A Ghost Story

Gluons and ghosts in the IR and the UV

Berlin, June 15, 2009

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III. Two solutions for Dyson Schwinger
IV. The UV and Λ_{MS}
V. The A^2 condensate
VI. Scaling
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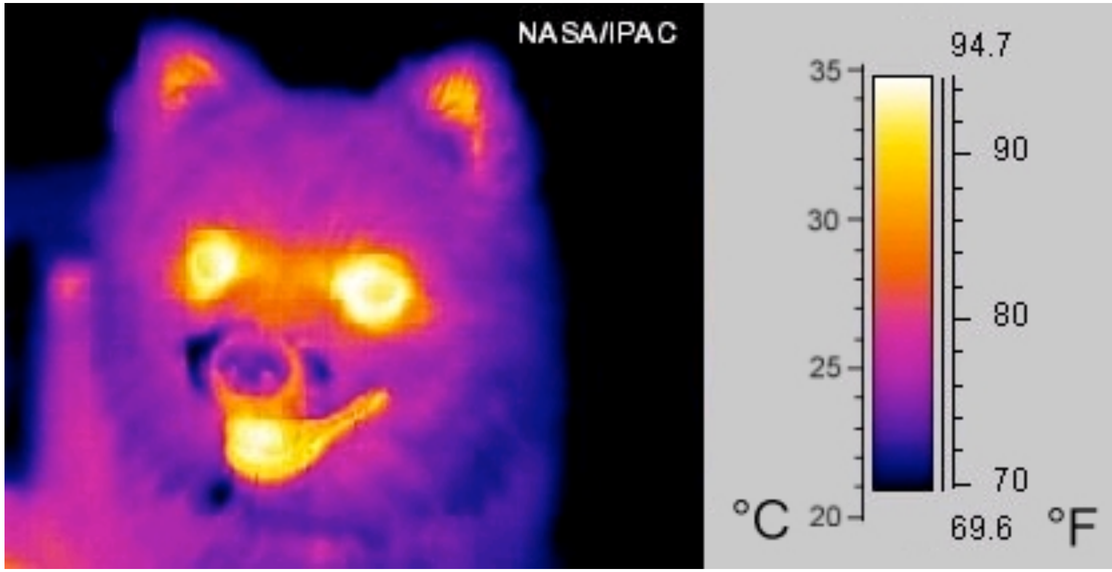
Infra-Red



- Infra-red dog

QCD in the IR

- QCD is « free » in the UV and confining in the IR. Hence the interest in IR behaviour. There exist different models for confinement which usually imply some consequences about the IR behaviour of Green functions.
- Zwanziger's conjecture that confinement has to do with Gribov horizon has such implications.

Existing tools ?

- There are two sets of very usefus analytic relations to learn about QCD in the IR: Ward-Slavnov-Taylor (WST) identities and the infinite tower of Dyson-Schwinger (DS) integral equations. Lattice QCD give also essential numerical indications.

The best would be to have an analytic solution, however this is not possible:

- WST relates Green-Functions, not enough constraints.
- DS are too complicated, highly non linear, it is not known how many solutions exist, but there is presumably a large number.

Common way out ?

- Use truncated DS with some hypotheses about vertex functions and sometimes compare the result to LQCD

WE PREFER

1- Combine informations from LQCD and analytic methods: not only using LQCD as an a posteriori check, but use it as an input for DSE. We believe that this allows a better control on systematic uncertainties of all methods.

2- Use WST identities (usually overlooked). This however leads today to an unsolved problem.

3- 1 and 2 are complemented with **mild** regularity assumptions about vertex functions

4- Take due care of the UV behaviour (known since QCD is asymptotically free) and use a well defined renormalisation procedure (no renormalisation at μ=0 because of possible IR singularities).

Notations *(In latin languages ghost is « fantômes, fantasmas »)*

- $G(p^2)$ is the bare gluon dressing function, $= p^2 G^{(2)}(p^2)$, $G^{(2)}(p^2)$ being the gluon propagator, G LIKE GLUON; $Z_3(\mu^2) = G(\mu^2)$ [MOM renormalisation constant of the gluon propagator]

(frequent notation (fn): $D(p^2)$instead of $G^{(2)}(p^2)$),

- $F(p^2)$ is the bare ghost dressing function, $= p^2 F^{(2)}(p^2)$, $F^{(2)}(p^2)$ being the ghost propagator, F LIKE FANTÔME; $\tilde{Z}_3(\mu^2) = F(\mu^2)$ [MOM renormalisation constant of the ghost propagator]

(fn: $G(p^2)$instead of $F^{(2)}(p^2)$)

- In the deep IR it is assumed $G(p^2) \propto (p^2)^{\alpha G}$

(fn: $p^2 D(p^2) \propto (p^2)^{\alpha_D}$ or $(p^2)^{\delta gl}$; $\alpha_G = 2\kappa$)

- In the deep IR it is assumed $F(p^2) \propto (p^2)^{\alpha F}$

(fn: $p^2 G(p^2) \propto 1/(p^2)^{\alpha_G}$ or $(p^2)^{\delta gh}$; $\alpha_F = -\kappa$)

Non-perturbative definitions of the strong coupling constant

- Compute a three-gluon or ghost-ghost-gluon Green function, in a well defined kinematics depending on a scale μ, and the gluon and ghost propagators.
- From there compute the corresponding bare vertex function Γ_B
- Then: $g_R(\mu^2) = g_0 G(\mu^2)^{3/2}\, \Gamma'_B$ or $g_R(\mu^2) = g_0\, F(\mu^2)\, G(\mu^2)^{1/2}\, \Gamma_B$
- Special and **preferred** case (Von Smekal) : one vanishing ghost momentum. Taylor: $\Gamma_B=1$

  $g_T(\mu^2) = g_0\, F(\mu^2)\, G(\mu^2)^{1/2}$

  $\alpha_T(\mu^2) = g_0^2/(4\pi)\, F(\mu^2)^2\, G(\mu^2)$

$F(p^2)^2G(p^2)$ is thus proportional to $\alpha_T(\mu^2)$

Lattice indicates $\alpha_G \sim 1$, $\alpha_F \sim 0_-$, $F(\mu^2)^2G(\mu^2) \to 0$, $g^f(\mu) \to 0$

BUT

A frequent analysis of the ghost propagator DS equation

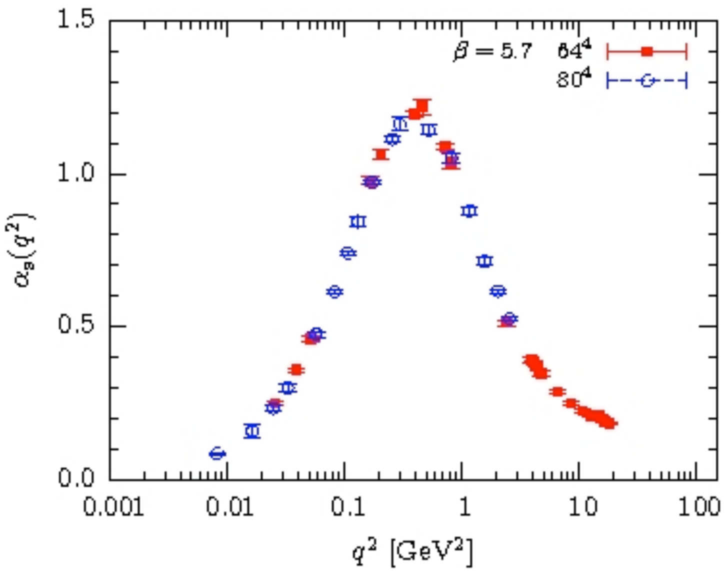
Leads to $2\alpha_F + \alpha_G = 0$ (fn: $\alpha_D = 2\,\alpha_G$ or $\delta_{gl} = -2\,\delta_{gl} = 2\kappa$) i.e. $F(p^2)^2G(p^2) \to ct$

