCD in the perturbative Regge limit?

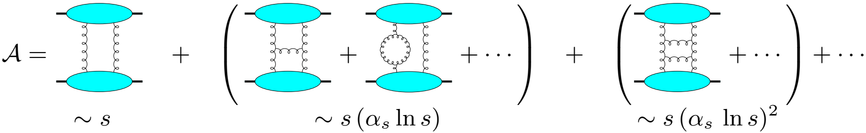
Some examples of processes

- inclusive: DIS (HERA), diffractive DIS, total $\gamma^*\gamma^*$ cross-section (LEP, ILC)
- semi-inclusive: forward jet and π^0 production in DIS, Mueller-Navelet double jets, diffractive double jets, high p_T central jet, in hadron-hadron colliders (Tevatron, LHC)
- exclusive: exclusive meson production in DIS, double diffractive meson production at e^+e^- colliders (ILC), ultraperipheral events at LHC ($\mathbb{P}$omeron, $\mathbb{O}$dderon)

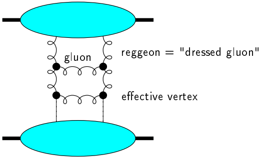
The specific case of QCD at large s

QCD in the perturbative Regge limit

- Small values of α_S (perturbation theory applies due to hard scales) can be compensated by large $\ln s$ enhancements. $\Rightarrow$ resummation of $\sum_n (\alpha_S \ln s)^n$ series (Balitski, Fadin, Kuraev, Lipatov)



- this results in the effective BFKL ladder

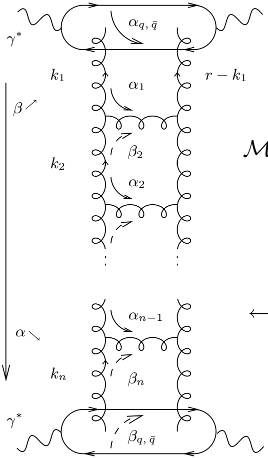


$$\Longrightarrow \sigma_{tot}^{h_1\, h_2 \to anything} = \frac{1}{s} Im\mathcal{A} \sim s^{\alpha_{\mathbb{P}}(0)-1}$$

with $\alpha_{\mathbb{P}}(0) - 1 = C\,\alpha_s \quad (C > 0)$ Leading Log Pomeron
Balitsky, Fadin, Kuraev, Lipatov

Opening the boxes: Impact representation $\gamma^* \gamma^* \to \gamma^* \gamma^*$ as an example

- Sudakov decomposition: $k_i = \alpha_i \, p_1 + \beta_i \, p_2 + k_{\perp i}$ $\quad (p_1^2 = p_2^2 = 0, \; 2p_1 \cdot p_2 = s)$
- write $\quad d^4 k_i = \frac{s}{2} \, d\alpha_i \, d\beta_i \, d^2 k_{\perp i}$ $\quad (\underline{k}$ = Eucl. $\leftrightarrow k_\perp$ = Mink.)
- $t-$channel gluons have non-sense polarizations at large s: $\epsilon_{NS}^{up/down} = \frac{2}{s} \, p_{2/1}$



$\Rightarrow$ set $\alpha_1 = 0$ and $\int d\beta_1 \;\Rightarrow\; \Phi^{\gamma^* \to \gamma^*}(\underline{k}_1, \underline{r} - \underline{k}_1)$

impact factor

$$\mathcal{M} = \frac{is}{(2\pi)^2} \int \frac{d^2 \underline{k}}{\underline{k}^2} \, \Phi^{up}(\underline{k}, \, \underline{r} - \underline{k}) \int \frac{d^2 \underline{k}'}{\underline{k}'^2} \, \Phi^{down}(-\underline{k}', \, -\underline{r} + \underline{k}')$$

$$\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0} \right)^{\omega} G_\omega(\underline{k}, \underline{k}', \underline{r})$$

$\longleftarrow$ multi-Regge kinematics

$\Rightarrow$ set $\beta_n = 0$ and $\int d\alpha_n \;\Rightarrow\; \Phi^{\gamma^* \to \gamma^*}(-\underline{k}_n, -\underline{r} + \underline{k}_n)$

higher order corrections

- Higher order corrections to BFKL kernel are known at NLL order (Lipatov Fadin; Camici, Ciafaloni), now for arbitrary impact parameter
  $\alpha_S \sum_n (\alpha_S \ln s)^n$ resummation
- impact factors are known in some cases at NLL
  - $\gamma^* \to \gamma^*$ at $t = 0$ (Bartels, Colferai, Gieseke, Kyrieleis, Qiao)
  - forward jet production (Bartels, Colferai, Vacca)
  - $\gamma_L^* \to \rho_L$ in the forward limit (Ivanov, Kotsky, Papa)

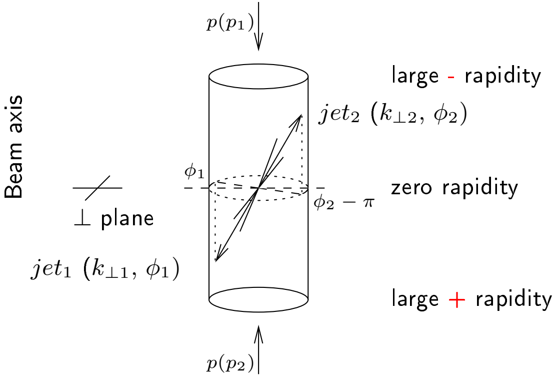
  note: for exclusive processes, some transitions may start at twist 3, for which almost nothing is known
    - The first computation of the $\gamma_T^* \to \rho_T$ twist 3 transition at LL has been performed only recently
      I. V. Anikin, D. Y. Ivanov, B. Pire, L. Szymanowski and S. W.
      Phys. Lett. B 688:154-167, 2010; Nucl. Phys. B 828:1-68, 2010.
    - successfull phenomenological application to H1 and ZEUS data for $\rho-$meson electroproduction
      I. V. Anikin, A. Besse, D. Y. Ivanov, B. Pire, L. Szymanowski and S. W.
      Phys. Rev. D 84 (2011) 054004
    - first dipole model suitable to saturation effects studies at twist 3
      A. Besse, L. Szymanowski and S. W.
      arXiv :1204.2281 [hep-ph]

