

# Virtual Compton Scattering in AdS/QCD

Samuel Wallon

Laboratoire de Physique Théorique  
Orsay

GDR Nucléon, 4ème Assemblée Plénière  
Saclay, SPhN, 26 novembre 2010

in collaboration with

Cyrille Marquet (CERN) and Claude Roiesnel (CPHT-X, Palaiseau)

JHEP04 (2010) 051 [arXiv:1002.0566[hep-ph]]

## 1 Introduction

## 2 Gauge/Gravity duality

- The duality
- AdS/QCD and Conformal breaking
- Soft-wall holographic model
- Correlation functions
- Holographic Compton amplitude

## 3 Compton Scattering

- Compton amplitude off an unpolarized target
- Relation with Deep Inelastic Scattering
- VCS amplitude
- Polarizabilities

# Introduction

## AdS/CFT versus AdS/QCD

- AdS/CFT correspondence:
  - strongly coupled 4D conformal field theory  
Conformal invariance group:  $SO(4, 2)$
  - weakly coupled 5D string theory  
weak coupling  $\longrightarrow$  classical metric background Anti-deSitter isometry group:  
 $SO(4, 2)$
- QCD not conformally invariant (dynamical mass scale) but
  - conformally invariant classical lagrangian
  - asymptotic freedom but
    - at high-energy stringy corrections are expected
    - semi-classical approximation no longer valid
- Does the AdS/CFT dictionary apply to QCD?

# The AdS/CFT correspondence

## String ↔ CFT correspondence

- correspondence between a IIB string theory compactified on  $AdS_5 \times S^5$  and a  $\mathcal{N} = 4$  super-Yang-Mills theory
- Field ↔ Operator correspondence: ( $\vec{x}$ =4d-vector,  $z$ =holographic direction)  
 $\mathcal{O}(\vec{x})$ : operator in CFT to create asymptotic states  
 $\phi_0(\vec{x})$ : source for that operator

### Duality:

- $\phi_0(\vec{x})$  = boundary value of the bulk field  $\phi(\vec{x}, z)$  (up to dimensionful prefactors)
- both generating function in CFT and AdS are related through:

$$\mathcal{Z}_{CFT}[\phi_0] = \langle e^{\int d^4x \phi_0(\vec{x}) \mathcal{O}(\vec{x})} \rangle_{CFT} = \mathcal{Z}_{string} \left[ \phi(\vec{x}, z) \Big|_{z=0} = \phi_0(\vec{x}) \right]$$

Maldacena; Witten; Gubser, Klebanov, Polyakov

# The AdS/CFT correspondence

## More on the $AdS_5 \times S^5$ dual string theory

- 5d metric with Lorentz group  $SO(1, 3)$  as isometry subgroup:

$$ds^2 = a^2(z) \left( dx_\mu dx^\mu + dz^2 \right), \quad g^{mn} = g_{mn}^{-1} = a^{-2}(z) \eta^{mn},$$

$$\eta = \text{diag}(-1, 1, 1, 1, 1), \quad m = 0, 1, 2, 3, 4.$$

- $AdS_5$  metric:  $a(z) = R/z$  (negative curvature, corresponding to a negative cosmological constant)  $\sqrt{-g} = R^5/z^5$  ( $R = 1$  from now on)
- UV limit  $z \rightarrow z_{min}$  : CFT boundary
- field content:
  - 5d vector  $U(1)$  field  $A_m(x, z)$ , dual to the electromagnetic current
  - 5d massive scalar field  $\Phi(z, x)$  dual to an operator creating the spinless target

# The AdS/CFT correspondence

## SUGRA ↔ CFT correspondence

In the large  $N$  and large 't Hooft coupling  $\lambda = g_{YM}^2 N$  limit, the IIB string theory reduces to a IIB SUGRA theory in 10 dimensions:

$$\frac{R^4}{l_s^4} \sim g_s N \sim g_{YM}^2 N \gg 1$$

$R$  :  $AdS_5$  and  $S^5$  radius

$l_s$  : string length

$g_s \ll 1$  and  $R \gg l_s$ : string theory → classical SUGRA  
(with decoupling of massive string excitations)