and $F(p^2) \to \infty$

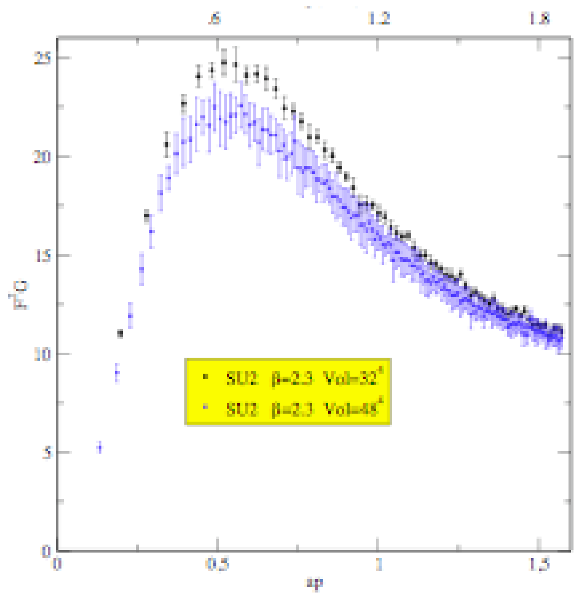
In contradiction with lattice

This is a strong, non truncated DS equation

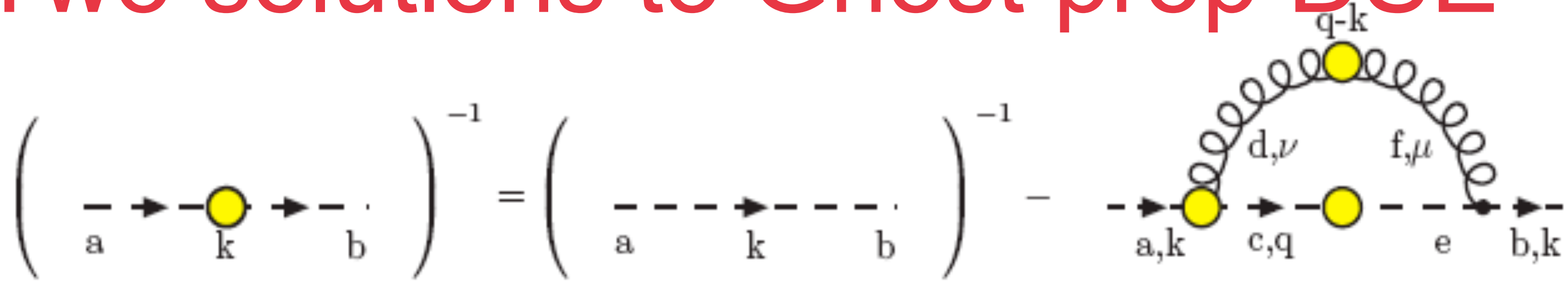
What is going on ??



Lattice gluodynamics computation of Landau gauge Green's functions in the deep infrared.
I.L. Bogolubsky , E.M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck arXiv:0901.0736



Two solutions to Ghost prop DSE



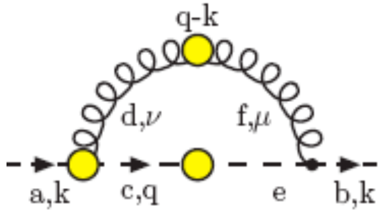
- The non-truncated ghost propagator DS equation
- It is also a WST equation !!!
- We will first prove that there are two types of solutions,

I. $2\alpha_F + \alpha_G = 0$, $\alpha_F < 0$ (fn: $\alpha_D = 2\alpha_G$ or or $\delta_{gl} = -2\,\delta_{gl} = 2\kappa$; « conformal solution »)

$F(p^2)^2 G(p^2) \to ct \neq 0$ and ; In disagreement with lattice

II. $\alpha_F = 0$ (fn: $\alpha_G = 0$, « disconnected solution ») $F(p^2) \to ct \neq 0$ In fair agreement with lattice, see recent large lattices: I.L. Bogolubsky, et al. arXiv:0710.1968 [hep-lat], A. Cucchieri and T Mendes arXiv:0710.0412 [hep-lat], and in agreement with WST

- We will next show via a numerical study that solution I (II) are obtained when the coupling constant is equal (non-equal) to a critical value.



hep-ph/0507104, hep-ph/06040

- From anomalous dimensions it is easy to see that the loop is UV divergent. It needs a careful renormalisation (the subscript R stands for renormalised)

or to use a subtracted DSE with two different external m omenta, thus cancelling the UV divergence.

$$\frac{1}{F_R(k^2)} - \frac{1}{F_R(k')^2} = -N_c g_R^2 \tilde{z}_1 \int \frac{d^4q}{(2\pi)^4} \left(1 - \frac{(k.q)^2}{k^2\, q^2}\right) \left[\frac{G_R((q-k)^2) H_{1R}(q,k)}{((q-k)^2)^2} - \frac{G_R((q-k')^2) H_{1R}(q,k')}{((q-k')^2)^2}\right] F_R(q^2)$$

- **$1/F-1/F' = g^2 \int (G-G')F$ ´ kinematics ⇒**

$2\alpha_F + \alpha_G = 0$ **(I). But if F=F' in the deep IR, i.e. if $F \rightarrow ct \neq 0$,**

the dimensional argument fails since the power α_F does not appear in the lhs.

This makes the point: $2\alpha_F + \alpha_G = 0$ (I) unless $\alpha_F = 0$

- Solution (I) also imposes an additional constraint on the value of g^2:

$$\frac{1}{F_R(k^2)} = \tilde{Z}_3 - N_c g_R^2 \tilde{z}_1 \int \frac{d^4q}{(2\pi)^4} \left(1 - \frac{(k.q)^2}{k^2\, q^2}\right) \left[\frac{G_R((q-k)^2) H_{1R}(q,k)}{((q-k)^2)^2}\right] F_R(q^2)$$

- $\tilde{Z}_3$ is the ghost prop renormalisation. It cancels the UV divergence.

  When $k \longrightarrow 0$ the lhs $\propto (k^2)^{-\alpha_F}$, $\tilde{Z}_3$ is independent of k.

I. If $\alpha_F < 0$, taking $k \rightarrow 0$, $\tilde{Z}_3$ has to be matched by the integral $\tilde{Z}_3 = g^2$ Integral(k=0),

   where $g^2 = N_c g_R^2 z_1$

   This leads to a well defined value for the coupling constant and the relation

J. C. R. Bloch, Few Body Syst. **33** (2003) 111 [arXiv:hep-ph/0303125]

$2\alpha_F + \alpha_G = 0$ (fn: $\alpha_D = 2\alpha_G$), $F(p^2)^2 G(p^2) \to ct \neq 0$, follows from a simple dimensional argument.

II. If $\alpha_F = 0$, the same integral is equal to: $-1/F_R(0) = g^2$ Integral(k=0),

the coupling constant now also deper $\tilde{Z}_3$ on $F_R(0)$ which is finite non zero.

In the small k region, $F_R(k^2) = F_R(0) + c\,(k^2)^{\alpha'_F}$ and now the dimensional argument gives $\alpha'_F = \alpha_G$.

If $\alpha_G = 1$ then $F_R(k^2) = F_R(0) + c\,k^2 \log(k^2)$

To summarise, *adding that* $G(p^2) \to 0$ (see lattice, late):

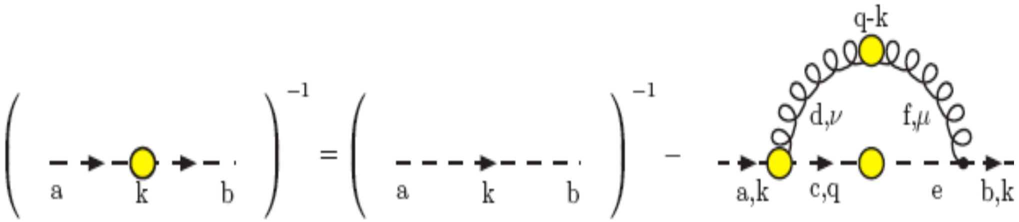
I. If $\alpha_F < 0$, $2\alpha_F + \alpha_G = 0$, $F(p^2)^2 G(p^2) \to ct \neq 0$ and fixed coupling constant at a finite scale; $\alpha_G = -2\alpha_F = 2\kappa$

From arXiv:0801.2762, Alkofer et al, $-0.75 \leq \alpha_F \leq -0.5$, $1 \leq \alpha_G \leq 1.5$

II. if $\alpha_F = 0$, $F(p^2) \to ct \neq 0$ $\alpha'_F = \alpha_G$ and no fixed coupling constant

Notice: solution II agrees rather well with lattice !!