    see Talk by A. Besse

Mueller-Navelet jets: Basics

Mueller Navelet jets

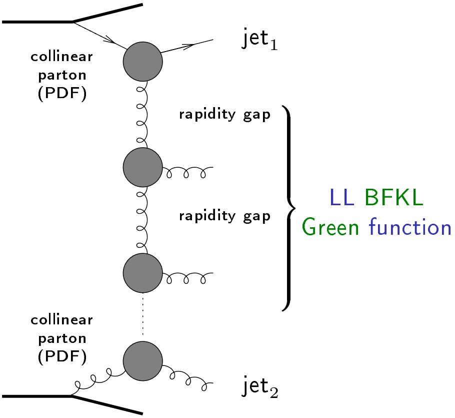
- Consider two jets (hadron paquet within a narrow cone) separated by a large rapidity, i.e. each of them almost fly in the direction of the hadron "close" to it, and with very similar transverse momenta
- in a pure LO collinear treatment, these two jets should be emitted back to back at leading order: $\Delta\phi - \pi = 0$ ($\Delta\phi = \phi_1 - \phi_2 =$ relative azimutal angle) and $k_{\perp 1}$=$k_{\perp 2}$. There is no phase space for (untagged) emission between them



Mueller-Navelet jets at LL fails

Mueller Navelet jets at LL BFKL

- in LL BFKL ($\sim \sum(\alpha_s \ln s)^n$), emission between these jets $\longrightarrow$ strong decorrelation between the relative azimutal angle jets, incompatible with $p\bar{p}$ Tevatron collider data
- a collinear treatment at next-to-leading order (NLO) can describe the data
- important issue: non-conservation of energy-momentum along the BFKL ladder. A LL BFKL-based Monte Carlo combined with e-m conservation improves dramatically the situation (Orr and Stirling)

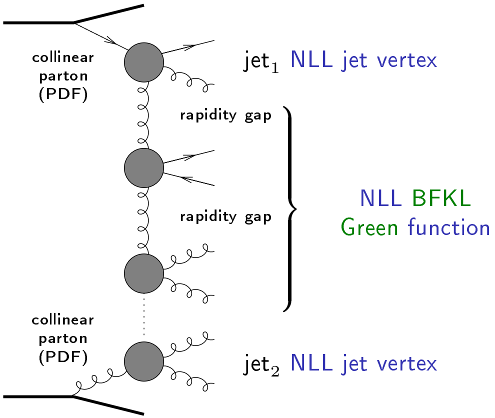


Multi-Regge kinematics (LL BFKL)

Studies at LHC: Mueller-Navelet jets

Mueller Navelet jets at NLL BFKL

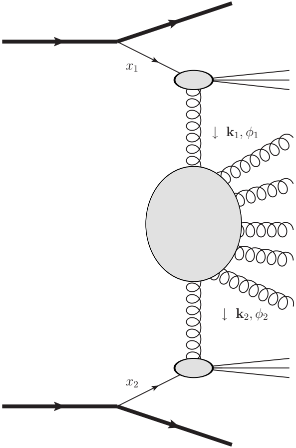
- up to now, the subseries $\alpha_s \sum (\alpha_s \ln s)^n$ NLL was included only in the exchanged $\mathbb{P}$omeron state, and not inside the jet vertices
  Sabio Vera, Schwennsen
  Marquet, Royon
- the common belief was that these corrections should not be important



Quasi Multi-Regge kinematics (here for NLL BFKL)

Master formulas

k_T-factorized differential cross-section



$$\frac{\mathrm{d}\sigma}{\mathrm{d}|\mathbf{k}_{J,1}|\,\mathrm{d}|\mathbf{k}_{J,2}|\,\mathrm{d}y_{J,1}\,\mathrm{d}y_{J,2}} = \int \mathrm{d}\phi_{J,1}\,\mathrm{d}\phi_{J,2} \int \mathrm{d}^2\mathbf{k}_1\,\mathrm{d}^2\mathbf{k}_2$$

$$\times\, \Phi(\mathbf{k}_{J,1}, x_{J,1}, -\mathbf{k}_1)$$

$$\times\, G(\mathbf{k}_1, \mathbf{k}_2, \hat{s})$$

$$\times\, \Phi(\mathbf{k}_{J,2}, x_{J,2}, \mathbf{k}_2)$$

with $\Phi(\mathbf{k}_{J,2}, x_{J,2}, \mathbf{k}_2) = \int \mathrm{d}x_2\, f(x_2) V(\mathbf{k}_2, x_2)$ $\quad f \equiv$ PDF $\quad x_J = \frac{|\mathbf{k}_J|}{\sqrt{s}} e^{y_J}$

Master formulas

Angular coefficients

$$\mathcal{C}_m \equiv \int \mathrm{d}\phi_{J,1}\, \mathrm{d}\phi_{J,2}\, \cos\left(m(\phi_{J,1} - \phi_{J,2} - \pi)\right)$$

$$\times \int \mathrm{d}^2\mathbf{k}_1\, \mathrm{d}^2\mathbf{k}_2\, \Phi(\mathbf{k}_{J,1}, x_{J,1}, -\mathbf{k}_1)\, G(\mathbf{k}_1, \mathbf{k}_2, \hat{s})\, \Phi(\mathbf{k}_{J,2}, x_{J,2}, \mathbf{k}_2).$$

- $m = 0 \implies$ cross-section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}|\mathbf{k}_{J,1}|\, \mathrm{d}|\mathbf{k}_{J,2}|\, \mathrm{d}y_{J,1}\, \mathrm{d}y_{J,2}} = \mathcal{C}_0$$

- $m > 0 \implies$ azimutal decorrelation

$$\langle \cos(m\varphi) \rangle \equiv \langle \cos\left(m(\phi_{J,1} - \phi_{J,2} - \pi)\right) \rangle = \frac{\mathcal{C}_m}{\mathcal{C}_0}$$