# From AdS/CFT to AdS/QCD correspondence

## Conformal invariance breaking due to confinement

- **Hard-wall model:** IR cut-off for  $z = 1/\Lambda \Rightarrow$  mass-gap  
Polchinski, Strassler
- **Soft-wall model:** the bulk field  $\Phi(z, x)$  is coupled to a background dilaton field  $\chi(z)$  (with the UV constraint that  $\chi(z) \rightarrow 0$  for  $z \rightarrow 0$ )  
 $\Rightarrow$  **Regge** trajectories  
Karch, Katz, Son, Stephanov

# The soft-wall model: two-point function from holographic principle

## Action in $AdS_5$

- action for the propagation of  $\Phi$  in this background:

$$S_\Phi = \frac{1}{2} \int d^4x dz \sqrt{-g} e^{-\chi} \left( g^{ij} \partial_i \Phi \partial_j \Phi + m_S^2 \Phi^2 \right) \quad (1)$$

- classical field equation for this free field  $\Phi^{(0)}$ :

$$z^2 \square \Phi^{(0)} + z^5 e^\chi \partial_z \left( z^{-3} e^{-\chi} \partial_z \Phi^{(0)} \right) = m_S^2 \Phi^{(0)}. \quad (2)$$

- looking for plane-wave solution in 4d space:

$$\Phi^{(0)}(x, z) = e^{ip \cdot x} \widehat{\Phi}(z)$$

- complete set of **Kaluza-Klein** solutions  $\{\widehat{\Phi}_n(z), n \in \mathbb{N}\}$  with

$$\langle \widehat{\Phi} | \widehat{\Phi}' \rangle = \int_0^\infty dz z^{-3} e^{-\chi(z)} \widehat{\Phi}^*(z) \widehat{\Phi}'(z)$$

- Green function:  $(\Delta_g - m_S^2) G(x, z; x', z') = \frac{e^\chi}{\sqrt{-g}} \delta^4(x - x') \delta(z - z')$

- Fourier transform: bulk to bulk propagator = **Källén-Lehman** spectral representation when summing over the **KK** excitations:

$$\widehat{G}(z, z'; p) = - \sum_{n=0}^{\infty} \frac{\widehat{\Phi}_n^*(z) \widehat{\Phi}_n(z')}{p^2 + m_n^2 - i\epsilon}.$$



# The soft-wall model: two-point function from holographic principle

## QCD two-point function

- Action of a classical solution leaves only the boundary term ( $a(z) = 1/z$ ):

$$S_{\text{cl}}[\Phi] = \frac{1}{2} \int_{z=z_{\text{min}}} d^4x \sqrt{-g} g^{ij} \Phi \partial_j \Phi = \frac{1}{2} a^3(z_{\text{min}}) \int d^4x \Phi(x, z_{\text{min}}) \left. \frac{\partial \Phi(x, z)}{\partial z} \right|_{z=z_{\text{min}}}.$$

- Green function relate bulk values of non-normalizable classical solution  $\Phi(x, z)$  to boundary values  $\Phi_S(x') \equiv \Phi(x', z_{\text{min}})$ :

$$\Phi(x, z) = \int d^4x' G_0(x, z; x', z_{\text{min}}) \Phi_S(x').$$

- Classical partition function:

$$Z_{5D}[\Phi_S] = \int_{\Phi(x, z_{\text{min}}) = \Phi_S(x)} e^{iS[\Phi]} \propto e^{iS_{\text{cl}}[\Phi_S]}.$$

- $\Rightarrow$  Holographic identification of two-point functions from AdS to CFT:

$$\begin{aligned} \langle O(x) O(x') \rangle &= \frac{1}{Z[\Phi_S]} \frac{\partial^2 Z[\Phi_S]}{\partial \Phi_S(x) \partial \Phi_S(x')} \approx a^3(z) \left. \frac{\partial}{\partial z} G(x-x'; z, z_{\text{min}}) \right|_{z=z_{\text{min}}} \\ &= \frac{1}{(2\pi)^4} \int_{-\infty}^{+\infty} d^4p e^{ip(x'-x)} \sum_{n=0}^{\infty} a^3(z_{\text{min}}) \frac{\partial_z \phi_n^*(z_{\text{min}}) \phi_n(z_{\text{min}})}{p^2 + m_n^2 - i\epsilon} \end{aligned}$$