Numerical solutions to Ghost prop DSE



$$\frac{1}{F_R(k^2)} - \frac{1}{F_R(k')^2} = -N_c g_R^2 \widetilde{z}_1 \int \frac{d^4q}{(2\pi)^4} \left(1 - \frac{(k.q)^2}{k^2\, q^2}\right) \left[\frac{G_R((q-k)^2) H_{1R}(q,k)}{((q-k)^2)^2} - \frac{G_R((q-k')^2) H_{1R}(q,k')}{((q-k')^2)^2}\right] F_R(q^2)$$

- To solve this equation one needs an input for the gluon propagator G_R (we take it from LQCD, extended to the UV via perturbative QCD) and for the ghost-ghost-gluon vertex H_{1R}: regularity is usually assumed from Taylor identity and confirmed by LQCD.
- To be more specific, we take H_{1R} to be constant, and G_R from lattice data interpolated with the α_G=1 IR power. For simplicity we subtract at k'=0. We take μ=1.5 GeV.The equation becomes

$$\frac{1}{\widetilde{F}(k^2)} = \frac{1}{\widetilde{F}(0)} - \int \frac{d^4q}{(2\pi)^4} \left(1 - \frac{(k.q)^2}{k^2\, q^2}\right) \left[\frac{G_R((q-k)^2)}{((q-k)^2)^2} - \frac{G_R((q)^2)}{((q)^2)^2}\right] \widetilde{F}(q^2)$$

where $\widetilde{F}(k)=\widetilde{g}(\mu)\, F_R(k, \mu)$. Notice that $\widetilde{F}(\mu)=g(\mu)$, with g defined as

$$g^2 = N_c g_R^2 \widetilde{z}_1 H_{1R} = N_c g_B^2 Z_3 \widetilde{Z}_3^2 H_{1B}$$

- We find one and only one solution for any positive value of $\tilde{F}(0)$. $F(0)=\infty$ corresponds to a critical value:

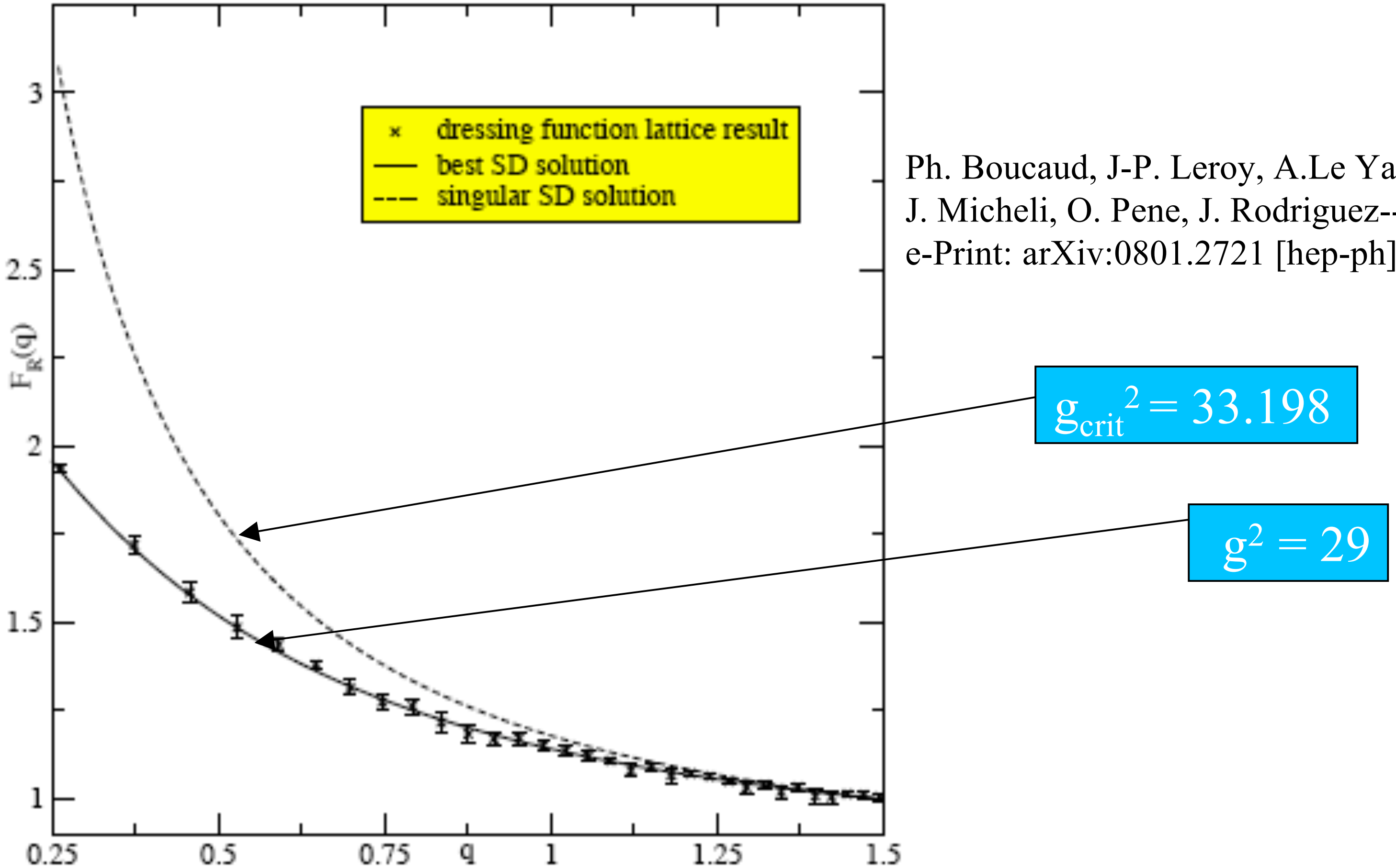
$g_c^2 = 10\pi^2/(F_R^2(0)\, G_R^{(2)}(0))$ (fn: $10\pi^2/(D_R(0) \lim p^2 G_R(p^2))$

J. C. R. Bloch, Few Body Syst. **33** (2003) 111 [arXiv:hep-ph/0303125]

- This critical solution corresponds to $F_R(0)=\infty$, It is the solution I, with $2\alpha_F + \alpha_G=0$, $F(p^2)^2G(p^2) \to ct \neq 0$, a diverging ghost dressing function and a fixed coupling constant.

- The non-critical solutions, have $F_R(0)$ finite, i.e. $\alpha_F = 0$, the behaviour $F_R(k^2)=F_R(0) + c\, k^2 \log(k^2)$ has been checked.

- Not much is changed if we assume a logarithmic divergence of the gluon propagator for $k \to 0$: $F_R(k^2)=F_R(0) - c'\, k^2 \log^2(k^2)$

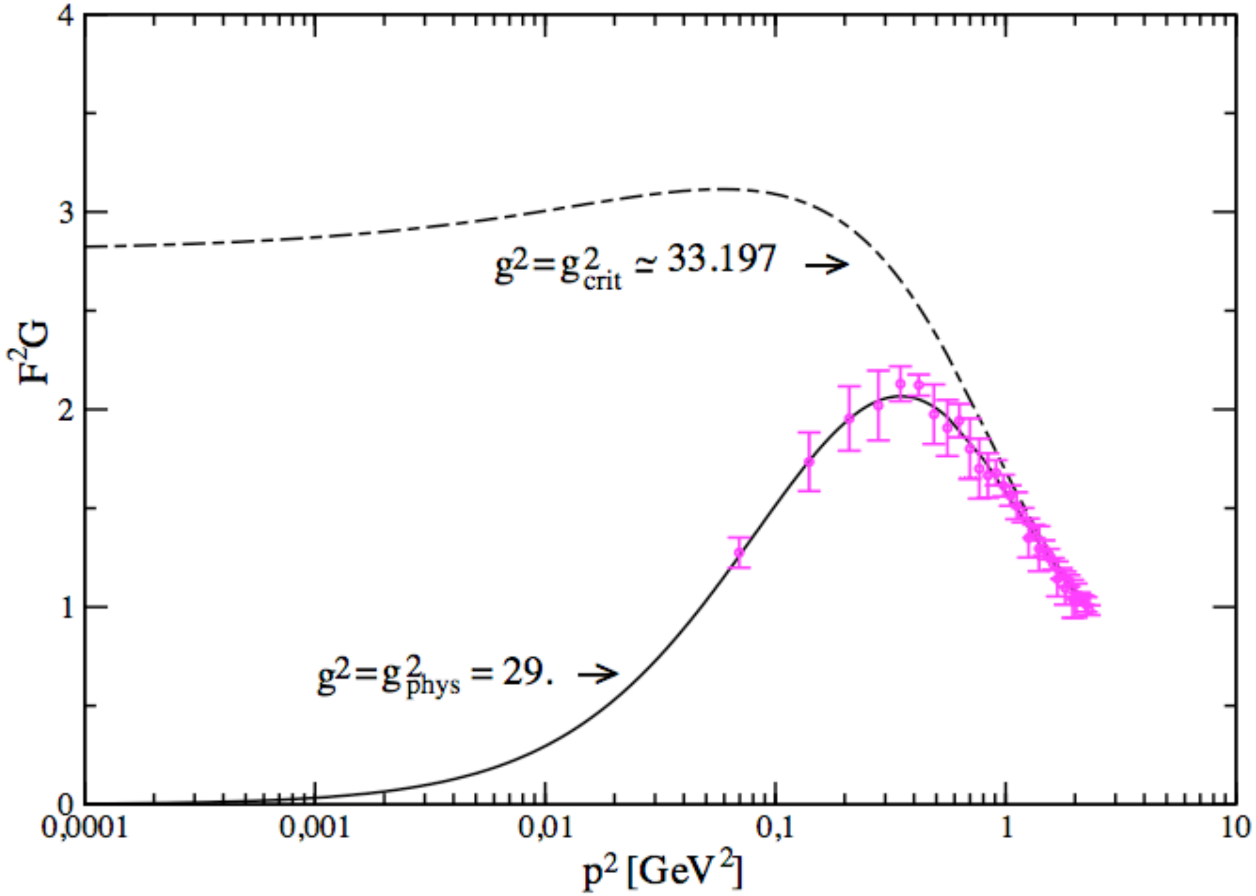


Ph. Boucaud, J-P. Leroy, A.Le Yaouanc, J. Micheli, O. Pene, J. Rodriguez--Quintero e-Print: arXiv:0801.2721 [hep-ph]