Master formulas in conformal variables

Rely on LL BFKL eigenfunctions

- LL BFKL eigenfunctions: $E_{n,\nu}(\mathbf{k}_1) = \frac{1}{\pi\sqrt{2}} \left(\mathbf{k}_1^2\right)^{i\nu-\frac{1}{2}} e^{in\phi_1}$
- decompose Φ on this basis
- use the known LL eigenvalue of the BFKL equation on this basis:

$$\omega(n,\nu) = \bar{\alpha}_s \chi_0 \left(|n|, \tfrac{1}{2} + i\nu\right)$$

with $\chi_0(n,\gamma) = 2\Psi(1) - \Psi\left(\gamma + \frac{n}{2}\right) - \Psi\left(1 - \gamma + \frac{n}{2}\right)$

$(\Psi(x) = \Gamma'(x)/\Gamma(x),\ \bar{\alpha}_s = N_c \alpha_s/\pi)$

- $\Longrightarrow$ master formula:

$$\mathcal{C}_m = (4 - 3\,\delta_{m,0}) \int \mathrm{d}\nu\, C_{m,\nu}(|\mathbf{k}_{J,1}|, x_{J,1})\, C^*_{m,\nu}(|\mathbf{k}_{J,2}|, x_{J,2}) \left(\frac{\hat{s}}{s_0}\right)^{\omega(m,\nu)}$$

with $\quad C_{m,\nu}(|\mathbf{k}_J|, x_J) = \int \mathrm{d}\phi_J\, \mathrm{d}^2\mathbf{k}\, \mathrm{d}x\, f(x) V(\mathbf{k}, x) E_{m,\nu}(\mathbf{k}) \cos(m\phi_J)$

- at NLL, same master formula: just change $\omega(m,\nu)$ and V (although $E_{n,\nu}$ are not anymore eigenfunctions)

Numerical implementation

In practice

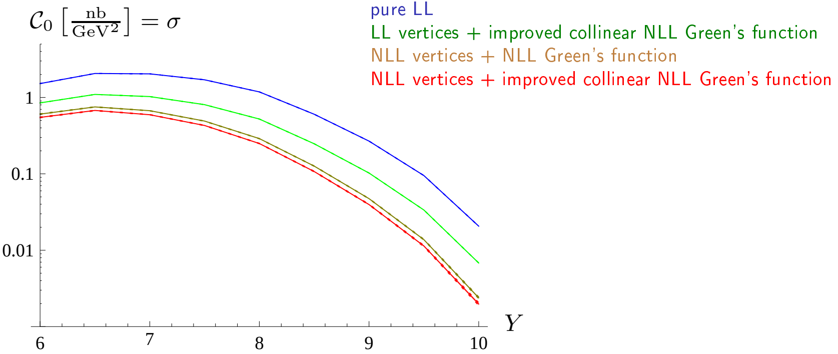
Two codes have been developed:

- a *Mathematica* code, exploratory, but rather slow
  - jet cone-algorithm with $R = 0.5$
  - MSTW 2008 PDFs (available as Mathematica packages)
  - $\mu_R = \mu_F$ (this is imposed by the MSTW 2008 PDFs)
  - two-loop running coupling $\alpha_s(\mu_R^2)$
  - We use a ν grid (with a dense sampling around 0)
  - we use Cuba integration routines (in practice Vegas): precision 10^{-2} for 500.000 max points per integration
  - mapping $|\mathbf{k}| = |\mathbf{k}_J| \tan(\xi\pi/2)$ for $\mathbf{k}$ integrations $\Rightarrow [0, \infty[\rightarrow [0, 1]$
  - although formally the results should be finite, it requires a special grouping of the integrand in order to get stable results

    $\Longrightarrow$ 14 minimal stable basic blocks to be evaluated numerically
- a *Fortran* code, $\simeq 20$ times faster, now available thanks to B. Ducloué
  - Check of our *Mathematica* based results
  - Detailled check of previous mixed studies (NLL Green's function + LL jet vertices)
  - Allows for k_J integration in a finite range
  - Stability studies (PDFs, etc...) made easier
  - A comparison with the recent small R study of D. Yu. Ivanov et al. has been performed

Results: symmetric configuration ($|\mathbf{k}_{J,1}| = |\mathbf{k}_{J,2}| = 35\,\mathrm{GeV}$) $\sqrt{s} = 14$ TeV

Cross-section



Differential cross section in dependence on Y for $|\mathbf{k}_{J,1}| = |\mathbf{k}_{J,2}| = 35\,\mathrm{GeV}$.
error bands=errors due to the Monte Carlo integration (2% to 5%)

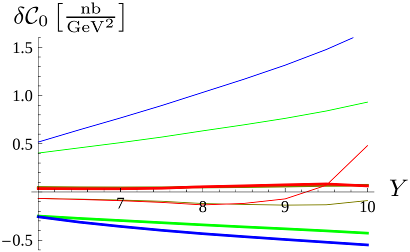
The effect of NLL vertex correction is very sizeable, comparable with NLL Green's function effects

Energy-momentum conservation not satisfied by BFKL-like approaches ⇒ validity restricted to $Y_{J,i} \ll \cosh^{-1} \frac{x_i\, E}{k_{J,i}}$, thus $Y = Y_1 + Y_2 \ll 9.8$ for $x \sim 1/3$

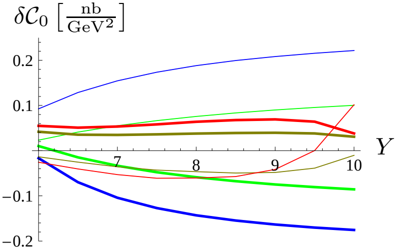
Results: symmetric configuration ($|\mathbf{k}_{J,1}| = |\mathbf{k}_{J,2}| = 35\,\mathrm{GeV}$) $\sqrt{s} = 14$ TeV

Cross-section: stability with respect to $\mu_R = \mu_F$ and s_0 changes

pure LL
LL vertices + improved collinear NLL Green's function
NLL vertices + NLL Green's function
NLL vertices + improved collinear NLL Green's function



Relative effect of changing $\mu_R = \mu_F$ by factors 2 (thick) and $1/2$ (thin)