# Calculation of $n$ -point functions in AdS/QCD

## AdS $\leftrightarrow$ QCD

- interaction between bulk  $\Phi$  and bulk  $U(1)$  fields:

$$S_{\text{AdS}}[\Phi, \Phi^*, A^m] = \int d^4x dz \sqrt{-g} \left( -\frac{1}{4} F^{mn} F_{mn} + e^{-\chi} \left( (D^m \Phi)^* D_m \Phi + m_S^2 \Phi^* \Phi \right) \right)$$

(no dilaton for  $A_m(x, z)$  : would break electromagnetic conformal invariance)

- rely on AdS/QCD correspondence:

$$\begin{aligned} Z_{\text{QCD}}(c, \bar{c}, n + \bar{n}) &= \left\langle \exp \left( \int d^4x (n_\mu + \bar{n}_\mu) J^\mu + \bar{c} O + c O^\dagger \right) \right\rangle_{\text{QCD}} \\ &= \exp \left( -S_{\text{AdS}}^{cl}[\Phi(c), \Phi^*(\bar{c}), A^m(n_\mu + \bar{n}_\mu)] \right) \end{aligned}$$

- $c, \bar{c}, n + \bar{n}$  : 4d sources for the CFT which appear as boundary conditions for the 5d *non-normalizable* bulk fields
- Correlation functions of CFT operators can be obtained by expanding to linear-order with respect to the sources
- QCD operators are coupled to asymptotic states which are boundary conditions of *normalizable* bulk fields  $\Phi^{(0)}, \Phi^{*(0)}$  (scalar probe)

Calculation of  $n$ -point functions in AdS/QCD

## Electromagnetic current

- the electromagnetic current is dual to a massless *non-normalizable* 5d vector field with a  $U(1)$  gauge invariance,  $A_m(x, z)$

- Maxwell equations:

$$\partial^m (z^{-1} F_{mn}) = 0, \quad \forall n$$

- we are looking for general plane-wave solutions:

$$A_\mu(x, z) = n_\mu e^{iq \cdot x} A(z), \quad A_z(x, z) = e^{iq \cdot x} A_0(z), \quad n^2 = 1$$

- in the Lorentz-like gauge condition

$$\partial^\mu A_\mu + z \partial_z (z^{-1} A_z) = 0$$

Maxwell equations reduce to

$$q^2 A(z) = z \partial_z (z^{-1} \partial_z A(z))$$

$$A_0(z) = -i \frac{q \cdot n}{q^2} \partial_z A(z)$$

- Boundary condition:  $\lim_{x \rightarrow 0} A(z) = 1$

$$\implies A(z) = Q z K_1(Qz) \quad (Q^2 = q \cdot q)$$

## 3-point functions in AdS/QCD

### 3-point function: application to DIS

- three-point correlation functions can be computed and applied for describing DIS

$$\text{Im } T^{\mu\nu} \sim \sum_X \delta(M_X^2 + (P + q)^2) \langle P | J^\nu(0) | P + q, X \rangle \langle P + q, X | J^\mu(0) | P \rangle$$

Polchinski, Strassler

Problem when trying to map with a partonic description:

Callan-Gross + Bjorken scaling not fulfilled

- extension for intermediate **vector states**  $X$  for a scalar probe
  - Pire, Roiesnel, Szymanowski, S.W. Phys.Lett.B670 (2008) 84
  - scalar + vector state: should in principle improve the partonic description through a parton-hadron duality
  - Callan-Gross relation requires a fine tuning of the 5d coupling
  - Bjorken scaling can be achieved with an unnatural conformal dimension  $\Delta = 1$  for the bulk field

## 4-point functions in AdS/QCD: Holographic Compton amplitude

### 4-point function: application to Compton scattering

- our aim is now to study the **Compton** scattering  $\gamma^{(*)}A \rightarrow \gamma^{(*)}A'$  on a spinless, or spin-averaged target
- this requires to study a 4-point function, this is the first non-trivial correlator involving the propagation inside the bulk
- using

$$\begin{aligned}
 Z_{\text{QCD}}(c, \bar{c}, n + \bar{n}) &= \left\langle \exp \left( \int d^4x (n_\mu + \bar{n}_\mu) J^\mu + \bar{c} O + c O^\dagger \right) \right\rangle_{\text{QCD}} \\
 &= \exp \left( -S_{\text{AdS}}^{cl}[\Phi(c), \Phi^*(\bar{c}), A^m(n_\mu + \bar{n}_\mu)] \right)
 \end{aligned}$$

one should thus extract the coefficient of  $\bar{c} n_\mu \bar{n}_\nu c$

Gao, Xiao; Marquet, Roiesnel, S.W.

