Discovering Primordial Gravitational Waves from Inflation with Bpol

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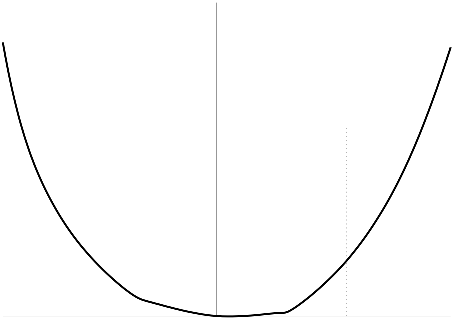
Single-Field Inflation

At the beginning there was a scalar field that dominated the universe. Everything came from this scalar field and there was nothing without the scalar field. The quantum fluctuations of this field (that is, those of the vacuum) generated small fluctuations that advanced or retarded the instant of re-heating. These were the seeds of the large-scale structure.



Perturbations generated during inflation

$$\boxed{\hbar = c = 1, M_{pl}^{-2}} \qquad \delta\phi \approx H \qquad \frac{\delta\rho}{\bar\rho} \approx H \cdot \delta t, \quad \delta t \approx \frac{\delta\phi}{\dot\phi}$$

$$H\dot\phi \approx V_{,\phi}, \quad \dot\phi \approx V_{,\phi}/H, \quad H^2 \approx \frac{1}{M_{pl}^2} V, \frac{\delta\rho}{\bar\rho} \approx \frac{V^{3/2}[\phi(k)]}{M_{pl}^3\ V_{,\phi}}$$

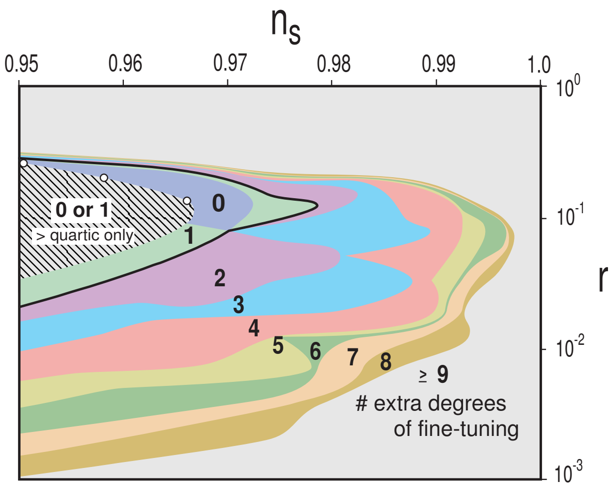
Scalar perturbations : $$\boxed{\mathcal{P}_S^{1/2}(k) \approx O(1) \cdot \frac{V^{3/2}[\phi(k)]}{M_{pl}^3\ V_{,\phi}[\phi(k)]}.}$$

Tensor perturbations : $$\boxed{\mathcal{P}_T^{1/2}(k) \approx O(1) \cdot \frac{H}{M_{pl}} \approx O(1) \cdot \frac{V^{1/2}}{M_{pl}^2}}$$

$\phi(k) \equiv$ value of ϕ at horizon crossing of the mode k

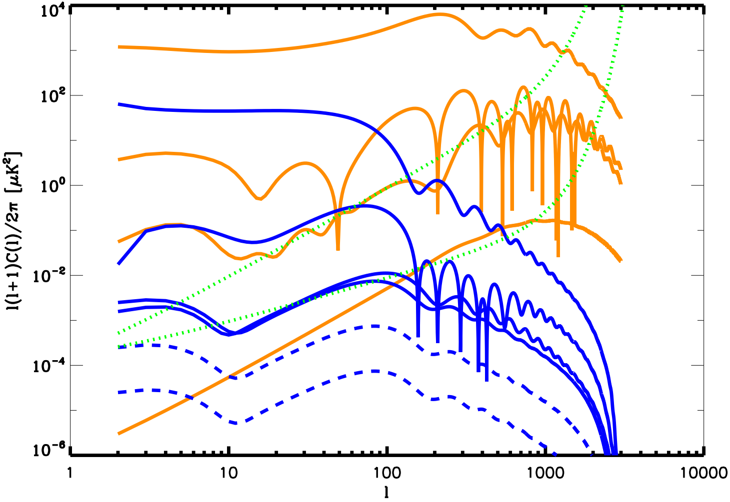
Reconstruction of the inflationary potential : the tensors measure the height of the potential, the scalars the slope.