Figure 2: Comparison between the lattice SU(3) data at $\beta = 5.8$ and with a volume 32^4 for the ghost form factor and our continuum SD prediction renormalised at $\mu = 1.5\ GeV$ for $\tilde{g}^2 = 29$. (solid line) ; the agreement is striking ; also shown is the singular solution at $\tilde{g}^2 = 33.198....$ (broken line), which is obviously excluded.

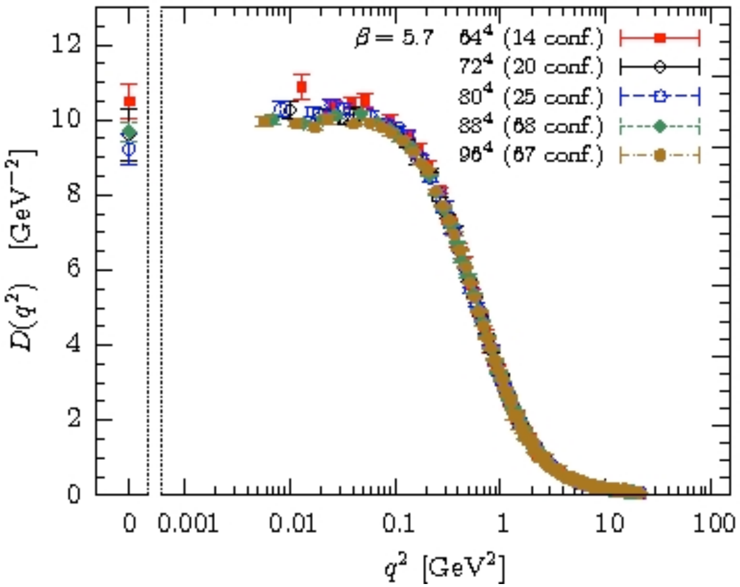
- The input gluon propagator is fitted from LQCD. The DSE is solved numerically for several coupling constants. The resulting F_R is compared to lattice results. For $g^2=29$, i.e. solution II ($F_R(0)$ finite, $\alpha_F=0$) the agreement is striking. The solution I ($F_R(0)$ infinite, $2\alpha_F + \alpha_G=0$) , dotted line, does not fit at all.

$F^2(p)G(p)$: the dotted line corresponds to the critical coupling constant. It is solution I, goes to a finite non zero value at $p \rightarrow 0$; the full line corresponds to the g2 which fits best lattice data. It corresponds to Solution II, $F^2(p)G(p) \rightarrow 0$ when $p \rightarrow 0$.

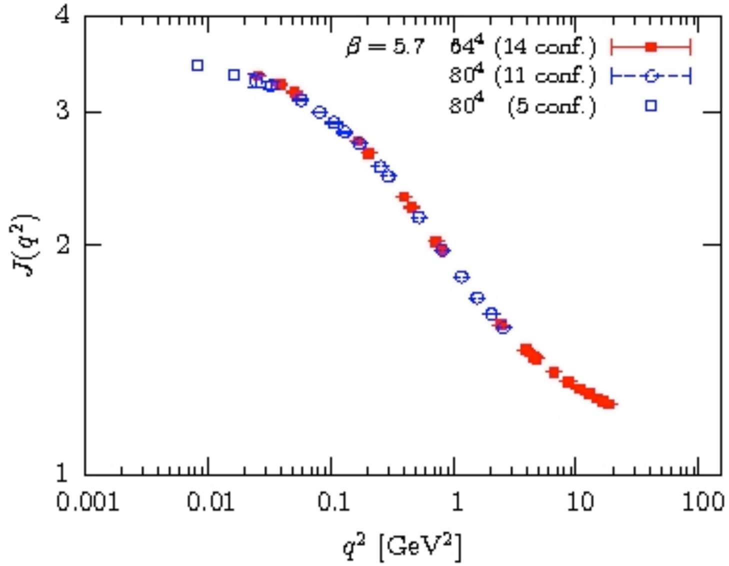


Ghost and gluon propagator from lattice,
(recent results)

Gluon propagator



Ghost dressing function



Lattice gluodynamics computation of Landau gauge Green's functions in the deep infrared.
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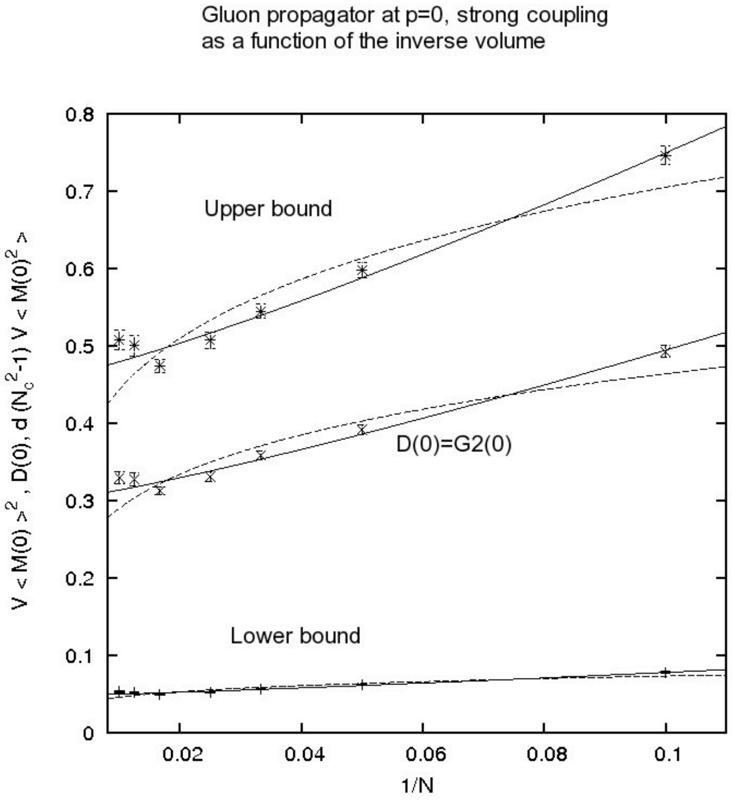
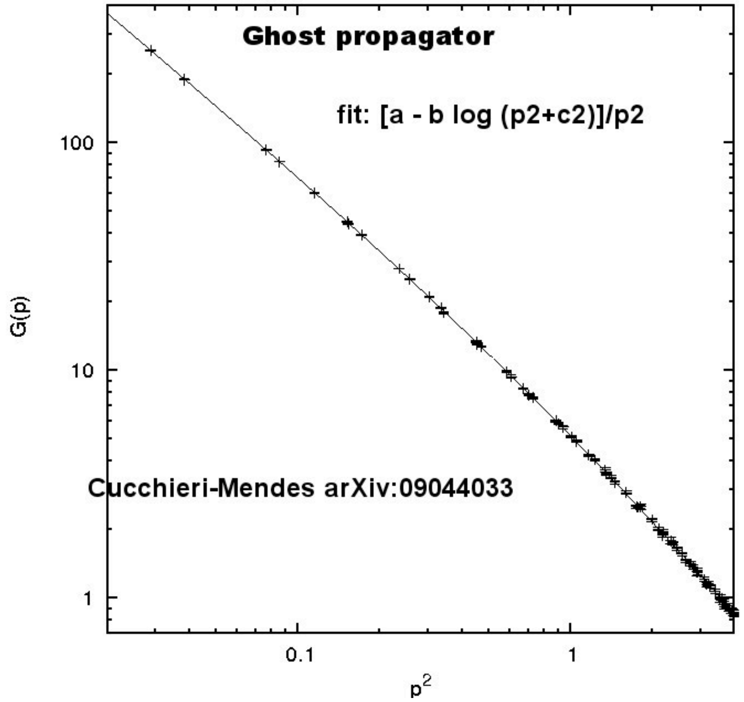
It results $\alpha_T(\mu^2) \propto \mu^2$ when $\mu \to 0$

Ghost and gluon propagator from lattice in strong coupling





Strong coupling: β=0

So what ?