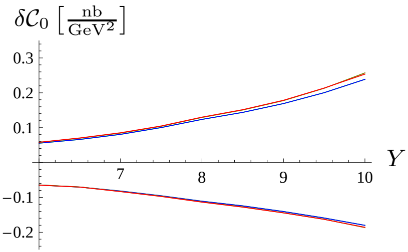


Relative effect of changing $\sqrt{s_0}$ by factors 2 (thick) and $1/2$ (thin)

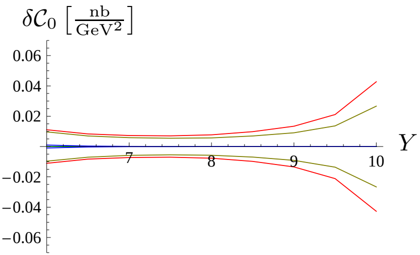
Results: symmetric configuration ($|\mathbf{k}_{J,1}| = |\mathbf{k}_{J,2}| = 35\,\mathrm{GeV}$) $\sqrt{s} = 14$ TeV

Cross-section: PDF and Monte Carlo errors

pure LL
LL vertices + improved collinear NLL Green's function
NLL vertices + NLL Green's function
NLL vertices + improved collinear NLL Green's function



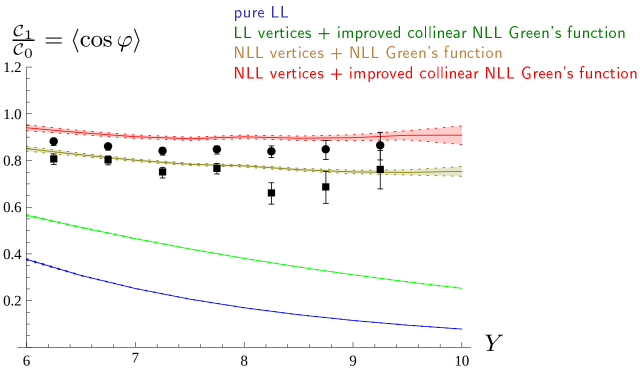
Relative effect of the PDF errors



Relative effect of the Monte Carlo errors

Results: symmetric configuration ($|\mathbf{k}_{J,1}| = |\mathbf{k}_{J,2}| = 35\,\mathrm{GeV}$) $\sqrt{s} = 14$ TeV

Azimuthal correlation

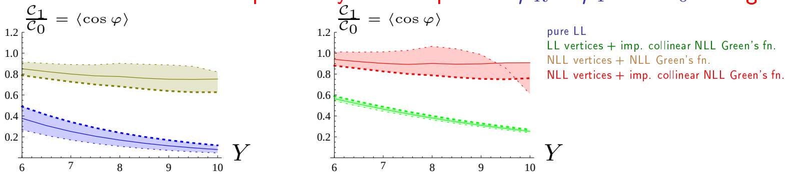


error bands	=	errors due to the Monte Carlo integration
dots	=	results obtained with *Pythia* (DGLAP LL MC)
squares	=	results obtained with *Herwig* (DGLAP LL MC)

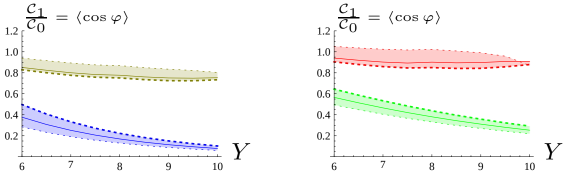
- LL $\longrightarrow$ NLL vertices change results dramatically
- At NLL, the decorrelation is very close to LL DGLAP type of Monte Carlo

Results: symmetric configuration ($|\mathbf{k}_{J,1}| = |\mathbf{k}_{J,2}| = 35\,\mathrm{GeV}$) $\sqrt{s} = 14$ TeV

Azimuthal correlation: dependency with respect to $\mu_R = \mu_F$ and s_0 changes



Effect of changing $\mu_R = \mu_F$ by factors 2 (thick) and $1/2$ (thin)

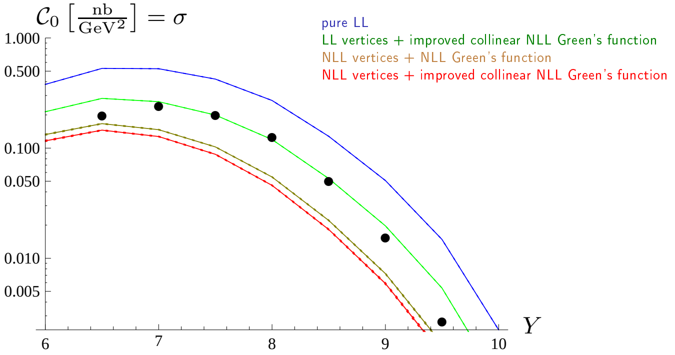


Effect of changing $\sqrt{s_0}$ by factors 2 (thick) and $1/2$ (thin)

- $\langle \cos\varphi \rangle$ is still rather $\mu_R = \mu_F$ and s_0 dependent
- collinear resummation can lead to $\langle \cos\varphi \rangle > 1(!)$ for small $\mu_R = \mu_F$
- based on NLL double-ρ production (Ivanov, Papa) one can expect that small scales are disfavored (Caporale, Papa, Sabio Vera)

Results: asymmetric configuration ($|\mathbf{k}_{J,1}| = 35\,\text{GeV}$, $|\mathbf{k}_{J,2}| = 50\,\text{GeV}$) $\sqrt{s} = 14$ TeV

Cross-section



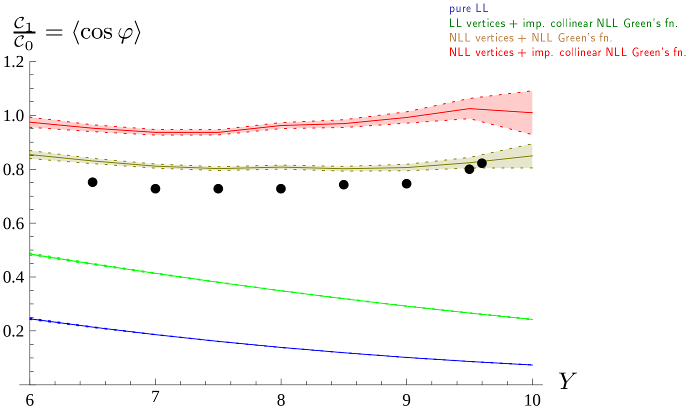
bands = errors due to the Monte Carlo integration

dots = based on the NLO DGLAP parton generator *Dijet* (thanks to M. Fontannaz)

