

Modeling of pedestrians

Cecile Appert-Rolland

Abstract Different families of models first developed for fluid mechanics have been extended to road, pedestrian, or intracellular transport. These models allow to describe the systems at different scales and to account for different aspects of dynamics. In this paper, we focus on pedestrians and illustrate the various families of models by giving an example of each type. We discuss the specificities of crowds compared to other transport systems.

1 Introduction

What is the common point between fluids, cars, pedestrians or molecular motors? Though they are quite different and evolve in systems of very different sizes, they all result into flows, and they all obey simple conservation laws. As a result, the families of models that have been developed in the past to describe fluids at different scales have also been adapted to describe highway traffic [1], crowds [2] or axonal transport [3, 4].

Let us consider first macroscopic models: At large scales, individuals are not visible anymore, and the state of the system can be characterized by locally averaged density and velocity. For fluids, Navier-Stokes equations express the conservation of mass and of momentum.

For road traffic, mass conservation is still relevant, and provides a first equation relating density and velocity. However, as vehicles are in contact with the road, momentum is not conserved. A second relation must be provided to close the equations. The simplest way is to give the (possibly data-based) fundamental diagram, relating the flow of vehicles and the density. The resulting model is a so-called first order model, a prominent example being the LWR model [14, 15]. The more sophisticated

Cecile Appert-Rolland
Lab. of Theoretical Physics, CNRS UMR 8627, Univ. Paris-Sud, Bât. 210, 91405 Orsay Cedex,
France. e-mail: Cecile.Appert-Rolland@th.u-psud.fr

Table 1 Correspondence of model families, for four different physical systems: fluids, road traffic, pedestrian traffic and intracellular traffic. We mention a few models (with their reference) as prominent and/or historical examples of a given model type. The scale at which the system is described increases as one goes down in the table.

Fluids	Road Traffic	Pedestrians	Molecular Motors
Molecular Dynamics [5, 6] $m \mathbf{a} = \sum \mathbf{f}$	Car-following [7, 8, 9] $a(\Delta V, \Delta x)$	Ped-following	Molecular Dynamics $m \mathbf{a} = \sum \mathbf{f}$
Kinetic theory $P(v,x,t)$	Kinetic theory $P(v,x,t)$	Kinetic theory $P(v,x,t,\xi)$	
Cellular automata <i>FHP Model</i> [10]	Cellular automata <i>Nagel-Schreckenberg model</i> [11]	Cellular automata <i>Floor Field model</i> [12]	Cellular automata <i>Langmuir kinetics</i> [13]
Continuous PDEs Conservation of mass and momentum <i>Navier-Stokes Eqs</i>	Continuous PDEs Conservation of mass + fundamental diagram $j(\rho)$ <i>LWR Model</i> [14, 15]	Continuous PDEs Conservation of mass and ...	Continuous PDEs Open system: balance of fluxes

second-order models [16, 17] express the fact that the adjustment of flow to density may not be instantaneous but rather takes place within a certain relaxation time. The second relation between density and velocity is then a second partial differential equation.

For pedestrians also, the mass conservation equation must be completed to provide a closed set of equations. However, the complexity is increased by the fact that pedestrians, first, walk in a two-dimensional space, and, second, do not necessarily all go in the same direction.

Within cells, intracellular transport also involves some “walkers”, i.e. some molecules equipped with some kind of legs that perform stepping along some cylindrical tracks called microtubules. In contrast with human pedestrians, these so-called molecular motors do not only walk along microtubules, they can also detach from the microtubules, diffuse around, and attach again. Thus, if one considers the density of motors on the microtubule, even mass conservation is not realized any more. The equations that determine the evolution of density and velocity must then rather express some balance of fluxes between different regions of the system.

In the same way as various macroscopic models can be proposed for all these systems, there are some equivalents of molecular dynamics or of cellular automata approaches that have been developed for road, pedestrian or intracellular traffic.

In most cases, for a given physical system, different types of models have been proposed independently to account for the behavior of the system at different scales, leading to large families of models. In some cases however, it is possible to relate

the models at the different scales and to understand how the macroscopic behavior can emerge from the individual dynamics.

In this paper, we shall focus on pedestrian modeling, and give an example for each family of models. Part of this work (sections 2, 3, and part of 4) was performed in the frame of the interdisciplinary PEDIGREE project [48]. The teams involved are presented in Table 2.

The work of section 5 was performed as part of the master and PhD of Julien Cividini, in collaboration with H. Hillhorst.