- Lattice favours solution II (finite ghost dressing function and vanishing coupling constant)
- Possible loopholes in lattice calculations in the deep IR ? The discussion turns a little « ideological ». We should stay cautious about the deep infrared, but the trends are already clear around 300 MeV. Could there be a sudden change at some significantly smaller scale ? Why not ? But this looks rather far-fetched.
- The Gribov-horizion based interpretation of confinement is then in bad shape.
- Today we can’t say more

Ultra-Violet

Theory stands here on a much stronger ground

The issue is to compute Λ_{QCD} to be compared to Experiment. There are several ways of computing Λ_{QCD}.

IS THIS UNDER CONTROL ?

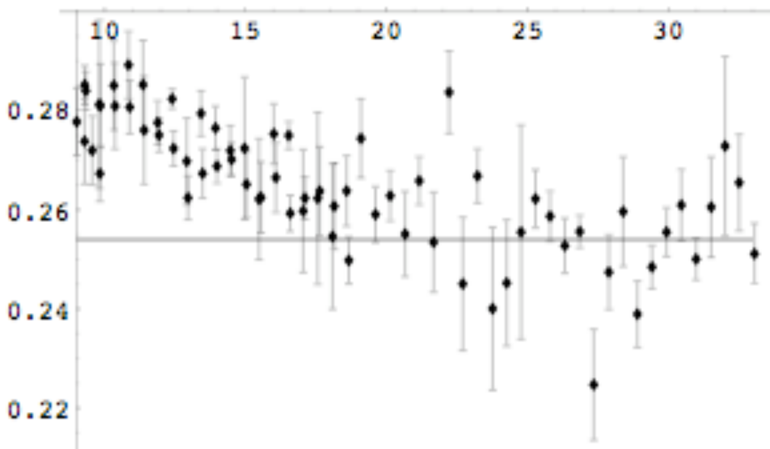
From $\alpha_T(\mu^2)$ to Λ_{MS}

Compute $\alpha_T(\mu^2) = g_0^2/(4\pi)\, F(\mu^2)^2\, G(\mu^2)$
from there compute Λ_{QCD}

$$\Lambda_{\widetilde{\mathrm{MOM}}} = \mu \exp\left\{\frac{-2\pi}{\tilde\beta_0 \alpha_{\widetilde{\mathrm{MOM}}}(\mu^2)}\right\} \left(\frac{\tilde\beta_0 \alpha_{\widetilde{\mathrm{MOM}}}(\mu^2)}{4\pi}\right)^{-\frac{\tilde\beta_1}{\tilde\beta_0^2}} \left(1 + \frac{\tilde\beta_1\ \alpha_{\widetilde{\mathrm{MOM}}}}{2\pi\tilde\beta_0} + \frac{\tilde\beta_2\ \alpha^2_{\widetilde{\mathrm{MOM}}}}{32\pi^2\tilde\beta_0}\right)^{\frac{\tilde\beta_1}{2\tilde\beta_0^2}}$$
$$\times \exp\left\{\frac{\tilde\beta_0\tilde\beta_2 - 4\tilde\beta_1^2}{2\tilde\beta_0^2\sqrt{\Delta}}\left[\arctan\left(\frac{\sqrt{\Delta}}{2\tilde\beta_1 + \tilde\beta_2\alpha_{\widetilde{\mathrm{MOM}}}/4\pi}\right) - \arctan\left(\frac{\sqrt{\Delta}}{2\tilde\beta_1}\right)\right]\right\}, \tag{21}$$

$$\frac{\Lambda_{\overline{\mathrm{MS}}}}{\Lambda_T} = e^{-\frac{c_1}{2\beta_0}} = e^{-\frac{507-40N_f}{792-48N_f}}.$$

$\Lambda_{\overline{MS}}$ (μ^2/[GeV^2]), *quenched*

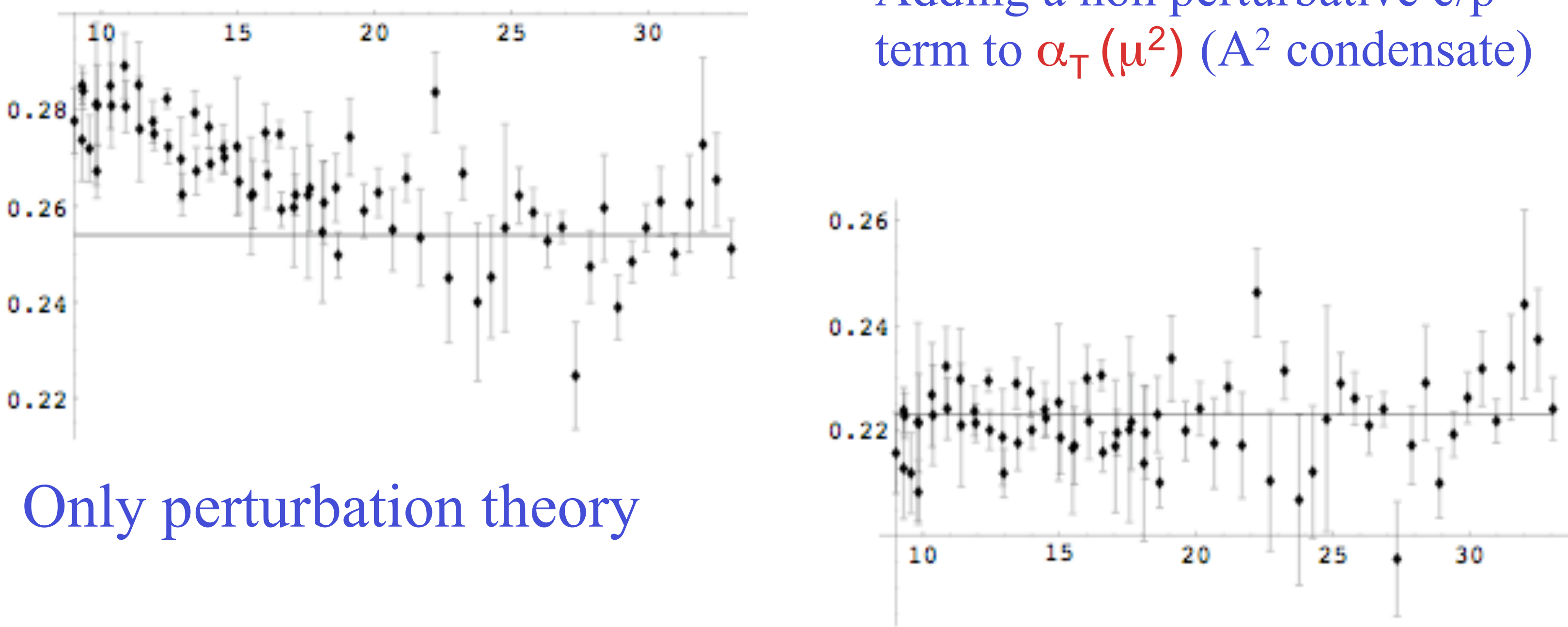


Only perturbation theory

Figure 2: (a) Plot of $\Lambda_{\overline{MS}}$ (in GeV) computed by the inversion of the four-loop perturbative formula eq. (23) as a function of the square of the momentum (in GeV^2); the coupling is estimated from the lattice data through eq. (9). (b) Same as plot (a) except for applying the non-perturbative formula eq. (33) for the coupling and looking for the gluon condensate generating the best plateau over $9 \lesssim p^2 \lesssim 33$ GeV^2.

$\Lambda_{\overline{MS}}$ $(\mu^2/[GeV^2])$, quenched

Adding a non perturbative c/p^2 term to $\alpha_T(\mu^2)$ (A^2 condensate)



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$<A^a_\mu A_a^\mu>$ condensate

- $<A^a_\mu A_a^\mu>$ is the only dimension-2 operator in Landau gauge. This makes it easier to apply Wilson expansion to different quantities. $<A^a_\mu A_a^\mu>$ should be the same for all quantities and the Wilson coefficient is calculable.

