
jungle component separation:
where is the snake?

CCA for BPOL and “hybrid” method

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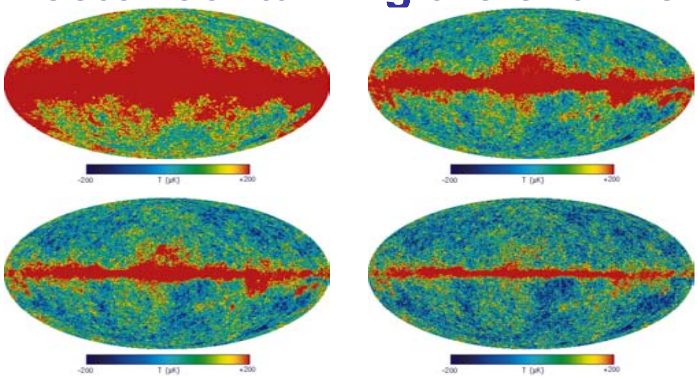
Cocktail party problem!

Mixing matrix (unknown)
but maybe…..

Noise (model)

$$\mathbf{x} = \mathbf{A} \cdot \mathbf{s} + \mathbf{n}$$

Vector containing the channel maps



Vector containing the components maps (Unknown in the interesting range of frequencies)

?

Different approaches…

- Blind approaches (e.g. FastICA)
- Previous knowledge regarding sources is not used at all
- Exploits statistical independencies of sources

- Non-blind approaches (e.g. MEM WF …)
- Assumption on the mixing matrix
- Exploits previous knowledge (spectral+ morphological)

- Hybrid approach (e.g. CCA)
- Estimation of the mixing matrix exploits previous knowledge about spectral behavior of sources
- Morphological priors can be added

CCA

- Estimation of the mixing matrix
- exploits information on the correlation of data to get an estimation of the mixing matrix,
- it permits to perform component separation with a minimum number of assumption regarding the sources spectral behavior

- Standard inversion through WF or MEM
- e.g. WF:
- $s^*=W\,x=$ $s^*=C_s A^T(A C_s A^T+C_n)^{-1}\,x$

$x=As+n$

CCA

Let us introduce the covariance matrix at a generic shift for a generic data vector **x**

$$\mathbf{C}_{\mathbf{x}}(\tau,\psi) = \left\langle [\mathbf{x}(\xi,\eta) - \mu_{\mathbf{x}}][\mathbf{x}(\xi+\tau,\eta+\psi) - \mu_{\mathbf{x}}]^T \right\rangle$$

From the data model we can easily derive:

$C_x(\tau,\psi) = A\ C_s(\tau,\psi)A^T + C_n(\tau,\psi)$ (II order statistic constraint)

This is a relation between covariance matrices and the mixing operator
With this we can estimate both the mixing operator and the covariance of sources from the covariance matrix of the data. To reduce the number of unknowns the mixing matrix can be parametrized. Exploiting the substantial auto-covariance of data we also consider this eq. for a sufficient number of shift pairs (τ,ψ) to have a good conditioning of the problem.

Implicit assumption: the mixing matrix is constant in the considered region.

Bedini et al. 2005

Parameterization of the mixing matrix

- It's a key aspect since it allows to substantially reduce the number of unknowns and to solve the problem
- It relies on the fact that the frequency scaling of the involved emissions is not completely unknown:
  1. CMB: blackbody law
  2. Free-free: power law
  3. Thermal dust: grey body law (dust spectral index, dust temperature)
  4. Synchrotron: power law (synchrotron spectral index)
- CCA is a very flexible method, since it permits to take advantage of prior informations
- The algorithm is very fast, and particularly efficient in those regions where the foreground signal is stronger than CMB, and different foregrounds are strongly correlated (… nicely complementarity with FastICA!)

Foregrounds scaling:

SYNCH

- Haslam map low resolution maps in the radio(1.4 Ghz)
- $T_\nu \propto n^\alpha$
- α~2.8 (?)

2.7 – 2.35

- polarized? yes! (up to 75% but more realistically ~20%)

FREE-FREE

- bremsstrahlung emissions
- $T_\nu \propto n^\alpha$
- α~2.15
- polarized? no!

another one?

- correlated with dust + scaling more similar to the synch scaling

Flat synch?

Parabolic ?

DUST

- IRAS map very high freq 3000GHz
- ~sum of gray bodies T~ 17- 21 K
- polarized? Yes

maybe ~5%

Details regarding parameterization…

The generic element of the mixing matrix **A** is

$$a_{dc} = F_c(\nu_d)\int b_d(\nu)d\nu$$

The frequency responses of the (radio-bolo)meters are normally known very well, whereas the astrophysical emission spectra are known only for special cases.

