Electromagnetic mass splittings in a SU(3) octet

Note : in Mathematica, greek letters are obtained by using the key "Esc" before and after typing the abreviation of the greek letter with a latine spelling

Theory

▪ octet content and properties

▪ octet of mesons $J^P = 0^-$

```
Φ = {{1 / Sqrt[2] π0 - 1 / Sqrt[6] η, πp, Kp},
  {πm, -1 / Sqrt[2] π0 - 1 / Sqrt[6] η, K0}, {Km, K0b, Sqrt[2 / 3] η}}
```

$$\begin{pmatrix} \frac{\pi 0}{\sqrt{2}} - \frac{\eta}{\sqrt{6}} & \pi p & \text{Kp} \\ \pi m & -\frac{\eta}{\sqrt{6}} - \frac{\pi 0}{\sqrt{2}} & \text{K0} \\ \text{Km} & \text{K0b} & \sqrt{\frac{2}{3}}\,\eta \end{pmatrix}$$

▪ octet of baryons $J^P = \frac{1}{2}^+$

```
Ψ = {{1 / Sqrt[2] Σ0 - 1 / Sqrt[6] Λ, Σp, p},
  {Σm, -1 / Sqrt[2] Σ0 - 1 / Sqrt[6] Λ, n}, {Ξm, Ξ0, Sqrt[2 / 3] Λ}}
```

$$\begin{pmatrix} \frac{\Sigma 0}{\sqrt{2}} - \frac{\Lambda}{\sqrt{6}} & \Sigma p & p \\ \Sigma m & -\frac{\Lambda}{\sqrt{6}} - \frac{\Sigma 0}{\sqrt{2}} & n \\ \Xi m & \Xi 0 & \sqrt{\frac{2}{3}}\,\Lambda \end{pmatrix}$$

One should be careful: Σm and Σp are not charge conjugated

```
Ψb = {{-(Λb / Sqrt[6]) + Σ0b / Sqrt[2], Σmb, Ξp},
  {Σpb, -(Λb / Sqrt[6]) - Σ0b / Sqrt[2], Ξ0b}, {pb, nb, Sqrt[2 / 3] * Λb}}
```

$$\begin{pmatrix} \frac{\Sigma 0b}{\sqrt{2}} - \frac{\Lambda b}{\sqrt{6}} & \Sigma mb & \Xi p \\ \Sigma pb & -\frac{\Lambda b}{\sqrt{6}} - \frac{\Sigma 0b}{\sqrt{2}} & \Xi 0b \\ pb & nb & \sqrt{\frac{2}{3}}\,\Lambda b \end{pmatrix}$$

the isospin, hypercharge and charge operators

```
Iz = 1 / 2 {{1, 0, 0}, {0, -1, 0}, {0, 0, 0}}
```

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
Y = 1 / 3 {{1, 0, 0}, {0, 1, 0}, {0, 0, -2}}
```

$$\begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix}$$

```
Q = 1 / 2 Y + Iz
```

$$\begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}$$

- **the isospin, hypercharge and charge of the octet of mesons $J^P = 0^-$**

η is isoscalar, (K^0,K^+) and (K^-, Kb^0) are two isodoublet, (π^-,π^0,π^+) is an isovector:

```
Simplify[Iz.Φ - Φ.Iz]
```

$$\begin{pmatrix} 0 & \pi\mathrm{p} & \frac{\mathrm{Kp}}{2} \\ -\pi\mathrm{m} & 0 & -\frac{\mathrm{K0}}{2} \\ -\frac{\mathrm{Km}}{2} & \frac{\mathrm{K0b}}{2} & 0 \end{pmatrix}$$

```
Simplify[Y.Φ - Φ.Y]
```

$$\begin{pmatrix} 0 & 0 & \mathrm{Kp} \\ 0 & 0 & \mathrm{K0} \\ -\mathrm{Km} & -\mathrm{K0b} & 0 \end{pmatrix}$$

```
Simplify[Q.Φ - Φ.Q]
```

$$\begin{pmatrix} 0 & \pi\mathrm{p} & \mathrm{Kp} \\ -\pi\mathrm{m} & 0 & 0 \\ -\mathrm{Km} & 0 & 0 \end{pmatrix}$$

- **the isospin, hypercharge and charge of octet of baryons $J^P = \frac{1}{2}^+$**

Λ^0 is an isoscalar, (n,p) and (Ξ^- ,Ξ^0)are two isodoublet, (Σ^-, Σ^0, Σ^+) is an isovector:

```
Simplify[Iz.Ψ - Ψ.Iz]
```

$$\begin{pmatrix} 0 & \Sigma p & \frac{p}{2} \\ -\Sigma m & 0 & -\frac{n}{2} \\ -\frac{\Xi m}{2} & \frac{\Xi 0}{2} & 0 \end{pmatrix}$$

```
Simplify[Y.Ψ - Ψ.Y]
```

$$\begin{pmatrix} 0 & 0 & p \\ 0 & 0 & n \\ -\Xi m & -\Xi 0 & 0 \end{pmatrix}$$

```
Simplify[Q.Ψ - Ψ.Q]
```

$$\begin{pmatrix} 0 & \Sigma p & p \\ -\Sigma m & 0 & 0 \\ -\Xi m & 0 & 0 \end{pmatrix}$$

Mass splittings

the four invariant operators

```
O1 = Ψb.Q.Q.Ψ;
```

```
O2 = Ψb.Ψ.Q.Q;
```

```
O3 = Ψb.Q.Ψ.Q;
```

```
O4 = Ψb.Ψ;
```

```
Oα = Expand[Tr[O1]]
```

$$\frac{n\,\text{nb}}{9} + \frac{4\,p\,\text{pb}}{9} + \frac{\Lambda\,\Lambda\text{b}}{6} + \frac{\Xi 0\,\Xi 0\text{b}}{9} + \frac{\Xi\text{m}\,\Xi\text{p}}{9} - \frac{\Lambda\text{b}\,\Sigma 0}{6\sqrt{3}} - \frac{\Lambda\,\Sigma 0\text{b}}{6\sqrt{3}} + \frac{5\,\Sigma 0\,\Sigma 0\text{b}}{18} + \frac{\Sigma\text{m}\,\Sigma\text{mb}}{9} + \frac{4\,\Sigma\text{p}\,\Sigma\text{pb}}{9}$$

```
Oβ = Expand[Tr[O2]]
```

$$\frac{n\,\text{nb}}{9} + \frac{p\,\text{pb}}{9} + \frac{\Lambda\,\Lambda\text{b}}{6} + \frac{\Xi 0\,\Xi 0\text{b}}{9} + \frac{4\,\Xi\text{m}\,\Xi\text{p}}{9} - \frac{\Lambda\text{b}\,\Sigma 0}{6\sqrt{3}} - \frac{\Lambda\,\Sigma 0\text{b}}{6\sqrt{3}} + \frac{5\,\Sigma 0\,\Sigma 0\text{b}}{18} + \frac{4\,\Sigma\text{m}\,\Sigma\text{mb}}{9} + \frac{\Sigma\text{p}\,\Sigma\text{pb}}{9}$$

```
Oγ = Expand[Tr[O3]]
```

$$\frac{n\,\text{nb}}{9} - \frac{2\,p\,\text{pb}}{9} + \frac{\Lambda\,\Lambda\text{b}}{6} + \frac{\Xi 0\,\Xi 0\text{b}}{9} - \frac{2\,\Xi\text{m}\,\Xi\text{p}}{9} - \frac{\Lambda\text{b}\,\Sigma 0}{6\sqrt{3}} - \frac{\Lambda\,\Sigma 0\text{b}}{6\sqrt{3}} + \frac{5\,\Sigma 0\,\Sigma 0\text{b}}{18} - \frac{2\,\Sigma\text{m}\,\Sigma\text{mb}}{9} - \frac{2\,\Sigma\text{p}\,\Sigma\text{pb}}{9}$$

Tr [Ψ Ψ b] is the identity operator:

```
Oid = Expand[Tr[O4]]
```

$$n\,\text{nb} + p\,\text{pb} + \Lambda\,\Lambda\text{b} + \Xi 0\,\Xi 0\text{b} + \Xi\text{m}\,\Xi\text{p} + \Sigma 0\,\Sigma 0\text{b} + \Sigma\text{m}\,\Sigma\text{mb} + \Sigma\text{p}\,\Sigma\text{pb}$$

the relevant mass operator

Note: since Oid is proportional to Id, it does not contribute to mass differences ; we anyway expand the mass operator as an expansion on the 4 invariants