Condensate

- $\alpha_T(\mu^2) = \alpha_T^{pert}(\mu^2)(1+9/\mu^2\ (\mathrm{Log}(\mu^2/\Lambda^2))^{-9/44} g_T^2/32 <A^2>_R$
- The Wilson coefficient has only been computed at leading log.
- We vary the coefficient multiplying $1/\mu^2$, compute $\Lambda_{MS}(\mu^2)$ from

$\alpha_T^{pert}(\mu^2) = \alpha_T^{latt}(\mu^2)/(1+c/\mu^2)$ using the three loop formula and fit c to get a plateau on the resulting $\Lambda_{MS}(\mu^2)$.

- This gives both an estimate of Λ_{MS} and of $<A^a_\mu A_a^\mu>$

$\langle A^a_\mu A_a^\mu \rangle$ condensate

- $\langle A^a_\mu A_a^\mu \rangle$ is the only dimension-2 operator in Landau gauge. This makes it easier to apply Wilson expansion to different quantities. $\langle A^a_\mu A_a^\mu \rangle$ should be the same for all quantities and the Wilson coefficient is calculable.

Condensate

- $\alpha_T(\mu^2) = \alpha_T^{pert}(\mu^2)\left(1 + \underbrace{9/\mu^2 \left(\mathrm{Log}(\mu^2/\Lambda^2)\right)^{-9/44} g_T^2/32}_{\text{Wilson coefficient}}\ \langle A^2 \rangle_R\right)$

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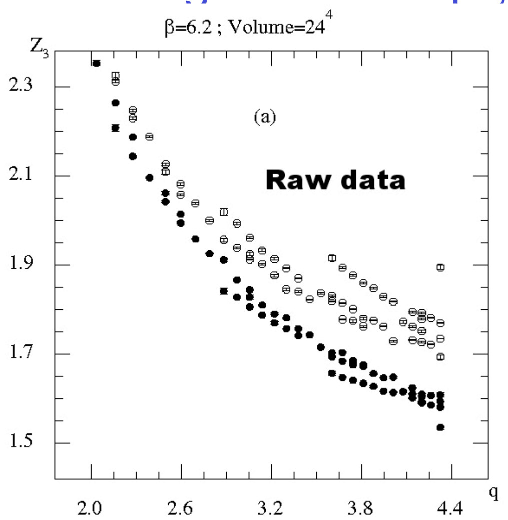
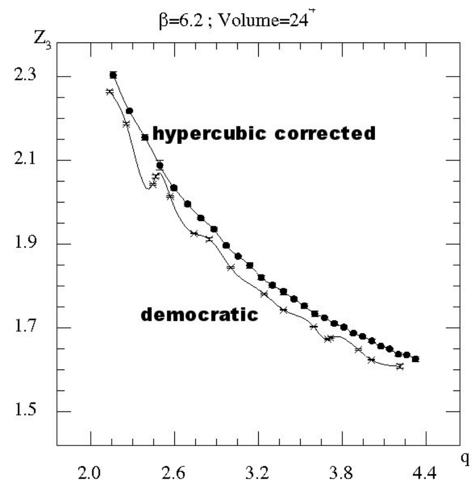
Lattice artefacts and scaling

- **Hypercubic artefacts:** The dependence of any lattice quantity as a function of p^2 is far from smooth, due to very different « geometries ». The H4 symmetry group of the lattice is only a subgroup of O(4). For example momenta $2\pi(2,0,0,0)/L$ and $2\pi(1,1,1,1)/L$ have the same p^2 but are not related by H4 symmetry.
- We define the H4 invariants $p^{[2n]}=\sum p_\mu^{2n}$ and expand, for example

$$\alpha_{meas}(p^2,p^{[4]},p^{[6]},..) = \alpha_{T,latt}(p^2) + c_4\, a^2 p^{[4]} + a^4 c_6\, p^{[6]} \ldots$$

- This being done the dependence in p^2 is very smooth below some limiting value of a^2p^2, but there are still O(4) invariant lattice artifacts





Lattice artifacts and scaling

- In order to have enough lever arm to study the dependence in μ, we combine several lattice spacing. The finer lattice spacings allow to go higher momenta where $1/\mu^4$ non-pertrubative contributions are reduced.
- We match the plots using ratios of lattice spacing taken from r_0, or we can fit the ratio of lattice spacings from the matching of $\alpha_T(\mu^2)$ from different lattice spacings. **Both methods agree fairly well.**

$\alpha_T(\mu^2/[\mathrm{GeV}^2])$

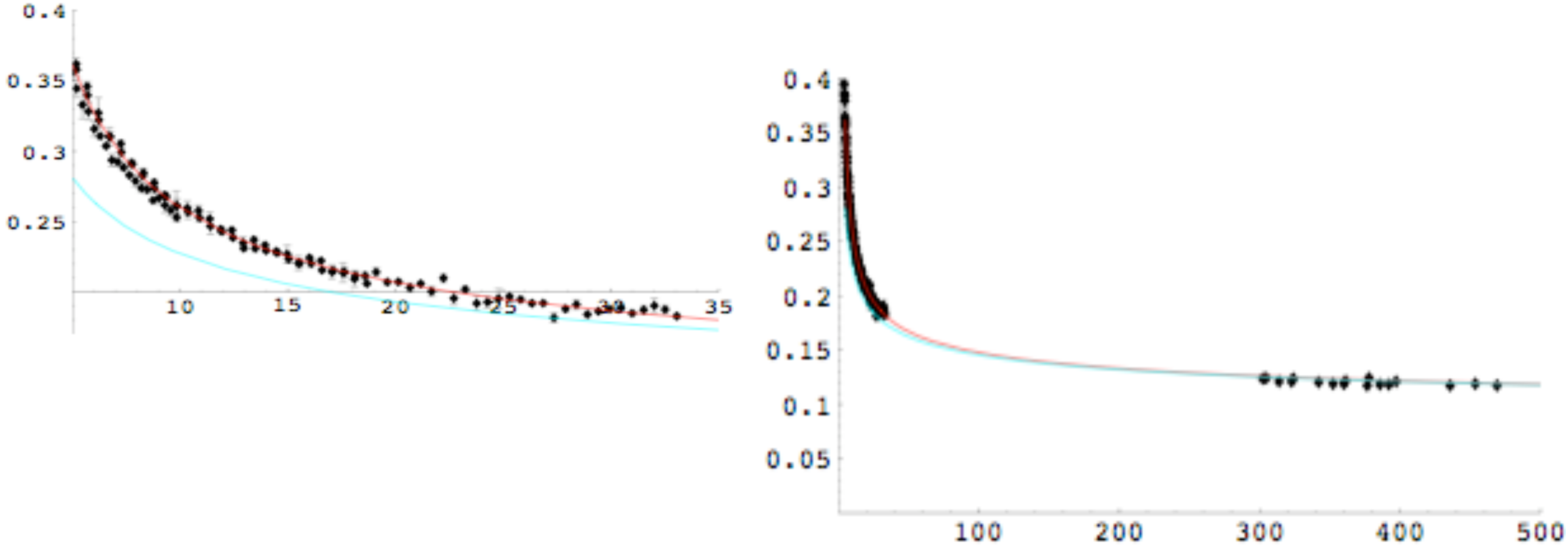


Figure 3: (a) Plot of α_T defined by eq. (9) in terms of the square of the renormalization momentum: the red solid line is computed with eq. (33) with $\Lambda_{\overline{MS}} = 224$ MeV, the blue one with eq. (23) for the same $\Lambda_{\overline{MS}}$ and the data are obtained from the lattice data set-up in table 1. (b) The same but with some additional lattice estimates for the coupling at very high momenta (300–500 GeV^2) taken from [9].

[9] A. Sternbeck, K. Maltman, L. von Smekal, A. G. Williams, E. M. Ilgenfritz and M. Muller-Preussker, PoS LAT2007 (2007) 256 [arXiv:0710.2965 [hep-lat]].

Different estimates of Λ_{MS} (μ^2/[GeV^2])

quenched case

$$\Lambda_{\overline{MS}}^{N_f=0} = 224^{+8}_{-5} \text{ MeV}$$
$$g_T^2 \langle A^2 \rangle_R = 5.1^{+0.7}_{-1.1} \text{ GeV}^2 .$$

- The non perturbative contribution is sizeable !!!