The cosmic microwave background spectrum is perfectly known

$$F_{CMB}(\nu) = \frac{\tilde{\nu}^2 \exp(\tilde{\nu})}{[\exp(\tilde{\nu})-1]^2}$$

Blackbody radiation

For the galactic dust radiation we have

$$F_{dust}(\nu) \propto \frac{\bar{\nu}^{m+1}}{\exp(\bar{\nu})-1}$$

For the synchrotron radiation we have

$$F_{syn}(\nu) \propto \nu^{-n_s}$$

All the elements of the mixing matrix are completely specified by the two parameters n_s and m. We thus have only two unknowns instead of the number of elements of the matrix

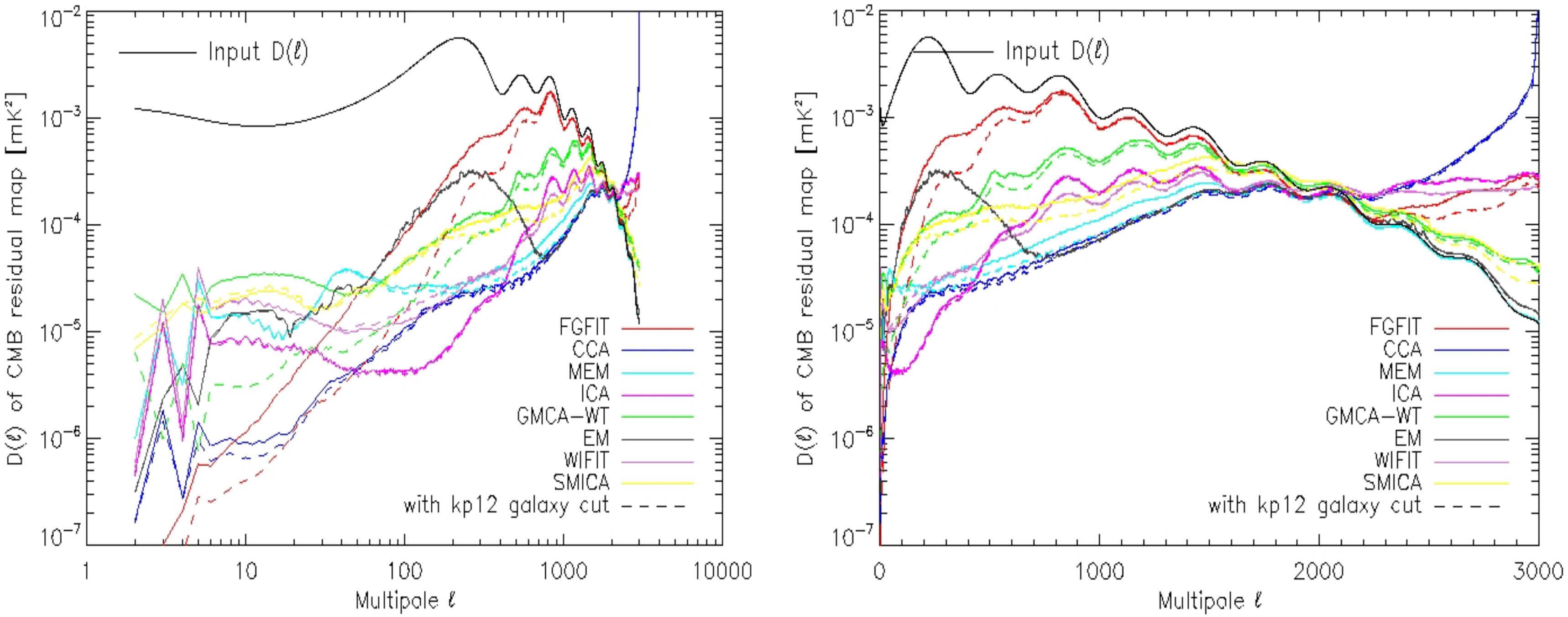
CCA (pixel space) total intensity: status

- Implemented
- Documented
- Tested on simulated data (with Planck specifications)

Bonaldi et al. 2006

- Tested in combination with harmonic WF (data released for the Planck 2º comp sep challenge)
- Tested on WMAP 3-years data (paper in preparation)

CCA is working!



Polarization

- Implementation is done
- Test ongoing (input ready - simulation set)
- In principal the estimation of the mixing matrix can be performed separately for Q and U maps exactly as for a Temperature map (pixel based method). The problem is more delicate in the inversion step (WF). In fact all the harmonic methods require to perform a deconvolution with the beam. Therefore we will need to develop an inversion technique that will take care of both Q and U simultaneously.
- 1^{o} test to perform: a similar mixing matrix parameterization


fine