Expected (T/S) From Inflation ?



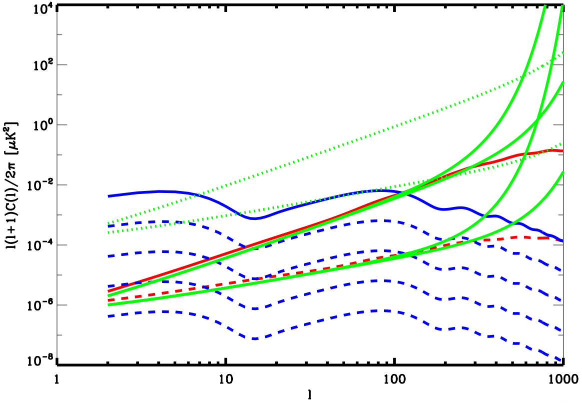
From Boyle, Steinhardt and Turok.

PLANCK and the CMB anisotropies



The detection of B modes

The B mode is that component that cannot be represented as a double gradient on the celestial sphere. In the linear approximation there is no B mode component arising from scalar degrees of freedom. The presence of the B mode would unambiguously signal the presence of primordial gravitational waves.



Lensing of the E mode into the B mode — $(E^{scalar} + \Phi \to B^{scalar})$

(Flat sky approximation : $(\ell m) \to \boldsymbol{\ell}, \quad \boldsymbol{\theta}, \boldsymbol{\ell} \in \mathcal{R}^2$.)

$$\delta\boldsymbol{\theta} = (\boldsymbol{\nabla}\Phi), \qquad \delta T(\boldsymbol{\theta}) = (\boldsymbol{\nabla}\Phi)\cdot(\boldsymbol{\nabla}T).$$

$$\delta T(\boldsymbol{\ell}_F) = \int \frac{d^2\boldsymbol{\ell}_L}{(2\pi)^2}\,(-\boldsymbol{\ell}_L)\cdot(\boldsymbol{\ell}_F - \boldsymbol{\ell}_L)\;\Phi(\boldsymbol{\ell}_L)\;T(\boldsymbol{\ell}_F - \boldsymbol{\ell}_L).$$

$$\left\langle T(\boldsymbol{\ell})\;T(\boldsymbol{\ell}')\right\rangle = (2\pi)^2\;\delta^2(\boldsymbol{\ell}+\boldsymbol{\ell}')\;C^{TT}(\ell)$$

$$C^{TT}(\boldsymbol{\ell}_F) = \int \frac{d^2\ell_L}{(2\pi)^2}\,[\boldsymbol{\ell}_L\cdot(\boldsymbol{\ell}_F - \boldsymbol{\ell}_L)]^2\;C^{\Phi\Phi}(\boldsymbol{\ell}_L)\;C^{TT}(\boldsymbol{\ell}_I = |\boldsymbol{\ell}_F - \boldsymbol{\ell}_L|)$$

$$C^{BB}(\boldsymbol{\ell}_B) = \int \frac{d^2\ell_L}{(2\pi)^2}\,[\boldsymbol{\ell}_L\cdot(\boldsymbol{\ell}_F - \boldsymbol{\ell}_L)]^2\;\sin^2[2\Theta(\boldsymbol{\ell}_B, \boldsymbol{\ell}_E)]C^{\Phi\Phi}(\boldsymbol{\ell}_L)\;C^{EE}(\boldsymbol{\ell}_E = |\boldsymbol{\ell}_B - \boldsymbol{\ell}_L|)$$

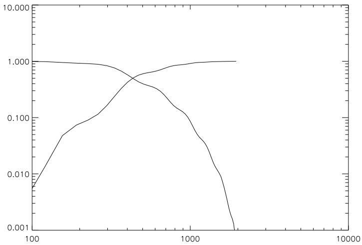
Lensing of the E mode into the B mode (II)

For small values of ℓ_B,

$$C^{BB}(\ell_B \approx 0) \sim \int_0^\infty \frac{d\ell}{\ell}\, \ell^6 C^{\Phi\Phi}(\ell)\; C^{EE}(\ell)$$