- such an asymmetric configuration is required by DGLAP like approaches, which are unstable for symmetric configurations
- energy-momentum issues in BFKL-like approaches $\Rightarrow Y \ll 9.4$

Results: asymmetric configuration ($|\mathbf{k}_{J,1}| = 35\,\mathrm{GeV}$, $|\mathbf{k}_{J,2}| = 50\,\mathrm{GeV}$) $\sqrt{s} = 14$ TeV

Azimuthal correlation: $\langle \cos \varphi \rangle$



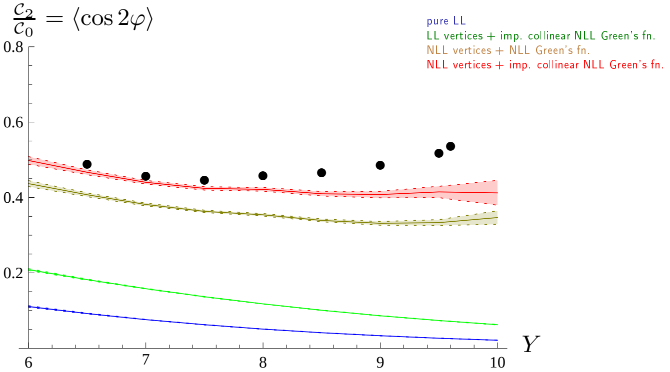
bands = errors due to the Monte Carlo integration

dots = based on the NLO DGLAP parton generator *Dijet* (thanks to M. Fontannaz)

- Both NLL and improved NLL results are almost flat in Y
- no significant difference between NLL BFKL and NLO DGLAP

Results: asymmetric configuration ($|\mathbf{k}_{J,1}| = 35\,\mathrm{GeV},\ |\mathbf{k}_{J,2}| = 50\,\mathrm{GeV}$) $\sqrt{s} = 14$ TeV

Azimuthal correlation: $\langle\cos 2\varphi\rangle$



bands = errors due to the Monte Carlo integration

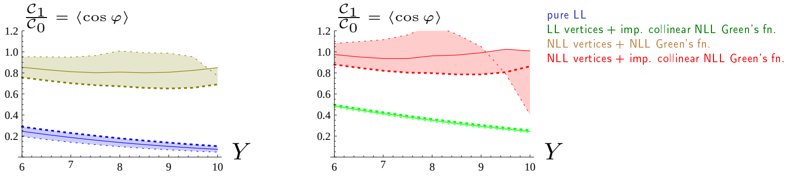
dots = based on the NLO DGLAP parton generator *Dijet* (thanks to M. Fontannaz)

Same conclusions:

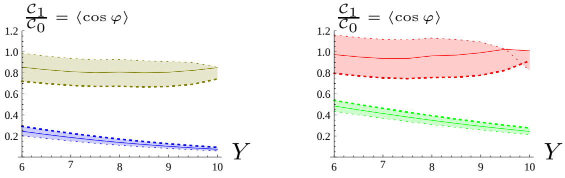
- Both NLL and improved NLL results are almost flat in Y
- no significant difference between NLL BFKL and NLO DGLAP

Results: asymmetric configuration ($|\mathbf{k}_{J,1}| = 35\,\mathrm{GeV}$, $|\mathbf{k}_{J,2}| = 50\,\mathrm{GeV}$) $\sqrt{s} = 14$ TeV

Azimuthal correlation: dependency with respect to $\mu_R = \mu_F$ and s_0 changes



Effect of changing $\mu_R = \mu_F$ by factors 2 (thick) and $1/2$ (thin)



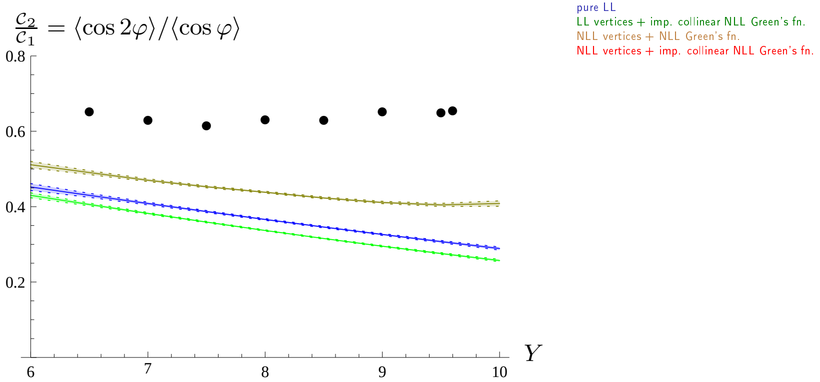
Effect of changing $\sqrt{s_0}$ by factors 2 (thick) and $1/2$ (thin)

Again:

- $\langle \cos\varphi \rangle$ is still rather $\mu_R = \mu_F$ and s_0 dependent
- collinear resummation can lead to $\langle \cos\varphi \rangle > 1(!)$ for small $\mu_R = \mu_F$

Results: asym. config. ($|\mathbf{k}_{J,1}| = 35\,\mathrm{GeV}$, $|\mathbf{k}_{J,2}| = 50\,\mathrm{GeV}$) $\sqrt{s} = 14$ TeV

Ratio of azimuthal correlations $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$



bands = errors due to the Monte Carlo integration
dots = based on the NLO DGLAP parton generator Dijet (thanks to Fontannaz)
NB: NLL collinear improved changed nothing wrt pure NLL

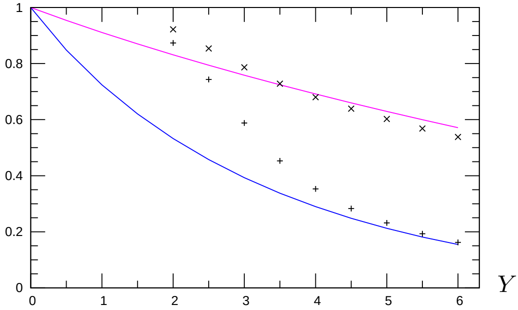
This is the only observable which might still differ noticeably between NLL BFKL and NLO DGLAP scenarii

