M2/CFP/Parcours of Physique Théorique

Invariances in physics and group theory

Electromagnetic mass splittings in an SU(3) octet

0. We easily get

$$\dim(E \otimes E) = d^2 \,,$$
$$\dim(E \otimes E)_S = \frac{d(d+1)}{2} \,,$$
$$\dim(E \otimes E)_A = \frac{d(d-1)}{2} \,.$$

a. One should distinguish the following multiplets : $(n,p)\,, (\Sigma^-,\,\Sigma^0,\,\Sigma^+)\,, (\Xi^-,\,\Xi^0)\,, (\Lambda^0)\,,$ therefore leading to 4 differences of masses, which can be chosen as $M_n - M_p\,,$ $M_{\Sigma^-} - M_{\Sigma^0}\,,\; M_{\Sigma^+} - M_{\Sigma^0}$ and $M_{\Xi^-} - M_{\Xi^0}\,.$

b. Based on Wigner-Eckart theorem, since j^μ transforms under $SU(3)$ as 8, and since B and B' are in the representation 8, on should determine the nomber of times that 8 appears in the product $8 \otimes 8 \otimes 8$, i.e. the number of invariants inside $8 \otimes 8 \otimes 8 \otimes 8$, or equivalently the number of times a given irreducible representation occurs in $8 \otimes 8$ and $8 \otimes 8$ (the equivalence of these various points of view are easily understood when playing with orthogonality formulas of characters). Now, from

$$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27$$

we thus deduce that this number equals $1 + 2^2 + 1 + 1 + 1 = 8\,.$

c. There are $8(8+1)/2 = 36$ independent symmetric tensors of rank 2 in the representation 8. This is in accordance with $\dim(1 \oplus 8 \oplus 27) = 36\,.$

d. i) There are thus, denoting m_R the multiplicity of representation R, $m_1 + m_8 + m_{27} = 1 + 2 + 1 = 4$ independent amplitudes.

ii) Among these, the one associated to $\mathbb{1}$ give the same contribution to all $\delta M_B\,.$ Thus, only 3 amplitudes contribute to the 4 independent mass differences $\delta M_B -$

$\delta M_{B'}$.

iii) One can consider for example the four operators

$$O_1 = \mathrm{tr}\bar{\mathrm{B}}\mathrm{Q}^2\mathrm{B}'$$
$$O_2 = \mathrm{tr}\bar{\mathrm{B}}\mathrm{Q}\mathrm{B}'\mathrm{Q}$$
$$O_3 = \mathrm{tr}\bar{\mathrm{B}}\mathrm{B}'\mathrm{Q}^2$$
$$O_4 = 1$$

among which $O_1\,, O_2$ and O_3 contribute to the mass differences.

e. i) Since there are 3 amplitudes contributing to 4 mass splittings, there should be a relation between these mass differences.

ii) See file DeltaM.nb

f. In the case of $J^P = 0^-$, the previous approach leads to 3 independent amplitudes, among which only two of them contribute to the mass splittings, $\mathrm{tr}\,\Phi^2\mathrm{Q}^2$ and $\mathrm{tr}(\Phi\mathrm{Q})^2$. We only have two independent mass differences : $m_{\pi^+} - m_{\pi^0} = m_{\pi^-} - m_{\pi^0}$ and $m_{K^+} - m_{K^0} = m_{K^-} - m_{K^0}$, the equalities coming from the identity of the mass of a particle and its antiparticle (from CPT invariance). Thus, we do not have anymore a relation between these mass differences!

g. One should evaluate the number of invariants in $10 \otimes \overline{10} \otimes (8 \otimes 8)_S$. Since $10 \otimes \overline{10} = 1 \oplus 8 \oplus 27 \oplus 64$ and $(8 \otimes 8)_S = 1 \oplus 8 \oplus 27$, there are thus 3 independent amplitudes, among which only those of the representations 8 and 27 do contribute to the mass splitting. We know two candidates for these invariants, namely Q and Q^2 . Thus $\Delta_{em} = \alpha\, Q + \beta\, Q^2$. The various mass splittings can organized as 6 independent ones, namely

$$M_{\Delta^-} - M_{\Delta^0} \quad , \; M_{\Delta^0} - M_{\Delta^+} \quad , \; M_{\Delta^{++}} - M_{\Delta^+} \; ,$$
$$M_{\Sigma^{*-}} - M_{\Sigma^{*0}} \; , \; M_{\Sigma^{*0}} - M_{\Sigma^{*+}} \; , \; M_{\Xi^{*-}} - M_{\Xi^{*0}} \; ,$$

which implies that

$$M_{\Delta^-} - M_{\Delta^0} = -\alpha + \beta \quad , \; M_{\Delta^0} - M_{\Delta^+} = -\alpha - \beta \quad , \; M_{\Delta^{++}} - M_{\Delta^+} = 0 \; ,$$
$$M_{\Sigma^{*-}} - M_{\Sigma^{*0}} = -\alpha + \beta \; , \; M_{\Sigma^{*0}} - M_{\Sigma^{*+}} = -\alpha - \beta \; , \; M_{\Xi^{*-}} - M_{\Xi^{*0}} = -\alpha + \beta \; ,$$

from which we deduce that

$$
\begin{aligned}
M_{\Delta^0} - M_{\Delta^+} &= M_{\Sigma^{*0}} - M_{\Sigma^{*+}}\,, \\
M_{\Delta^-} - M_{\Delta^0} &= M_{\Sigma^{*-}} - M_{\Sigma^{*0}} \quad = M_{\Xi^{*-}} - M_{\Xi^{*0}}\,, \\
M_{\Delta^{++}} &= M_{\Delta^+}\,.
\end{aligned}
$$

The experimental values (see PDG 2013) do not constraint very well the first and the third equality, since $M_{\Delta^0} - M_{\Delta^+}$ and $M_{\Delta^{++}} - M_{\Delta^+}$ are not known with a good precision. Still, it seems that the third equality is probably significantly violated. For the second set of equalities, one can only consider the last one (because $M_{\Delta^-} - M_{\Delta^0}$ is badly known). From PDG 2013, $M_{\Sigma^{*-}} - M_{\Sigma^{*0}} \simeq 3.5$ MeV and $M_{\Xi^{*-}} - M_{\Xi^{*0}} \simeq 3.2$ MeV, with an error band leading to a very good compatibility of the two mass splitting.