Putting it altogether - The CKM matrix

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Abstract. I review our current understanding of the Cabibbo-Kobayashi-Maskawa matrix within the Standard Model, with a focus on some current tensions between the available observables. I discuss the extension of the CKMfitter approach for \( \Delta F = 1 \) New Physics, namely the two-Higgs doublet model of type II.

Keywords: Electroweak interactions, CKM matrix, Two-Higgs doublet

In the Standard Model (SM), the weak charged-current transitions mix quarks of different generations, which is encoded in the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix. In the case of three generations of quarks, the physical content of this matrix reduces to four real parameters, among which one phase, the only source of CP violation in the Standard Model (the lepton sector can also exhibit similar sources of CP violation once masses, provided by New Physics (NP), are considered). One can define these four real parameters as:

\[
\lambda^2 = \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2}, \quad A^2 \lambda^4 = \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2}, \quad \tilde{\rho} + i\tilde{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}. \quad (1)
\]

This parametrisation is exact, unitary to all orders in \( \lambda \) and independent of phase conventions. A Wolfenstein-like parametrisation of the CKM matrix can be derived up to an arbitrary power in the Cabibbo angle \( \lambda = \sin(\theta_C) \), using the unitarity of the matrix to determine all its elements. A challenge for both experimentalists and theorists consists in extracting information on these parameters and the underlying mechanism of CP violation from the wealth of data currently available, in the presence of the strong interaction that binds quarks into hadrons. The CKMfitter group pursue this goal using the Rfit frequentist approach [1].

Since the era of \( B \)-factories is coming to its end, it is quite natural that experimental updates are scarcer nowadays. An interesting recent update came concerning \( B \rightarrow \rho \rho \) decays, since the world averages for the branching ratio and the longitudinal polarisation for \( \rho^+\rho^0 \) have increased due to a new BaBar value [3]: \( \mathcal{B}(\rho^+\rho^0) = (18.2 \pm 3.0) \times 10^{-6} \rightarrow (24.0 \pm 1.9) \times 10^{-6} \) and \( f_L(\rho^+\rho^0) = 0.912 \pm 0.044 \rightarrow 0.950 \pm 0.015 \). As is well known, \( \alpha \) can be extracted from \( B \rightarrow \pi\pi, \rho\pi \) or \( \rho\rho \) decays. First, an effective value \( \alpha_{eff} \) is determined from the time-dependent CP-asymmetry in \( \bar{B} \rightarrow h^+h^- \). The analysis of the other modes can be done based on isospin relations to eliminate the pollution from penguin contributions and determine: \( \alpha_{eff} - \alpha \), through a geometric reconstruction of isospin triangles up to an eight-fold ambiguity. Comparing the three different cases (\( \pi\pi, \rho\pi \) and \( \rho\rho \)), one finds that the smallest penguin pollution, and thus the most stringent bound, comes from \( \rho\rho \).
FIGURE 1. On the left: 95% CL constraints on the unitarity triangle from the global fit. On the right: constraints on alpha from $B \to \pi \pi, \rho \pi, \rho \rho$ decays, showing a good agreement among the three channels and with the global fit expectation.

The BaBar update forces the $\rho \rho$ isospin triangles not to close for the central values, which means that one prefers flat triangles with degenerate mirror solutions. This has an impact on the determination of $\alpha$, which used to $(88.2^{+6.1}_{-4.8})^\circ$ in Summer 08 and is now $\alpha = (89.0^{+4.4}_{-4.2})^\circ$. $\alpha$ becomes a precise measurement (5%) at same level as $\beta$ (4.2%). Through a toy Monte-Carlo using the best fit for central values, we have studied the distribution of the 1$\sigma$ error on $\alpha$ for $\rho \rho$: the average toy error is $7.5^\circ$ (observed: $5.4^\circ$), with a long asymmetric tail due to configurations with non-degenerate mirror solutions, and 68% of the toys have a larger error than the data. We have also modeled isospin-breaking effects and checked that even a sizeable isospin breaking contribution to $A_{+0}$ has only a very mild impact on the determination of $\alpha$ at 95% CL.

The global fit exhibits currently a slight tension between the various observables. Indeed, the global fit $\chi^2_{\text{min}}$ drops by $\sim 2.4\sigma$ if $\sin 2\beta_{\overline{c}c}$ or $B \to \tau \nu$ is removed. Before claiming any effect from new physics, one should consider the possibility of a fluctuation in the experimental measurements or in the theoretical inputs. On the experimental side, both $\sin 2\beta_{\overline{c}c}$ or $B \to \tau \nu$ show a good agreement between BaBar and Belle. On the theoretical side, the issue is not restricted to the value of $f_{B_d}$ since one observes a $2.4\sigma$ discrepancy from the ratio

$$\frac{\mathcal{B}(B \to \tau \nu)}{\Delta m_d} = \frac{3\pi}{4} \frac{m^2_{\tau B}}{m^2_W} \eta_{B S}[\chi_t] \left(1 - \frac{m^2_{\tau}}{m^2_{B}}\right)^2 \sin^2 \beta \frac{\sin^2(\alpha + \beta)}{\sin^2(\alpha + \beta)} \frac{1}{|V_{ud}|^2 B_{B_d}}. \tag{2}$$

Two possible ways of escaping the current tension would be a change in the measured branching ratio of $B \to \tau \nu$ or a correlated change in lattice estimates of $f_{B_d}$ and $B_{B_d}$.