Table 2 The PEDIGREE Project involved four French teams listed below.

Laboratory	IMT	INRIA	CRCA	LPT
Team Leader	P. Degond	J. Pettré	G. Theraulaz	C. Appert-Rolland
Participants	J. Fehrenbach J. Hua S. Motsch J. Narski	S. Donikian S. Lemerrier	O. Chabiron E. Guillot M. Moreau M. Moussaïd	J. Cividini A. Jelić

2 Ped-following model

Fluids can be described at the level of molecules, by taking into account all the interaction potentials between atoms in a more or less refined way, as is done in molecular dynamics simulations [5, 6]. When vehicles or pedestrians are considered, two main difficulties arise. First the interaction potential is not known - actually the interaction cannot in general be written as deriving from a potential. Second, the interaction is in general highly non-isotropic, and does not depend only on the position but also on the velocity and on the target direction of each individual.

In road traffic, cars naturally follow lanes. This feature greatly simplifies the problem. Each car has a single well-defined predecessor on its lane. Apart from lane changes, a car driver can only adjust its speed. He will do so depending on the conditions in front (distance, velocity, acceleration of the predecessor). Actually several cars ahead could be taken into account (and indeed some empirical studies [18] have shown that a driver may take into account several of its leaders). But still, there is a clear hierarchy among the leaders, given by their order in the lane.

In pedestrian traffic, individuals evolve in a two-dimensional space, and may interact with several pedestrians at the same time, without a clear hierarchy. Besides, the combination of interactions is in general not a simple sum of one-by-one interactions. However, there are situations where the flow is organized in such a way that it is quasi one dimensional.

For example in corridors, all pedestrians mostly go in the same direction. Even if two opposite flows are considered, it is known that some lanes are formed spontaneously, and within each lane the flow is again quasi one dimensional and one directional.

The way pedestrians follow each other is even more clear when pedestrians walk in a line. Such a configuration can be met for example in very narrow corridors. It has been realized in several experiments [19, 20, 21], in order to study how pedestrians react when they can only adjust their speed. One may then wonder how the acceleration of a pedestrian is related to the distance, velocity, acceleration of its predecessor, and how the behavior of a pedestrian differs from the one of a car. However, in order to evaluate the following behavior of a pedestrian, one needs to be able to track at the same time, and on long enough time windows, the trajectory of both the pedestrian under consideration and its predecessor.

Such an experiment has been realized in the frame of the PEDIGREE project [48]. Pedestrians were asked to walk as a line, i.e. to follow each other without passing [22]. Their trajectory was circular, in order to avoid boundary effects. The motion of all pedestrians was tracked with a high precision motion capture device (VICON) [49]. As a result, the trajectories of all pedestrians were obtained for the whole duration of the experiment (from 1 to 3 minutes).

Various combinations of the dynamic coordinates of the predecessor have been tested against the acceleration a of the follower. It turned out that the best correlation was obtained [22, 23] for the relation

$$a(t) = C \frac{\Delta v(t - \tau)}{[\Delta x(t)]^\gamma} \quad (1)$$

where v is the velocity of the predecessor, and Δx the distance between the predecessor and its follower.

One important difference with car traffic is the time delay τ introduced in the velocity: While the follower is able to evaluate quite instantaneously the position of his predecessor, he needs some time delay τ to evaluate his velocity.

Another difference with car traffic is the ability of pedestrians to flow even at very large local densities [24]. In the aforementioned experiment, the velocity was still of the order of 1 or 2 dm/s at local densities as high as 3 ped/m. This can be achieved thanks to the ability of pedestrians to keep walking even at very low densities: they can reduce the amplitude of their steps almost to zero while still keeping a stepping pace almost constant [25].

In contrast to cars, pedestrians can also take advantage of any space left by the predecessor, synchronizing partially their steps as was observed in previous experiments [19]. Surprisingly, this synchronization effect is also observed for pedestrians walking at a larger distance [25], probably as a result of the tendency of pedestrians to synchronize with external rhythmic stimuli [21].

Here we have presented a model for one-dimensional pedestrian flows. In general, pedestrians move in a two-dimensional space, and various agent based models have been proposed which we shall not review here.