```
M = α Oα + β Oβ + γ Oγ + δ Oid
```

$$\gamma\left(\frac{n\,\text{nb}}{9}-\frac{2\,p\,\text{pb}}{9}+\frac{\Lambda\,\Lambda\text{b}}{6}+\frac{\Xi0\,\Xi0\text{b}}{9}-\frac{2\,\Xi\text{m}\,\Xi\text{p}}{9}-\frac{\Lambda\text{b}\,\Sigma0}{6\sqrt{3}}-\frac{\Lambda\,\Sigma0\text{b}}{6\sqrt{3}}+\frac{5\,\Sigma0\,\Sigma0\text{b}}{18}-\frac{2\,\Sigma\text{m}\,\Sigma\text{mb}}{9}-\frac{2\,\Sigma\text{p}\,\Sigma\text{pb}}{9}\right)+$$
$$\beta\left(\frac{n\,\text{nb}}{9}+\frac{p\,\text{pb}}{9}+\frac{\Lambda\,\Lambda\text{b}}{6}+\frac{\Xi0\,\Xi0\text{b}}{9}+\frac{4\,\Xi\text{m}\,\Xi\text{p}}{9}-\frac{\Lambda\text{b}\,\Sigma0}{6\sqrt{3}}-\frac{\Lambda\,\Sigma0\text{b}}{6\sqrt{3}}+\frac{5\,\Sigma0\,\Sigma0\text{b}}{18}+\frac{4\,\Sigma\text{m}\,\Sigma\text{mb}}{9}+\frac{\Sigma\text{p}\,\Sigma\text{pb}}{9}\right)+$$
$$\alpha\left(\frac{n\,\text{nb}}{9}+\frac{4\,p\,\text{pb}}{9}+\frac{\Lambda\,\Lambda\text{b}}{6}+\frac{\Xi0\,\Xi0\text{b}}{9}+\frac{\Xi\text{m}\,\Xi\text{p}}{9}-\frac{\Lambda\text{b}\,\Sigma0}{6\sqrt{3}}-\frac{\Lambda\,\Sigma0\text{b}}{6\sqrt{3}}+\frac{5\,\Sigma0\,\Sigma0\text{b}}{18}+\frac{\Sigma\text{m}\,\Sigma\text{mb}}{9}+\frac{4\,\Sigma\text{p}\,\Sigma\text{pb}}{9}\right)+$$
$$\delta\,(n\,\text{nb}+p\,\text{pb}+\Lambda\,\Lambda\text{b}+\Xi0\,\Xi0\text{b}+\Xi\text{m}\,\Xi\text{p}+\Sigma0\,\Sigma0\text{b}+\Sigma\text{m}\,\Sigma\text{mb}+\Sigma\text{p}\,\Sigma\text{pb})$$

```
Mp = Coefficient[M, p pb]
```

$$\frac{4\,\alpha}{9}+\frac{\beta}{9}-\frac{2\,\gamma}{9}+\delta$$

```
Mn = Coefficient[M, n nb]
```

$$\frac{\alpha}{9}+\frac{\beta}{9}+\frac{\gamma}{9}+\delta$$

```
MΛ = Coefficient[M, Λ Λb]
```

$$\frac{\alpha}{6}+\frac{\beta}{6}+\frac{\gamma}{6}+\delta$$

```
MΣp = Coefficient[M, Σp Σpb]
```

$$\frac{4\,\alpha}{9}+\frac{\beta}{9}-\frac{2\,\gamma}{9}+\delta$$

```
MΣ0 = Coefficient[M, Σ0 Σ0b]
```

$$\frac{5\,\alpha}{18}+\frac{5\,\beta}{18}+\frac{5\,\gamma}{18}+\delta$$

```
MΣm = Coefficient[M, Σm Σmb]
```

$$\frac{\alpha}{9}+\frac{4\,\beta}{9}-\frac{2\,\gamma}{9}+\delta$$

```
MΞ0 = Coefficient[M, Ξ0 Ξ0b]
```

$$\frac{\alpha}{9}+\frac{\beta}{9}+\frac{\gamma}{9}+\delta$$

```
MΞm = Coefficient[M, Ξm Ξp]
```

$$\frac{\alpha}{9}+\frac{4\,\beta}{9}-\frac{2\,\gamma}{9}+\delta$$

```
{MΞm - MΞ0, MΣm - MΣp + Mp - Mn}
```

$$\left\{\frac{\beta}{3}-\frac{\gamma}{3},\frac{\beta}{3}-\frac{\gamma}{3}\right\}$$

Conclusion : MΞ m- MΞ 0= MΣ m- MΣ p+ Mp - Mn

Phenomenology

Experimental data for masses (in MeV/c^2) of the octet of baryons $J^P = \frac{1}{2}^+$

```
MnE = 939.56;
MpE = 938.27;
MΞmE = 1321.71;
MΞ0E = 1314.86;
MΣmE = 1197.45;
MΣ0E = 1192.64; MΣpE = 1189.37;
```

Comparison with theory

```
{MΞmE - MΞ0E, MΣmE - MΣpE + MpE - MnE}
```

{6.85, 6.79}

```
(MΞmE - MΞ0E) / (MΣmE - MΣpE + MpE - MnE)
```

1.00884