The bulk of the integral is concentrated around $\ell \approx 300$.
White noise spectrum up to $\ell \lesssim 300$

$$F(\ell_{max}) = \frac{\int_0^{\ell_{max}} \frac{d\ell}{\ell}\, \ell^6 C^{\Phi\Phi}(\ell)\; C^{EE}(\ell)}{\int_0^\infty \frac{d\ell}{\ell}\, \ell^6 C^{\Phi\Phi}(\ell)\; C^{EE}(\ell)}$$

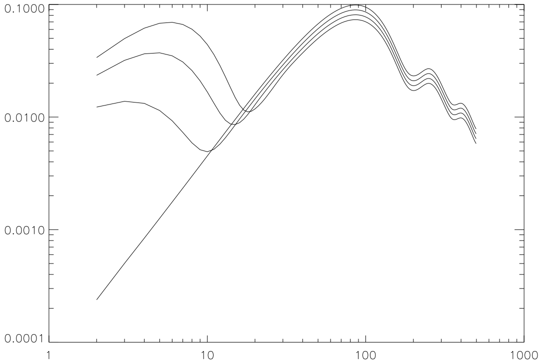


Capabilities of BPol

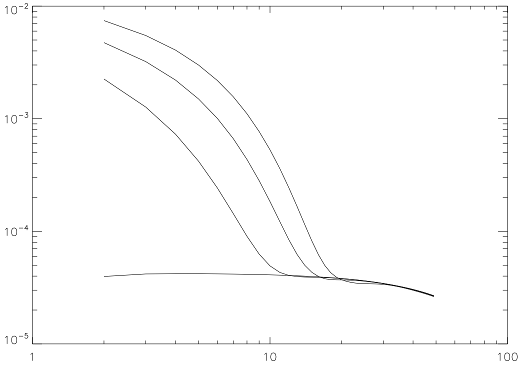
(T/S)	$2 < \ell < 15$	$16 < \ell < 100$	$101 < \ell < 200$
PLANCK	1.6×10^{-2}	0.25	1.0
BPol	1.5×10^{-4}	2.2×10^{-3}	8.8×10^{-3}
Cosmic variance	8.6×10^{-5}	1.2×10^{-3}	5.0×10^{-3}

TAB.: Thresholds for a 3σ detection of primordial gravity waves with PLANCK, BPol and an ideal cosmic variance limited full-sky polarization survey. (First column assumed first year WMAP value of $\tau = 0.17$.)

The Reionization Bump (I)

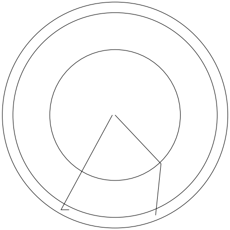


The Reionization Bump (II)



Amplification of the B mode signal relative to the non reionized case by a factor of about 50,100, and 150 at $\tau = 0.05$, $\tau = 0.10$, and $\tau = 0.15$, respectively.

The Reionization Bump (III)



It turns out that

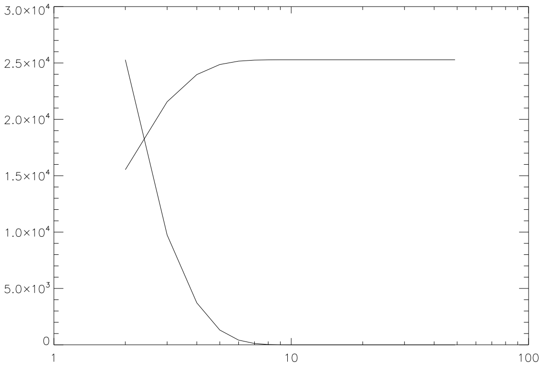
$$P \propto (1-\tau) d^2_{lastscatter} \frac{\partial^2 T}{\partial x^2}$$

is small compared to

$$P \propto \tau d^2_{reion} \frac{\partial^2 T}{\partial x^2}$$

even when τ is small.

The Reionization Bump (III)



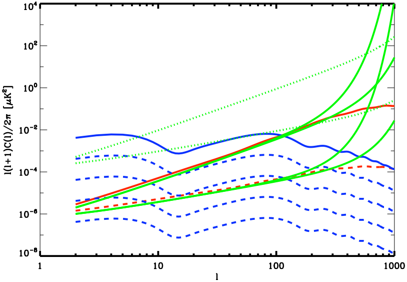
Information is concentrated at the very lowest multipoles.