Results: integrated k_J at Tevatron ($|\mathbf{k}_{J,1}| > 20\,\mathrm{GeV}$, $|\mathbf{k}_{J,2}| > 50\,\mathrm{GeV}$) $\sqrt{s} = 1.8$ TeV

Comparison in the simplified NLL Green's function + LL jet vertices scenario

- The integration $\int_{k_{J\,min}}^{\infty} dk_J$ can be performed analytically
- A comparison with the numerical integration based on code provides a good test of stability, valid for large Y

$$\frac{\mathcal{C}_1}{\mathcal{C}_0} = \langle \cos\varphi \rangle$$



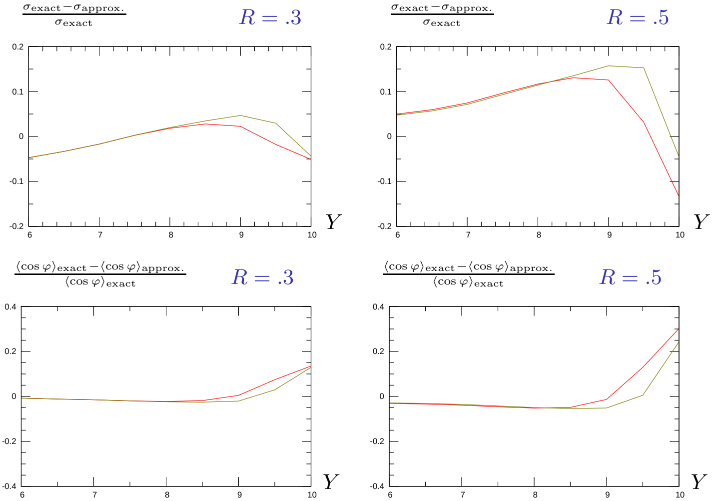
blue: LL

magenta: NLL Green's function + LL jet vertices scenario Sabio Vera, Schwennsen

$\times$: numerical dk_J integration $k_{J,1} > 20$ GeV and $k_{J,2} > 50$ GeV

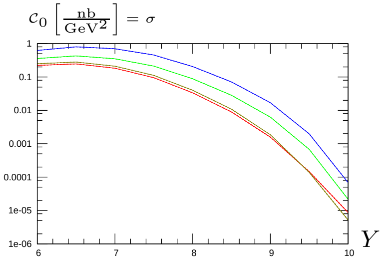
Results: asym. config. $(|\mathbf{k}_{J,1}| = 35\,\mathrm{GeV},\ |\mathbf{k}_{J,2}| = 50\,\mathrm{GeV})$ $\sqrt{s} = 7$ TeV

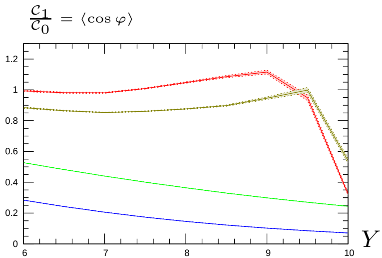
Comparison between the exact R and approximated small R treatments



Note: $Y \ll 8$ for BFKL validity (e-m conservation issues)

What to do at LHC?





$k_{J,1} = 30$ GeV and $k_{J,1} = 35$ GeV: BFKL applicable for $Y \ll 8.6$, for $x_i \sim 1/3$

- Having access to low k_J enlarges the BFKL domain: $Y_{J,i} \ll \cosh^{-1} \frac{x_i E}{k_{J,i}}$
- a measurement at CMS down to $k_{J,1} = 25$ GeV and $k_{J,1} = 30$ GeV seems feasible: BFKL applicable for $Y \ll 8.9$
- note that:
  - k_J integration reduces the Y domain between jets
  - x_i integration weighted by PDFs reduces the Y domain between jets
- Using LHC data for different $\sqrt{s}$ values: get rid of PDFs $\Rightarrow$ direct access to Pomeron trajectory
- Implementing energy-momentum conservation: direct iteration of NLL BFKL kernel?

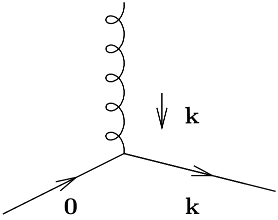
Conclusion

- We have performed the first complete NLL analysis of Mueller-Navelet jets
- The effect of NLL jets corrections have is dramatic, similar to the NLL Green function corrections
- For the cross-section:
  - makes prediction much more stable with respect to variation of parameters (factorization scale, scale s_0 entering the rapidity definition, PDF)
  - close to NLO DGLAP (although surprisingly a bit below!)
- Surprisingly small decorrelation effect:
  - very close to NLO DGLAP
  - very flat in rapidity Y
  - still rather dependent on these parameters
- Pure NLL BFKL and collinear improved NLL BFKL lead to similar results
- Collinear improved NLL BFKL faces some puzzling behaviour for the azimuthal correlation
- except for $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$, the difference between NLL BFKL and NLO DGLAP based observables is quite small
- Mueller Navelet jets are thus probably a much more complicate observable to see the perturbative Regge effect of QCD than expected
- Energy-momentum conservation could modify the picture, in particular for maximal values of Y

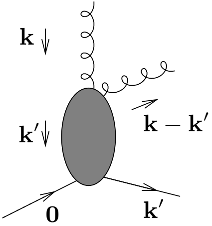
Jet vertex: LL versus NLL

$\mathbf{k}, \mathbf{k}' =$ Euclidian two dimensional vectors

LL jet vertex:



NLL jet vertex:



Jet vertex: jet algorithms

Jet algorithms

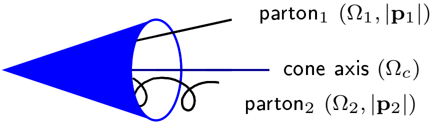
- a jet algorithm should be IR safe, both for soft and collinear singularities
- the most common jet algorithm are:
  - k_t algorithms (IR safe but time consuming for multiple jets configurations)
  - cone algorithm (not IR safe in general; can be made IR safe at NLO: Ellis, Kunszt, Soper)