## 4-point functions in AdS/QCD: Holographic Compton amplitude

## The Compton amplitude from holographic principle

- Solve iteratively the coupled classical equations, in terms of the free *non-normalizable* bulk fields  $A^{(0)}$ ,  $\Phi^{*(0)}$ ,  $\Phi^{(0)}$
- Deduce the classical action, up to  $e^2$   
the notation  $^{(0)}$  is now dropped for clarity;  $y \equiv (x, z)$  and  $dy \equiv d^4x dz$

$$S_{int}^{cl} = ie \int dy \sqrt{-g} e^{-\chi} A^m (\Phi^* \partial_m \Phi - \Phi \partial_m \Phi^*) + e^2 \int dy \sqrt{-g} e^{-\chi} A^m A_m \Phi^* \Phi$$

$$+ e^2 \int dy dy' \sqrt{-g} e^{-\chi} \sqrt{-g'} e^{-\chi'} A^m(y) A^n(y')$$

$$\left\{ \left( \Phi^*(y) \partial_m - (\partial_m \Phi^*(y)) \right) \left( \Phi(y') \partial'_n - (\partial'_n \Phi(y')) \right) \right.$$

$$\left. + \left( \Phi(y) \partial_m - (\partial_m \Phi(y)) \right) \left( \Phi^*(y') \partial'_n - (\partial'_n \Phi^*(y')) \right) \right\} G_0(y; y') + (e^3)$$

- boundary condition at  $z = 0$  for  $A^{(0)}$ ,  $\Phi^{*(0)}$ ,  $\Phi^{(0)}$  are  $n_\mu + \bar{n}_\mu$ ,  $c$ , and  $\bar{c}$
- they enter in a linear way in these fields

$$\Rightarrow \bar{c} n_\mu \bar{n}_\nu c \text{ coefficient} = A^{(0)} A^{(0)} \Phi^{*(0)} \Phi^{(0)} \text{ coefficient}$$



## Compton amplitude off an unpolarized target

## Compton kinematics

- Compton amplitude off an unpolarized target  $A$ :

$$\gamma^*(q_1) + P(p_1) \rightarrow \gamma^*(q_2) + P(p_2),$$

$$T_{\mu\nu} = i \int d^4x e^{i\frac{1}{2}(q_1+q_2)\cdot x} \langle p_2 | T \{ J_\mu(x/2) J_\nu(-x/2) \} | p_1 \rangle$$

- Choose  $q_1$ ,  $q_2$  and  $p = p_1 + p_2$  as the three independent momenta  
 $\implies$  10 independent parity-conserving tensors of rank 2
- Electromagnetic gauge invariance:

$$q_1^\mu T_{\mu\nu} = T_{\mu\nu} q_2^\nu = 0.$$

- $\implies$  5 linear independent constraints



## Compton amplitude off an unpolarized target

## Compton form factors

⇒ **5 linear independent form factors**, function of **6 independent scalar invariants**:

$$\begin{aligned}
 T^{\mu\nu} &= V_1 \left( g^{\mu\nu} - \frac{q_1^\mu q_1^\nu}{q_1^2} - \frac{q_2^\mu q_2^\nu}{q_2^2} + q_1^\mu q_2^\nu \frac{(q_1 \cdot q_2)}{q_1^2 q_2^2} \right) \\
 &+ V_2 \left( p^\mu - q_1^\mu \frac{(p \cdot q_1)}{q_1^2} \right) \left( p^\nu - q_2^\nu \frac{(p \cdot q_2)}{q_2^2} \right) \\
 &+ V_3 \left( q_2^\mu - q_1^\mu \frac{(q_1 \cdot q_2)}{q_1^2} \right) \left( q_1^\nu - q_2^\nu \frac{(q_1 \cdot q_2)}{q_2^2} \right) \\
 &+ V_4 \left( p^\mu - q_1^\mu \frac{(p \cdot q_1)}{q_1^2} \right) \left( q_1^\nu - q_2^\nu \frac{(q_1 \cdot q_2)}{q_2^2} \right) \\
 &+ V_5 \left( q_2^\mu - q_1^\mu \frac{(q_1 \cdot q_2)}{q_1^2} \right) \left( p^\nu - q_2^\nu \frac{(p \cdot q_2)}{q_2^2} \right), \\
 &= \sum_{i=1}^5 V_i(p_1^2, p_2^2, q_1^2, q_2^2, s, u) \mathcal{V}_i^{\mu\nu}(p, q_1, q_2).
 \end{aligned}$$

## Compton amplitude off an unpolarized target

## The Virtual Compton Scattering and Real Compton Scattering amplitudes

- VCS amplitude:  $q_2^2 = 0$ :

$$\begin{aligned} V_1 + (q_1 \cdot q_2) V_3 + (p \cdot q_2) V_5 &= 0, \\ (p \cdot q_2) V_2 + (q_1 \cdot q_2) V_4 &= 0, \end{aligned}$$

⇒ 3 independent Compton form factors

- Real compton amplitude:  $q_1^2 = q_2^2 = 0$ :

$$V_1 + (q_1 \cdot q_2) V_3 = -(p \cdot q_1) V_4 = -(p \cdot q_2) V_5 = \frac{(p \cdot q_1)(p \cdot q_2)}{q_1 \cdot q_2} V_2$$

⇒ 2 independent Compton form factors

## Compton amplitude off an unpolarized target

## Amplitude from the semi-classical expansion

- 3 Tree-level diagrams at order  $e^2$ :

$$\begin{aligned} \epsilon_\mu T^{\mu\nu} \epsilon_\nu^* &= 2e^2 \int dy \sqrt{-g} e^{-\chi} A^m(y) A_m^*(y) \Phi^*(y) \Phi(y) \\ &+ e^2 \int dy dy' \sqrt{-g} e^{-\chi} \sqrt{-g'} e^{-\chi'} (A^m(y) A^{*n}(y') + A^{*m}(y) A^n(y')) \\ &\times \left( \Phi(y) \partial_m - (\partial_m \Phi(y)) \right) \left( \Phi^*(y') \partial'_n - (\partial'_n \Phi^*(y')) \right) G_0(y; y'), \end{aligned}$$

- Initial and final states:

$$\begin{aligned} \Phi(x, z) &= e^{ip_1 \cdot x} \phi_i(z), & \Phi^*(x', z') &= e^{-ip_2 \cdot x'} \phi_f(z'), \\ A_\mu(x, z) &= \epsilon_\mu e^{iq_1 \cdot x} A_1(z), & A_z(x, z) &= -i \frac{\epsilon \cdot q_1}{q_1^2} e^{iq_1 \cdot x} \partial_z A_1(z), \\ A_\nu^*(x', z') &= \epsilon_\nu e^{-iq_2 \cdot x'} A_2(z'), & A_z^*(x', z') &= i \frac{\epsilon \cdot q_2}{q_2^2} e^{-iq_2 \cdot x'} \partial_{z'} A_2(z'). \end{aligned}$$

with

$$A_1(z) = Q_{1z} K_1(Q_1 z), \quad A_2(z) = Q_{2z} K_1(Q_2 z)$$

## Compton amplitude off an unpolarized target

## Result for the on-shell Compton amplitude

- On-shell gauge-invariant amplitude ( $p_1^2 = p_2^2 = -m^2$ ):

$$T^{\mu\nu} = e^2 (2C_1 \mathcal{V}_1^{\mu\nu} + C_+ (\mathcal{V}_2^{\mu\nu} + \mathcal{V}_3^{\mu\nu}) + C_- (\mathcal{V}_4^{\mu\nu} + \mathcal{V}_5^{\mu\nu}))$$

where

$$C_{\pm}(m^2, q_1^2, q_2^2, s, u) = \mathcal{F}_1(m^2, q_1^2, q_2^2, s) \pm \mathcal{F}_1(m^2, q_2^2, q_1^2, u).$$