$\alpha_T(\mu^2) \sim \alpha_T^{pert}(\mu^2)(1+1.4/\mu^2)$ (1.4 % at 10 GeV)

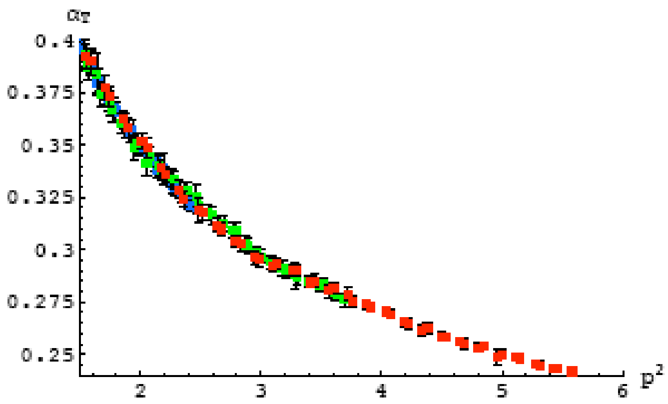
- There is a fair agreement from very different estimates.

	F^2G (this work)	Asym. 3-g [8]	Sym. 3-g [8]	F/G [22]	[2]
$\Lambda_{\overline{MS}}$ (MeV)	224^{+8}_{-5}	260(18)	233(28)	270(30)	238(19)
$\sqrt{\langle A^2 \rangle_{R\mu}}$ (GeV)	1.64(17)	2.3(6)	1.9(3)	1.3(4)	–

Table 3: Comparison of estimates of $\Lambda_{\overline{MS}}$ obtained from the analysis of the ghost-gluon vertex in this work (first column), the asymmetric 3-gluon vertex (second), the symmetric 3-gluon vertex (third), the ratio of gluon and ghost dressing functions (fourth) and with the Schrödinger functional method (last). The gluon condensate $\langle A^2 \rangle_{R\mu}$ has been obtained at the renormalization momentum $\mu = 10$ GeV, for the sake of comparison with the other estimates, from eq. (35) by applying $g^2(\mu^2 = 100 \text{ GeV}^2)/4\pi = 0.15$.

[2] M. Luscher, R. Sommer, P. Weisz and U. Wolff, Nucl. Phys. B 413 (1994) 481; S. Capitani, M. Luscher, R. Sommer and H. Wittig [ALPHA Collaboration], Nucl. Phys. B 544 (1999) 669 [arXiv:hep-lat/9810063].

Unquenched case, Nf=2 twisted mass configurations (ETMC)



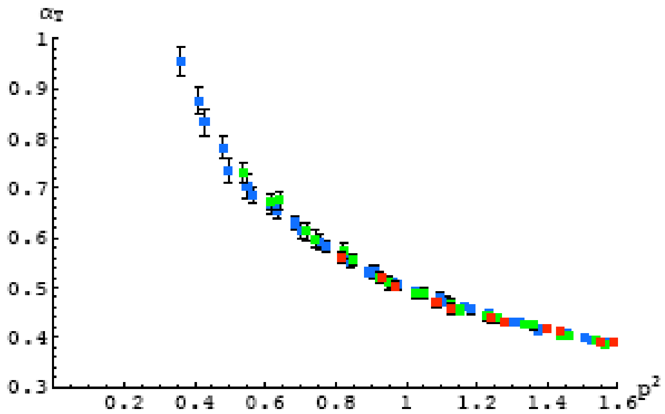


Figure 1: the QCD coupling defined by Eq. (3) from the three lattice data sets employed: red squares stand for $\beta = 4.2$, green ones for $\beta = 4.05$ and blue for $\beta = 3.90$. Right (left) plot shows estimates for momenta above (below) 10 GeV^2. the physical value of the momentum in x-axis is obtained by applying the ratios of lattice sizes in tab. 1 and $a(3.9)^{-1} = 2.301 GeV$.

	This note	String tension	deviation (%))
$a(3.9)/a(4.05)$	1.223(4)	1.277	4.2
$a(3.9)/a(4.2)$	1.504(4)	1.547	2.8

Unquenched Λ_{MS} (μ^2/[GeV^2])

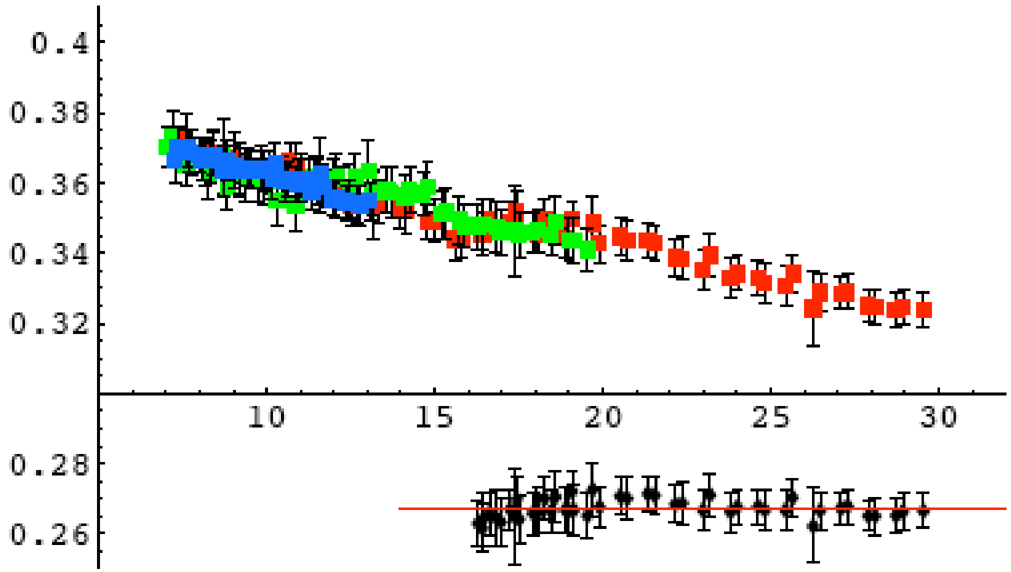


Figure 2: $\Lambda_{\overline{MS}}$ derived from confronting the lattice value of α_T with the perturbative+OPE prediction, in terms of the momentum where α_T is estimated from the lattice, as described in ref. [1].

$$\Lambda_{\overline{MS}} = 267 \pm 11 \text{ MeV} ;$$

$g_T{}^2\langle A^2\rangle=9.6(5)$ GeV^2

Even larger than in quenched

Conclusions

- **IR:** The ghost propagator Dyson-Schwinger equation allows for two types of solutions,

I) with a divergent ghost dressing function and a finite non zero F^2G, i.e. the relation $2\alpha_F + \alpha_G = 0$ (fn: $\alpha_D = 2\,\alpha_G$), « conformal solution »;

II) with a finite ghost dressing function and the relation $\alpha_F = 0$ (fn: $\alpha_G = 0$), « decoupled solution » and a vanishing F^2G

Lattice QCD clearly favors II)

- **UV:** $\alpha_T(\mu^2)$ is computed from gluon and ghost propagators.

Discretisation errors seem under control. Perturbative scaling is achieved only at about 3 GeV (small lattice spacings) *provided a sizeable contribution of <A2> condensate is taken into account*. Different estimates of Λ_{MS} and of the condensate agree failry well.

Extension under way to the unquenched case

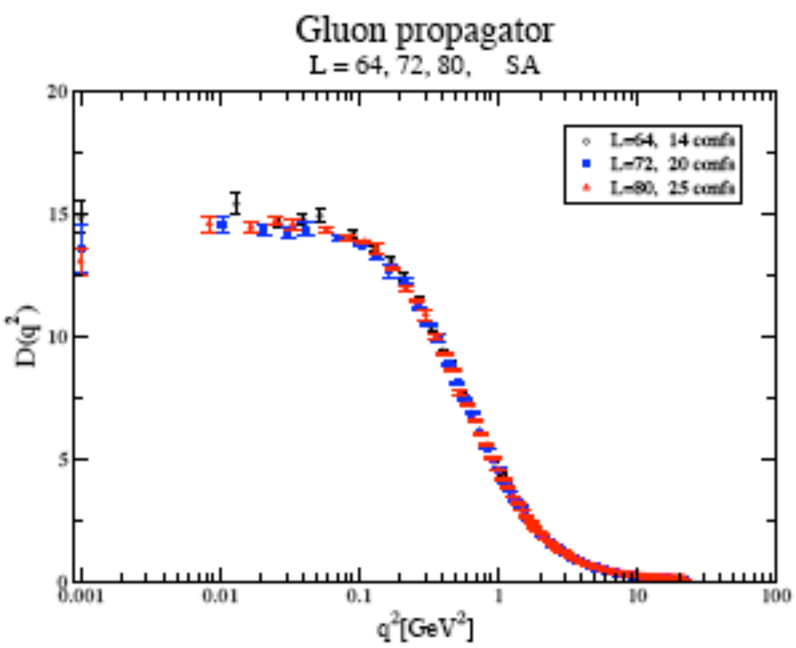
Back-up slide What do we learn from big lattices ?