Another topic discussed recently consists in $\varepsilon_K$, which is the only input from (indirect) CP-violation in $K$ sector. It is dominated by long distances and can be evaluated on the...
lattice $\hat{B}_K = 0.721 \pm 0.005 \pm 0.040$, with the usual relation:

$$|\varepsilon_K| = C_\varepsilon \hat{B}_K \lambda^2 \eta^2 |V_{cb}|^2 \left[|V_{cb}|^2 (1 - \bar{\rho}) \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_t, x_c) - \eta_{cc} S_0(x_c)\right]$$  \(3\)

with the well-known Inami-Lim functions $S_0$ of $x_q = m_q^2 / m_W^2$ and $C_\varepsilon$ normalisation. Recently [2], it was advocated that a multiplicative correction $\kappa_\varepsilon$ should be applied, because of neglected contributions from related to the $\pi\pi$ strong phase $\phi \neq \pi/4$ and the amplitude $ImA_0 \neq 0$. If the estimate $\kappa_\varepsilon = 0.92 \pm 0.02$ is currently scrutiny, it is interesting to notice how the presence (or the absence) of a tension in a fit depends on the treatment of errors. Indeed, if we include this correction and treat the uncertainty on $\kappa_\varepsilon$ in a Gaussian way, we observe a tension ($\sim 1.5 \sigma$) in the global fit. But if we stick to our usual and more conservative Rfit treatment of systematics, we observe no tension, as can be seen in the above figure.

Flavour physics and CP-violation data provide stringent constraints on the parameters of extensions of the Standard Model, which yield generally fine-tuning problems. From this point a view, a rather minimal extension of the SM, with a limited number of new parameters to fix, consists in two-Higgs doublets models (2HDM). The additional parameters needed to describe this SM extension are the masses of the additional Higgs bosons $H^\pm, H^0$ and $A$, the ratio of vacuum expectation values $\tan\beta$ and an angle describing $h^0 - H^0$ mixing. A particularly alluring version of 2HDM is type II, where a doublet couples to down-type quarks and the other one to up-type quarks (and to leptons), because of its resemblance with the SM in the quark sector: the CKM matrix remains the only source of flavour changing interactions and there are no flavour-changing neutral currents at tree level. But there are new flavour-changing charged interactions, corresponding to the exchange of a charged Higgs rather than a $W$ (obviously, there are also interactions with neutral Higgs fields).

We have collected measured decays potentially sensitive to contributions from charged Higgs, for which a good control of the theoretical hadronic uncertainties can be
achieved: a) the leptonic decays $\Gamma[K \to \mu \nu]/\Gamma[\pi \to \mu \nu]$, $\mathcal{B}[D \to \mu \nu]$, $\mathcal{B}[D_s \to \mu \nu]$, $\mathcal{B}[B \to \tau \nu]$, and $\mathcal{B}[B \to \pi \nu]$, b) the semileptonic decays $B \to D \tau \nu$, through the ratio $\mathcal{B}[B \to D \tau \nu]/\mathcal{B}[B \to D \ell \nu]$, and $K \to \pi \nu$, through the ratio $\mathcal{B}[K \to \pi \nu]/\mathcal{B}[K \to \pi \nu]$, c) the $Z$ partial width into $b$ quarks, $R_b = \Gamma[Z \to bb]/\Gamma[Z \to \text{hadrons}]$, d) the FCNC radiative decay $b \to s \gamma$ through the ratio $\mathcal{B}[\bar{B} \to X_s \gamma]/\mathcal{B}[\bar{B} \to X_c \bar{\nu}]$.

The most stringent constraints come from leptonic decays and from $b \to s \gamma$ [4]. The minimum $\chi^2$ of $\chi^2_{\text{min}} \simeq 11$ is obtained for $m_{H^+} > 600$ GeV. At high $H^+$ masses and irrespective of the value of $\tan \beta$ (decoupling limit), the charged Higgs contribution becomes negligible for all the processes we are considering in this analysis, so that the SM predictions are recovered. The large $\chi^2_{\text{min}}$ value is due to the tension between: the large measured $\mathcal{B}[B \to \tau \nu]$ which favours a fine-tuned solution at low $m_{H^+}$ values, and all the other observables, in fair agreement with the SM expectations which select large values for the charged Higgs mass. The global fit hence selects large values of the charged Higgs mass, but with a penalty from the $\mathcal{B}[B \to \tau \nu]$ branching ratio. A lower limit of the charged Higgs mass can be inferred: $m_{H^+} > 323$ GeV at 95% CL, while no significant constraint is obtained for $\tan \beta$. This can be compared to the bound obtained from direct searches at LEP for any value of $\tan \beta$: $m_{H^+} > 78.6$ GeV at 95% CL.

REFERENCES

4. The CKMfitter Group, in preparation