Jet vertex: jet algorithms

Cone jet algorithm at NLO (Ellis, Kunszt, Soper)

- Should partons $(|\mathbf{p}_1|, \phi_1, y_1)$ and $(\mathbf{p}_2|, \phi_2, y_2)$ combined in a single jet? $|\mathbf{p}_i|$ =transverse energy deposit in the calorimeter cell i of parameter $\Omega = (y_i, \phi_i)$ in $y - \phi$ plane
- define transverse energy of the jet: $p_J = |\mathbf{p}_1| + |\mathbf{p}_2|$
- jet axis:

$$\Omega_c \begin{cases} y_J = \dfrac{|\mathbf{p}_1|\, y_1 + |\mathbf{p}_2|\, y_2}{p_J} \\ \\ \phi_J = \dfrac{|\mathbf{p}_1|\, \phi_1 + |\mathbf{p}_2|\, \phi_2}{p_J} \end{cases}$$



$\Omega = (y_i, \phi_i)$ in $y - \phi$ plane

If distances $|\Omega_i - \Omega_c|^2 \equiv (y_i - y_c)^2 + (\phi_i - \phi_c)^2 < R^2$ $(i = 1$ and $i = 2)$

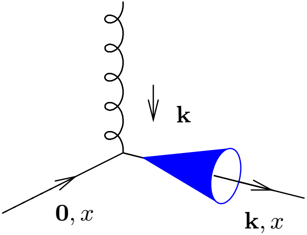
$\Longrightarrow$ partons 1 and 2 are in the same cone Ω_c

combined condition: $|\Omega_1 - \Omega_2| < \dfrac{|\mathbf{p}_1| + |\mathbf{p}_2|}{max(|\mathbf{p}_1|, |\mathbf{p}_2|)} R$

Jet vertex: LL versus NLL and jet algorithms

LL jet vertex and cone algorithm

$\mathbf{k}, \mathbf{k}'$ = Euclidian two dimensional vectors



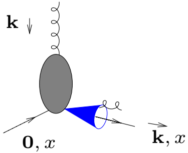
$$\mathcal{S}_J^{(2)}(k_\perp; x) = \delta\left(1 - \frac{x_J}{x}\right) |\mathbf{k}|\, \delta^{(2)}(\mathbf{k} - \mathbf{k}_J)$$

Jet vertex: LL versus NLL and jet algorithms

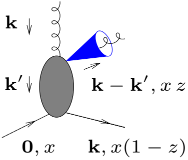
NLL jet vertex and cone algorithm

$\mathbf{k}, \mathbf{k}'$ = Euclidian two dimensional vectors

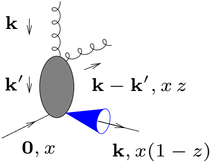
$$\mathcal{S}_J^{(3,\text{cone})}(\mathbf{k}', \mathbf{k}-\mathbf{k}', xz; x) =$$



$$\mathcal{S}_J^{(2)}(\mathbf{k}, x)\, \Theta\left(\left[\frac{|\mathbf{k}-\mathbf{k}'|+|\mathbf{k}'|}{\max(|\mathbf{k}-\mathbf{k}'|,|\mathbf{k}'|)} R_{\text{cone}}\right]^2 - [\Delta y^2 + \Delta\phi^2]\right)$$



$$+\, \mathcal{S}_J^{(2)}(\mathbf{k}-\mathbf{k}', xz)\, \Theta\left([\Delta y^2 + \Delta\phi^2] - \left[\frac{|\mathbf{k}-\mathbf{k}'|+|\mathbf{k}'|}{\max(|\mathbf{k}-\mathbf{k}'|,|\mathbf{k}'|)} R_{\text{cone}}\right]^2\right)$$



$$+\, \mathcal{S}_J^{(2)}(\mathbf{k}', x(1-z))\, \Theta\left([\Delta y^2 + \Delta\phi^2] - \left[\frac{|\mathbf{k}-\mathbf{k}'|+|\mathbf{k}'|}{\max(|\mathbf{k}-\mathbf{k}'|,|\mathbf{k}'|)} R_{\text{cone}}\right]^2\right),$$

Mueller-Navelet jets at NLL and finiteness

Using a IR safe jet algorithm, Mueller-Navelet jets at NLL are finite

- UV sector:
  - the NLL impact factor contains UV divergencies $1/\epsilon$
  - they are absorbed by the renormalization of the coupling: $\alpha_S \longrightarrow \alpha_S(\mu_R)$
- IR sector:
  - PDF have IR collinear singularities: pole $1/\epsilon$ at LO
  - these collinear singularities can be compensated by collinear singularities of the two jets vertices and the real part of the BFKL kernel
  - the remaining collinear singularities compensate exactly among themselves
  - soft singularities of the real and virtual BFKL kernel, and of the jets vertices compensates among themselves

This was shown for both quark and gluon initiated vertices (Bartels, Colferai, Vacca)

BFKL Green's function at NLL

NLL Green's function: rely on LL BFKL eigenfunctions

- NLL BFKLkernel is not conformal invariant
- LL $E_{n,\nu}$ are not anymore eigenfunction
- this can be overcome by considering the eigenvalue as an operator with a part containing $\frac{\partial}{\partial\nu}$
- it acts on the impact factor

$$\omega(n,\nu) = \bar{\alpha}_s \chi_0\left(|n|, \frac{1}{2}+i\nu\right) + \bar{\alpha}_s^2\left[\chi_1\left(|n|, \frac{1}{2}+i\nu\right)\right.$$

$$\left. -\frac{\pi b_0}{2N_c}\chi_0\left(|n|, \frac{1}{2}+i\nu\right)\underbrace{\left\{-2\ln\mu_R^2 - i\frac{\partial}{\partial\nu}\ln\frac{C_{n,\nu}(|\mathbf{k}_{J,1}|, x_{J,1})}{C_{n,\nu}(|\mathbf{k}_{J,2}|, x_{J,2})}\right\}}_{2\ln\frac{|\mathbf{k}_{J,1}|\cdot|\mathbf{k}_{J,2}|}{\mu_R^2}}\right],$$