⇒ only 3 Compton form factor out of a possible 5

- Form factors  $\mathcal{F}_1(m^2, q_1^2, q_2^2, k^2)$  and  $C_1(m^2, q_1^2, q_2^2)$  from the AdS/QCD correspondence:

$$\begin{aligned} \mathcal{F}_1(m^2, q_1^2, q_2^2, k^2) &= \iint dz_1 dz_2 z_1^{-3} e^{-\chi(z_1)} A_1(z_1) \phi_i(z_1) \\ &\quad \times \widehat{G}_0(z_1, z_2, k^2) \phi_f^*(z_2) A_2^*(z_2) z_2^{-3} e^{-\chi(z_2)} \\ C_1(m^2, q_1^2, q_2^2) &= \int dz z^{-3} e^{-\chi(z)} A_1(z) A_2^*(z) \phi_i(z) \phi_f^*(z) \end{aligned}$$

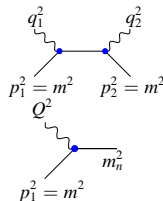
# Relation with Deep Inelastic Scattering

## Doubly spacelike Compton amplitude

$$\gamma^* A \rightarrow \gamma^* A'$$

- Form factor  $\mathcal{F}_1$  through the **Källén-Lehman** spectral representation of the Green function:

$$\mathcal{F}_1(m^2, q_1^2, q_2^2, k^2) = \sum_{n=0}^{\infty} \frac{\Gamma(m^2, m_n^2, q_1^2) \Gamma^*(m^2, m_n^2, q_2^2)}{k^2 + m_n^2 - i\epsilon}$$



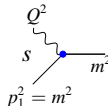
$$\Gamma(m^2, m_n^2, Q^2) = Q \int dz z^{-2} e^{-\chi(z)} K_1(Qz) \phi_m(z) \phi_n^*(z)$$

- Vertex function  $\Gamma$ :

$$\langle p_2 | J^\mu(0) | p_1 \rangle = \Gamma(p_1^2, p_2^2, k^2) \left( p^\mu - \frac{p_2^2 - p_1^2}{k^2} k^\mu \right) \quad p = p_1 + p_2, \quad k = p_2 - p_1$$

- Electromagnetic form factor:

$$F_\gamma(Q^2) = \Gamma(m^2, m^2, Q^2) = \mathcal{C}_1(m^2, Q^2, 0)$$



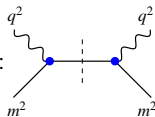
$$= Q \int dz z^{-2} e^{-\chi(z)} K_1(Qz) |\phi_m(z)|^2$$

# Relation with Deep Inelastic Scattering

From doubly spacelike Compton amplitude to DIS:  
from  $\gamma^* A \rightarrow \gamma^* A$  to  $\gamma^* A \rightarrow X$

- Absorptive part of the forward Compton scattering amplitude:

$$x_{\text{Bjorken}} = -\frac{q^2}{p \cdot q}$$



$$\begin{aligned} \text{Im } T^{\mu\nu}(q^2, s) &= e^2 \left( p^\mu + \frac{1}{x} q^\mu \right) \left( p^\nu + \frac{1}{x} q^\nu \right) \sum_{n=0}^{\infty} \delta(s + m_n^2) \left| \Gamma(m^2, m_n^2, q^2) \right|^2, \\ &\approx e^2 \left( \frac{\partial m_n^2}{\partial n} \right)^{-1} \Big|_{m_n^2 = -s} \left| \Gamma(m^2, -s, q^2) \right|^2 \left( p^\mu + \frac{1}{x} q^\mu \right) \left( p^\nu + \frac{1}{x} q^\nu \right) \end{aligned}$$

$\implies$  only  $F_L$  survives (i.e.  $F_1 = 0$ )

- Scaling properties ( $\chi(z) \xrightarrow{z \rightarrow 0} 0$ ): governed by

$$\Gamma(m^2, m_n^2, Q^2) \approx \int_0^{1/Q} \frac{dz}{z^3} e^{-\chi(z)} \phi(z) \phi_n^*(z)$$