3 One-dimensional bi-directional macroscopic model for crowds

At the other extreme, when seen from a distance, crowds can be described as continuous fluids. As mentioned in the introduction, one important difference with fluids is that pedestrians have a target - which may not be the same for all of them. A simple configuration is met in corridors: the flow is quasi one-dimensional, but pedestrians can walk in both directions. There is thus a need to distinguish two densities ρ_{\pm} of pedestrians, one for each walking direction. Each density obeys a conservation law:

$$\begin{aligned}\partial_t \rho_+ + \partial_x(\rho_+ u_+) &= 0, \\ \partial_t \rho_- + \partial_x(\rho_- u_-) &= 0,\end{aligned}$$

where u_{\pm} is the locally averaged velocity of pedestrians going in the \pm direction.

Two other relations are needed to determine the four unknown densities and velocities. This is achieved by writing two other differential equations for the momentum [26, 27]

$$\begin{aligned}\partial_t(\rho_+ u_+) + \partial_x(\rho_+ u_+ u_+) &= -\rho_+ \left(\frac{d}{dt} \right)_+ [p(\rho_+, \rho_-)], \\ \partial_t(\rho_- u_-) + \partial_x(\rho_- u_- u_-) &= \rho_- \left(\frac{d}{dt} \right)_- [p(\rho_-, \rho_+)],\end{aligned}$$

in which, by analogy to the pressure in fluid mechanics, the interactions between pedestrians are described by a term $p(\rho_{\pm}, \rho_{\mp})$. There is however a major difference with fluid mechanics: following [17], the derivative

$$(d/dt)_{\pm} = \partial_t + u_{\pm} \partial_x \quad (2)$$

is taken in the referential of the walking pedestrians, and not in the fixed frame as for fluids. Indeed, pedestrians react to their perception of the surrounding density as they see it while walking.

The term $p(\rho_{\pm}, \rho_{\mp})$ is actually not a pressure as in fluid mechanics, but rather a velocity offset between the achieved velocity u_{\pm} , and another quantity w_{\pm} which, as it is conserved along each pedestrian trajectory, can be interpreted as the desired velocity that the pedestrian would have if he was alone. In other words,

$$\begin{aligned}u_+ &= w_+ - p(\rho_+, \rho_-) \\ -u_- &= w_- - p(\rho_-, \rho_+)\end{aligned}$$

where w_{\pm} are Riemann invariants

$$\begin{aligned}\partial_t w_+ + u_+ \partial_x w_+ &= 0 \\ \partial_t w_- + u_- \partial_x w_- &= 0\end{aligned}$$

conserved along the trajectories of \pm pedestrians. The function $p(\rho_{\pm}, \rho_{\mp})$ can be determined from experimental measurements.

One difficulty coming from the fact that pedestrians do not necessarily have the same target site is that they may converge towards the same region, leading in the simulations to density divergences. A special treatment is thus required in order to limit the density to physical values. The solution retained in [26, 27] was to let $p(\rho_{\pm}, \rho_{\mp})$ diverge when the density approaches its upper limit value.

4 Kinetic models

In kinetic models, instead of describing explicitly the presence of a particle (molecule, vehicle or pedestrian) at a given location with a given velocity, one deals with the corresponding probability.

For example, in [28], with G. Schehr and H. Hilhorst, we have considered the case of a bidirectional two-lane road. Cars had different desired velocities, but the overtaking can occur only if there is enough space on the other lane (see Fig. 1). Assuming translation invariance of the probability distributions along the road, the problem results in finding the distribution of effective velocities as a function of the distribution of desired velocities. The solution of this problem requires to evaluate the overtaking probability. To do so, we assume that the probability to find a sufficient empty interval in the other lane at a given time and given place is equal to the average probability (mean-field assumption). Under these assumptions, we find that a symmetry breaking can occur between the lanes. This model developed for road traffic could be seen as a first attempt to model pedestrians in a corridor when lanes are formed, i.e. at high enough densities. However, this organization into lanes will not be as stable as in road traffic.

To account more completely for pedestrian flows, one has to consider the full joint probability distribution $f(\mathbf{v}, \mathbf{x}, t, \xi)$ of finding in position \mathbf{x} at time t a pedestrian with velocity \mathbf{v} and target site located in ξ . It is out of scope yet to find universal equations for this distribution. However, one may try to derive those from microscopic models. Two such derivations have been proposed by P. Degond et al in [29] and [30], starting from agent based models in which pedestrians modify their direction (and possibly velocity modulus) in order to avoid possible collisions in the near future, while still trying to keep as close as possible to their target direction.