LL substraction and s_0

- one sums up $\sum(\alpha_s \ln \hat{s}/s_0)^n + \alpha_s \sum(\alpha_s \ln \hat{s}/s_0)^n \quad (\hat{s} = x_1\, x_2\, s)$
- at LL s_0 is arbitrary
- natural choice: $s_0 = \sqrt{s_{0,1}\, s_{0,2}}$ $\quad s_{0,i}$ for each of the scattering objects
  - possible choice: $s_{0,i} = (|\mathbf{k}_J| + |\mathbf{k}_J - \mathbf{k}|)^2$ (Bartels, Colferai, Vacca)
    - but depend on $\mathbf{k}$, which is integrated over
    - $\hat{s}$ is not an external scale ($x_{1,2}$ are integrated over)
  - we prefer

$$\left.\begin{array}{l} s_{0,1} = (|\mathbf{k}_{J,1}| + |\mathbf{k}_{J,1} - \mathbf{k}_1|)^2 \;\rightarrow\; s'_{0,1} = \dfrac{x_1^2}{x_{J,1}^2}\mathbf{k}_{J,1}^2 \\[2ex] s_{0,2} = (|\mathbf{k}_{J,2}| + |\mathbf{k}_{J,2} - \mathbf{k}_2|)^2 \;\rightarrow\; s'_{0,2} = \dfrac{x_2^2}{x_{J,2}^2}\mathbf{k}_{J,2}^2 \end{array}\right\} \quad \frac{\hat{s}}{s_0} \;\rightarrow\; \frac{\hat{s}}{s'_0} = \frac{x_{J,1}\, x_{J_2}\, s}{|\mathbf{k}_{J,1}|\,|\mathbf{k}_{J,2}|} = e^{y_{J,1} - y_{J,2}} \equiv e^Y$$

- $s_0 \rightarrow s'_0$ affects
  - the BFKL NLL Green function
  - the impact factors:

$$\Phi_{\rm NLL}(\mathbf{k}_i; s'_{0,i}) = \Phi_{\rm NLL}(\mathbf{k}_i; s_{0,i}) + \int \mathrm{d}^2\mathbf{k}'\, \Phi_{\rm LL}(\mathbf{k}'_i)\, \mathcal{K}_{\rm LL}(\mathbf{k}'_i, \mathbf{k}_i) \frac{1}{2} \ln \frac{s'_{0,i}}{s_{0,i}} \qquad (1)$$

- numerical stabilities (non azimuthal averaging of LL substraction) improved with the choice $s_{0,i} = (\mathbf{k}_i - 2\mathbf{k}_{J,i})^2$
  (then replaced by $s'_{0,i}$ after numerical integration)
- (1) can be used to test $s_0 \rightarrow \lambda\, s_0$ dependence

Collinear improvement at NLL

Collinear improved Green's function at NLL

- one may improve the NLL BFKLkernel for $n = 0$ by imposing its compatibility with DGLAP in the collinear limit
  Salam; Ciafaloni, Colferai
- usual (anti)collinear poles in $\gamma = 1/2 + i\nu$ (resp. $1 - \gamma$) are shifted by $\omega/2$
- one practical implementation:
  - the new kernel $\bar{\alpha}_s \chi^{(1)}(\gamma, \omega)$ with shifted poles replaces

    $\bar{\alpha}_s \chi_0(\gamma, 0) + \bar{\alpha}_s^2 \chi_1(\gamma, 0)$
  - $\omega(0, \nu)$ is obtained by solving the implicit equation

    $$\omega(0, \nu) = \bar{\alpha}_s \chi^{(1)}(\gamma, \omega(0, \nu))$$

    for $\omega(n, \nu)$ numerically.
- there is no need for any jet vertex improvement because of the absence of γ and $1 - \gamma$ poles (numerical proof using Cauchy theorem "backward")

Numerical implementation

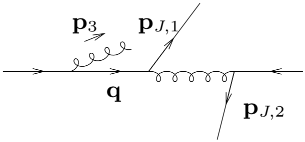
In practice

- MSTW 2008 PDFs (available as Mathematica packages)
- $\mu_R = \mu_F$ (this is imposed by the MSTW 2008 PDFs)
- two-loop running coupling $\alpha_s(\mu_R^2)$
- We use a ν grid (with a dense sampling around 0)
- all numerical calculations are done in Mathematica
- we use Cuba integration routines (in practice Vegas): precision 10^{-2} for 500.000 max points per integration
- mapping $|\mathbf{k}| = |\mathbf{k}_J| \tan(\xi\pi/2)$ for $\mathbf{k}$ integrations $\Rightarrow [0, \infty[\rightarrow [0, 1]$
- although formally the results should be finite, it requires a special grouping of the integrand in order to get stable results

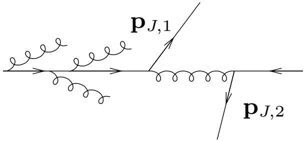
$\Longrightarrow$ 14 minimal stable basic blocks to be evaluated numerically

Motivation for asymmetric configurations

- Initial state radiation (unseen) produces divergencies if one touches the collinear singularity $\mathbf{q}^2 \to 0$

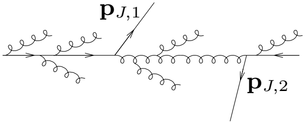


- they are compensated by virtual corrections
- this compensation is in practice difficult to implement when for some reason this additional emission is in a "corner" of the phase space (dip in the differential cross-section)
- this is the case when $\mathbf{p}_1 + \mathbf{p}_2 \to 0$
- this calls for a resummation of large remaing logs $\Rightarrow$ Sudakov resummation



Motivation for asymmetric configurations

- since these resummation have never been investigated in this context, one should better avoid that region
- note that for BFKL, due to additional emission between the two jets, one may expect a less severe problem (at least a smearing in the dip region $|\mathbf{p}_1| \sim |\mathbf{p}_2|$)



- this may however not mean that the region $|\mathbf{p}_1| \sim |\mathbf{p}_2|$ is perfectly trustable even in a BFKL type of treatment
- we now investigate a region where NLL DGLAP is under control