## Scaling behavior of vertex function

- $Q^2 \gg m^2$ ,  $Q^2 \gtrsim m_n^2$ :

hadron state:  $\phi(z) \underset{z \rightarrow 0}{\sim} z^\Delta$ ,

$$\Gamma(m^2, m_n^2, Q^2) \propto C(m_n) \left(\frac{1}{Q}\right)^\Delta F\left(\frac{Q}{m_n}\right)$$

$$\frac{x}{Q^2} F_2(Q^2, x) \propto \left(\frac{1}{Q^2}\right)^\Delta F^2(x), \quad x = \frac{Q^2}{Q^2 - s}, \quad s = m_n^2.$$

$$\implies F_2(Q^2, x) \propto \left(\frac{1}{Q^2}\right)^{\Delta-1} \quad \text{calls for } \Delta = 1$$

- $Q^2 \gg m^2$ ,  $Q^2 \gg m_n^2$ :

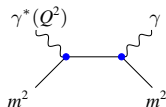
$$\phi(z) \underset{z \rightarrow 0}{\sim} z^\Delta, \quad \phi_n(z) \underset{z \rightarrow 0}{\sim} z^\Delta$$

$$\implies F_\gamma(Q^2) \propto \left(\frac{1}{Q^2}\right)^{\Delta-1} \quad \text{calls for } \Delta = 2$$

Brodsky de Teramond

- the same scaling obtained for structure function and form factors are generic to AdS/QCD correspondence, **impossible to reconcile with a partonic picture**

## Virtual Compton scattering

 $\gamma^* A \rightarrow \gamma A'$  from AdS/QCD correspondence

- Virtual incoming photon ( $q_1^2 = Q^2$ ), real outgoing photon ( $q_2^2 = 0$ ):

$$2\mathcal{C}_1 + (q_1 \cdot q_2) \mathcal{C}_+ + (p \cdot q_2) \mathcal{C}_- = 0,$$

$$(p \cdot q_2) \mathcal{C}_+ + (q_1 \cdot q_2) \mathcal{C}_- = 0.$$

- AdS/QCD VCS amplitude has the same tensorial structure as point-like scalar electrodynamics:

$$T_{\mu\nu}^{\text{VCS}} = e^2 \mathcal{C}_1(m^2, q_1^2, 0) \left( 2\mathcal{V}_{1,\mu\nu} - \frac{2m^2 + s + u}{(m^2 + s)(m^2 + u)} (\mathcal{V}_{2,\mu\nu} + \mathcal{V}_{3,\mu\nu}) + \frac{s - u}{(m^2 + s)(m^2 + u)} (\mathcal{V}_{4,\mu\nu} + \mathcal{V}_{5,\mu\nu}) \right).$$

- Electromagnetic form factor is the unique form factor of VCS amplitude (out a possible 3), and contains the whole internal structure of the hadron:

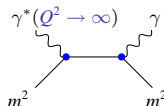
$$\mathcal{C}_1(m^2, q_1^2, 0) = \int_0^\infty dz z^{-3} e^{-\chi(z)} \left| \widehat{\Phi}_m(z) \right|^2 A_1(z)$$

$$(\mathcal{C}_1(m^2, 0, 0) = 1)$$



## Virtual Compton scattering

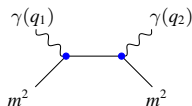
## Deeply Virtual Compton Scattering



- In perturbative QCD, these form factors can in principle be related, through factorization, to **generalized parton distributions (GPDs)**
- Partonic interpretation based on the convolution of **real GPDs** with coefficient functions which contain **both a real and an imaginary part**  $\implies$  **problem with the holographic DVCS amplitude which has no absorptive part** calling for stringy corrections?
- Asymptotic behavior in  $Q^2$  of the holographic DVCS cross-section governed by the power-law behavior of the electromagnetic form factor.
- Problem with the partonic interpretation of a power-law behavior in accordance with the dimensional counting rules: **contradiction between the  $\gamma$  form-factor scaling** (calling for  $\Delta = 2$ ) **and scaling of the DVCS amplitude** (calling for  $\Delta = 1$ )