N^3	a_G	N^4	a_G	N^3	a_G	N^4	a_G
140^3	0.073(4)	48^4	0.093(7)	140^3	0.13(2)	48^4	0.19(4)
200^3	0.051(3)	56^4	0.063(6)	200^3	0.06(2)	56^4	0.18(4)
240^3	0.003(3)	64^4	0.049(9)	240^3	0.10(2)	64^4	0.17(4)
320^3	-0.021(9)	80^4	0.052(5)	320^3	0.01(5)	80^4	0.15(2)
		112^4	0.038(6)			112^4	0.10(7)
		128^4	0.016(5)			128^4	0.06(3)

Table 1: Table for the ghost propagator IR exponent a_G, in the 3d and 4d cases, obtained using either the two smallest nonzero momenta (left) or the third and fourth smallest nonzero momenta (right).

Attilio Cucchieri, Tereza Mendes.Published in PoS (LATTICE 2007) 297. arXiv:0710.0412 [hep-lat]

here $\alpha_G = -\alpha_F$ (in our notations) $= -\delta_{gh}$

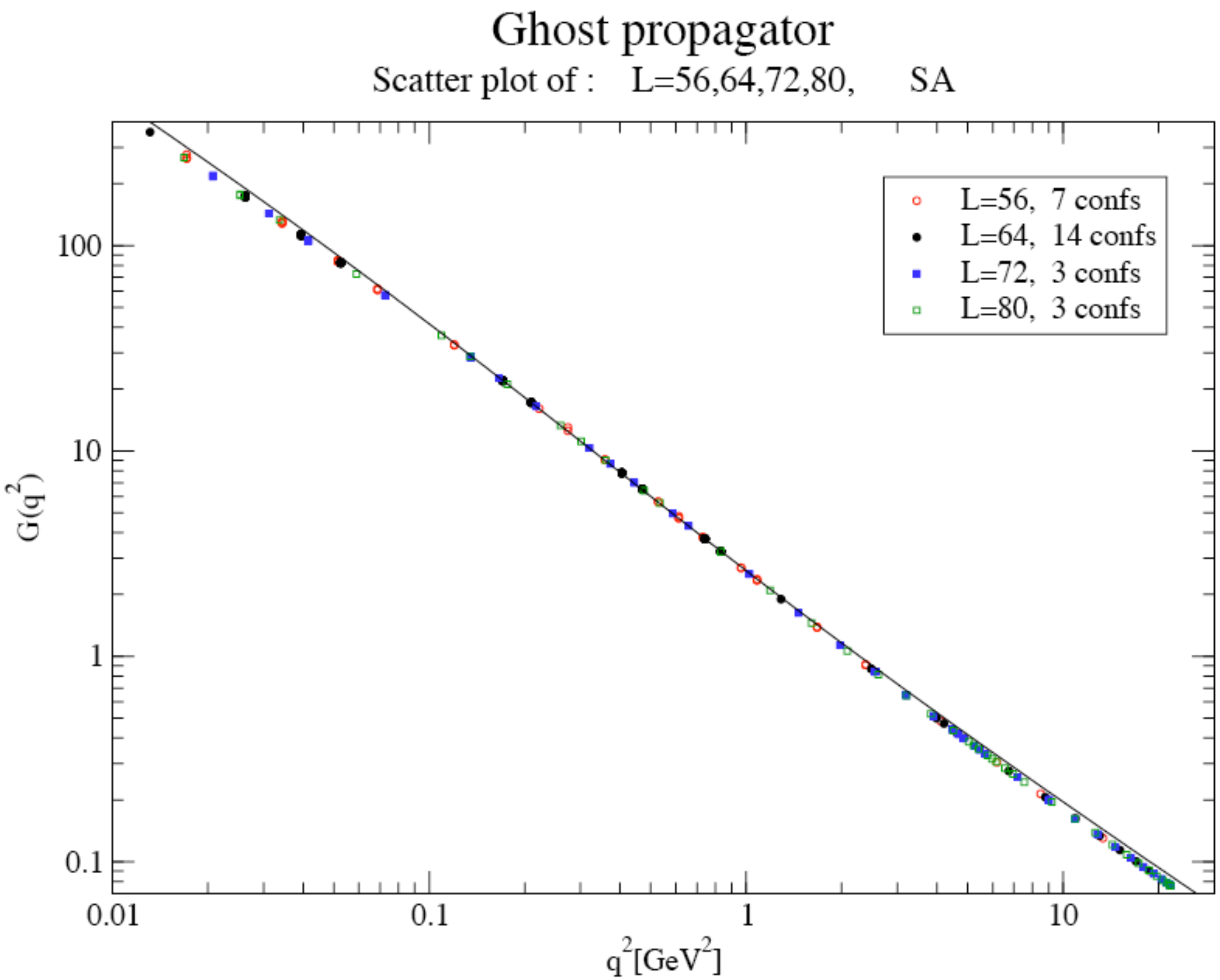


I.L. Bogolubsky, E.M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck Published in PoS(LATTICE-2007)290. arXiv:0710.1968 [hep-lat]

They find α_F=-0.174 which seems at odds with both α_F=0 and $\alpha_F \leq$-0.5

But the fit is delicate, the power behaviour is dominant, if ever, only on a small domain of momenta.

Back-up slide

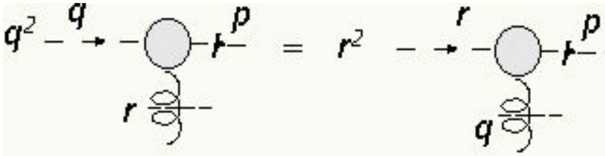


comparison of the lattice data of ref arXiv:0710.1968 with our solution arXiv:0801.2721.

Back-up slide

IR Ghost propagator from WST identities

hep-ph/0007088, hep-ph/0702092



$$F(p^2)X(q,p;r) = F(r^2)X(q,r;p)$$

$$X(q,p;r) \; = \; a(q,p;r) - (r \cdot p)\, b(q,p;r) + (r \cdot q)\, d(q,p;r)$$

Cut lines: the external propagator has been cut; p,q,r : momenta

Identities valid for all covariant gauges

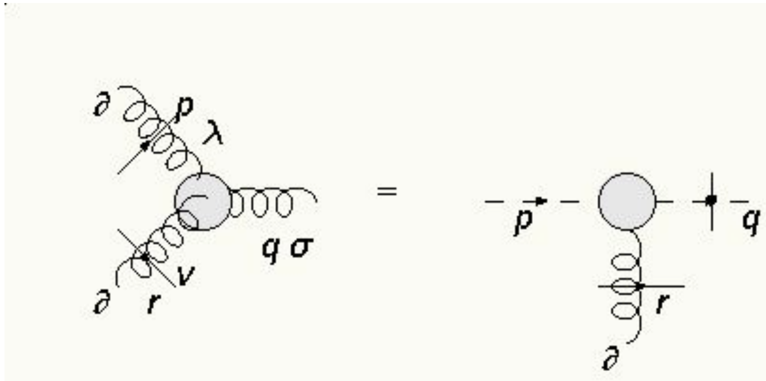
- Assuming X regular when one momentum vanishes, the lhs is regular when r → 0, then the ghost dressing has to be finite non zero:

F(0) finite non zero, α_F=0 (fn: $p^2G(p^2)$ →finite≠0 ,α_G=0 or δ_{gh} = 0, κ =0)

- *There is almost no way out, unless the ghost-gluon vertex is singular* when only one momentum vanishes (difficult without violating Taylor's theorem)
- Does this contradict DS equations ? Lattice ? We do not believe, see later

Back-up slide

IR Gluon propagator from WST identities, hep-ph/0507104, hep-ph/0702092



∂: longitudinal propagator, the third gluon is transverse.

Cut lines: the external propagator has been cut,

p,q,r : momenta

λνσ: gluons polarisation

When q →0 it is easy to prove that the lhs vertex function vanishes under mild regularity assumptions. The rhs goes to a finite limit (see Taylor theorem).Then the transverse gluon propagator must diverge:

$G^{(2)}(0)=\infty$,

α_G<1 (frequent notation: $D(p^2) \to \infty$ α_D <1 or δ_{gl} <1; κ < 0.5)

or **α_G=1** with very mild divergences, to fit with lattice indications: $G^{(2)}(0)$ = finite$\neq$0

unless

The three gluon vertex diverges for one vanishing momentum. It might diverge only in the limit of Landau gauge.

In arXiv:0801.2762, Alkofer et al, it is proven from DS that α_G **>=** 0, **OK**