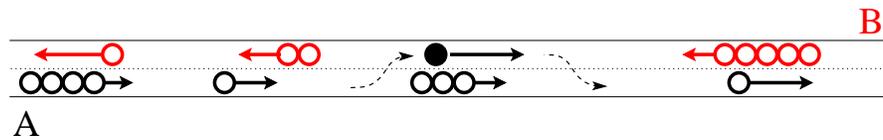


Fig. 1 Kinetic model for a bidirectional two-lane road. The vehicles (red and black circles) have different desired velocities, leading to platoon formation. Overtaking can take place only if there is enough free space on the other lane. From [28].

Some mean-field approximations have to be done to go from the microscopic discrete models to the kinetic ones. The smoothing due to these mean-field expressions has to be balanced by the introduction of some appropriate noise in the equations for the probability distribution f .

Once they have been obtained, these kinetic models can themselves be taken as a starting point to derive macroscopic models [29, 30] in two dimensions.

5 Cellular automata

To complete our comparison between pedestrian and fluid models, we shall now consider cellular automata models. For fluids, the story started in 1986, with the FHP model [10], in which pointlike particles were hopping onto a hexagonal lattice, and undergoing collisions at the nodes of the lattice. Providing that these collisions conserve mass and momentum but still mix enough the particle distributions, and that the lattice has enough symmetries, the resulting lattice gas was found to obey equations very close to Navier-Stokes equations [31]. Hence direct simulations of this lattice gas were providing solutions of the (almost) Navier-Stokes equations - a breakthrough given the difficulty to solve the latter.

A similar approach was proposed in 1992 for road traffic by Nagel and Schreckenberg [11]. Of course in this case there is no momentum conservation anymore, but rather some rules expressing the increase of velocity up to some maximal velocity, under the constraint of collision avoidance.

For pedestrians, interactions can be quite long ranged and one has to combine interactions taking place in any direction. It is thus a priori quite complex to develop a cellular automaton based on interactions between neighboring cells. A solution inspired from ants was provided by the use of some effective pheromones, which mediate the interactions between pedestrians [32, 12, 33].

Apart from the geometry of the lattice and the evolution rules, a cellular automaton is also defined by the sequence under which lattice sites are updated. Processes in continuous time, with independent events occurring with given rates, are well described by random sequential updates, in which a site is chosen at random and updated at each micro-time step. However this update leads to large fluctuations (the same site can be chosen twice in a row while another one will be ignored for a long time). Thus, for traffic applications, more regular updates are preferred. In particular the parallel update in which all the sites are updated in parallel at discrete time steps ensures a certain regularity in the flow. It introduces a time scale (the aforementioned time step) which can be interpreted as the reaction time of individuals.

Parallel update is widely used in road traffic modeling. It is also employed for pedestrians [34, 35], but requires to be complemented by extra rules. Indeed, in two-dimensional flows, two pedestrians may chose the same target site, resulting in a conflict that has to be solved by ad-hoc rules. Though these conflicts may be

given a physical meaning in terms of friction [35], some other updates which do not require these extra rules have been proposed.

The random shuffle update [36, 37, 38, 39] ensures that each site will be updated exactly once per time step, but in an order that is randomly chosen at each time step. This update has indeed been used in early cellular automata simulations of pedestrian evacuations [40, 39]¹.

The frozen shuffle update [41, 42] associates to each pedestrian a fixed phase τ , i.e. a real number between 0 and 1, and updates pedestrians once per time step in the order of increasing phases. This update allows to have higher fluxes than parallel update, reproducing the tendency of pedestrians to flow even at high densities. Besides, the phase τ can be given a physical meaning. It can represent the phase in the stepping cycle. It allows also to some extent to map the cellular automaton dynamics onto a continuous time/continuous space dynamics [42].

Cellular automata simulations can be useful to simulate large systems [39]. They can also help to understand some pattern formation [43], for example lane formation in counterflows [44, 12], or diagonal patterns at the crossing of two perpendicular flows [45, 46].

6 Conclusion

In this paper we have reviewed a few models for pedestrians proposed in the past years, to illustrate the various families of models that span over the different physical systems considered at the TGF conference.

For completeness, we must mention that at even larger scales than considered in this paper, models for road or pedestrian traffic must be supplemented with route choice models, as in [47].

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¹ Note that in some communities, *random shuffle update* is called *random sequential update*, as done in [40]. We shall stick to the denomination used in physics, for which *random sequential update* rather refers to an update close to continuous time.

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