## Polarizabilities

## Real Compton scattering



- Real Compton scattering ( $q_1^2 = q_2^2 = 0$ ):

$$T_{RCS}^{\mu\nu} = V_1 \left( g^{\mu\nu} - \frac{q_1^\nu q_2^\mu}{q_1 \cdot q_2} \right) + V_2 \left( p^\mu - \frac{p \cdot q_1}{q_1 \cdot q_2} q_2^\mu \right) \left( p^\nu - \frac{p \cdot q_2}{q_1 \cdot q_2} q_1^\nu \right)$$

- corrections to Thomson scattering which are quadratic in the energy of the photons  $\equiv$  static polarizabilities defined in lab-frame  $\vec{p}_1 = \vec{0}$ :

$$q_i = \omega_i(1, \hat{q}_i), \quad \omega_i^2 \ll m_\pi^2$$

$$A(\gamma\pi \rightarrow \gamma\pi) = 2e^2 \vec{\epsilon}_1 \cdot \vec{\epsilon}_2 + 8\pi m_\pi \omega_1 \omega_2 (\alpha_E \vec{\epsilon}_1 \cdot \vec{\epsilon}_2 + \beta_M (\vec{\epsilon}_1 \times \hat{q}_1) \cdot (\vec{\epsilon}_2 \times \hat{q}_2))$$

- Relation to Compton form factors:

$$\text{electric:} \quad 8\pi m \alpha_E = \left. \frac{\partial^2}{\partial \omega_1 \partial \omega_2} (V_1 + (V_3 - V_2) \vec{q}_1 \cdot \vec{q}_2) \right|_{\omega_1 = \omega_2 = 0}$$

$$\text{magnetic:} \quad 8\pi m \beta_M = (V_2 - V_3)|_{\omega_1 = \omega_2 = 0}.$$

## Polarizabilities

## Static polarisabilities from AdS/QCD

- Holographic Compton amplitude  $\implies$  **Static polarizabilities vanish**
- **This contradict the most recent experimental values**
- Threshold theorem:

$$C_1(m^2, 0, 0) = \int_0^\infty dz z^{-3} e^{-\chi(z)} \left| \widehat{\Phi}_m(z) \right|^2 = 1.$$

- Real Compton scattering on a scalar target in holographic models with minimal coupling to the photon = point-like scalar electrodynamics in the tree level approximation!
- **Non-vanishing polarizabilities  $\implies$  non-minimal coupling(s) to the photon**

## Polarizabilities

## Static polarisabilities from AdS/QCD in non-minimal SUGRA

### Chiral perturbation theory analogy

- Same problematics encountered in the calculation of the pion polarizabilities in  $\chi$ PT:

$$\alpha_E = \frac{4\alpha}{m_\pi F_\pi^2} (L_9^r + L_{10}^r), \quad \alpha_E + \beta_M = 0,$$

$$-iL_9 F_{\mu\nu} \text{Tr} \left( Q D^\mu U (D^\nu U)^\dagger + Q (D^\mu U)^\dagger D^\nu U \right) + L_{10} F_{\mu\nu} F^{\mu\nu} \text{Tr} \left( Q U Q U^\dagger \right),$$

$$D_\mu U = \partial_\mu U + ie A_\mu [Q, U]$$

$\alpha_E \neq 0$  and  $\beta_M \neq 0$  at order 4 in  $\chi$ PT

- Phenomenological 5d Lagrangians: **non-minimal coupling to be added**

$$\Delta S_{\text{AdS}}[\Phi, \Phi^*, A^m] = \int d^4x dz \sqrt{-g} e^{-\chi} \left( ig_1 \frac{e}{2} F_{mn} [D^m \Phi (D^n \Phi)^* - (D^m \Phi)^* D^n \Phi] \right. \\ \left. + g_2 \frac{e^2}{4} F_{mn} F^{mn} \Phi^* \Phi \right).$$

# Conclusion

- AdS/QCD Compton amplitude is **trivial in the low-energy limit**
- AdS/QCD Compton amplitude has **no partonic interpretation** in the high-energy limit
- Most popular AdS/QCD models incorporating flavor symmetry do not cure all the problems
- Neglecting stringy corrections is maybe not satisfactory (e.g.: might be the only source of absorptive parts)