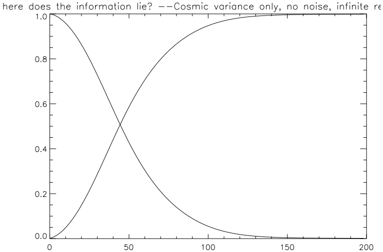
Pro : There is comparatively a very large signal.

Drawback : It may be very hard to rule out a galactic explanation given the large role of the lowest ℓ. No way to jacknife the data. (Cf. Controversy regarding the significance of the WMAP low quadrapole.)

Where does the information on (T/S) lie ?

$$\delta c_{\ell,measurable} \sim \frac{c_{\ell,parasite} + n_\ell}{\ell}$$





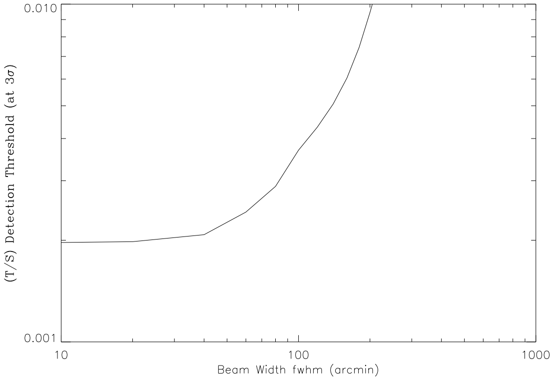
Conclusion : Approx. 80 % of the information (excluding the reionization bump) lies between $\ell = 20$ and $\ell = 80$.

BPol Capabilities : Fisher matrix analysis

	Fiducial model	BPol 20′ fwhm, $(5\mu\mathrm{K}\cdot\mathrm{arcmin})^2$ (No reionization)					
		TT	TE	EE	BB	BB+EE	All
r	0.0	1.57×10^{-1}	7.19×10^{-2}	1.32×10^{-2}	1.58×10^{-3}	6.71×10^{-4}	6.60×10^{-4}
$\delta A_S/A_S$	1.00×10^{0}	3.14×10^{-1}	6.15×10^{-3}	3.81×10^{-3}	2.70×10^{0}	3.57×10^{-3}	1.27×10^{-3}
H	7.20×10^{1}	9.96×10^{-2}	6.62×10^{-2}	9.91×10^{-2}	5.94×10^{0}	8.06×10^{-2}	3.93×10^{-2}
Ω_b	5.00×10^{-2}	3.43×10^{-4}	2.68×10^{-4}	6.01×10^{-4}	3.91×10^{-2}	4.88×10^{-4}	1.39×10^{-4}
Ω_c	2.50×10^{-1}	3.73×10^{-5}	5.03×10^{-5}	1.34×10^{-4}	3.45×10^{-2}	1.28×10^{-4}	2.68×10^{-5}
n_s	1.00×10^{0}	3.47×10^{-3}	6.69×10^{-3}	3.98×10^{-3}	1.49×10^{-1}	3.74×10^{-3}	1.84×10^{-3}
Ω_k	0.0	5.53×10^{-4}	4.37×10^{-4}	3.08×10^{-4}	3.04×10^{-2}	3.01×10^{-4}	1.87×10^{-4}
τ	0.0	1.73×10^{-1}	8.00×10^{-4}	1.19×10^{-5}	1.60×10^{0}	5.96×10^{-6}	5.96×10^{-6}

TAB.: 1σ errors resulting from the fit of an eight parameter family of cosmological models for a detector rms white noise amplitude of $5\mu\mathrm{K}\cdot\mathrm{arcmin}$ and a resolution of 20 arc minute. (Note that $\delta A_S/A_S$ denotes the variation of the normalization of the scalar power spectrum as compared to that inferred from COBE data.) This table

Dependence of sentivitity to T/S on angular resolution



Here the reionization bump has been turned off by setting $\tau = 0$.

Conclusion : (excluding foreground and instrumental considerations) A resolution coarser than $\approx 1.5^\circ$ will entail a significant loss of information from the non-reionization bump signal.

Requirements for a B Mode Polarization Mission

- **Sensitivity in the neighborhood of** $5\mu K \cdot arcmin$. Chosen equal to contaminant lensing signal. Approximately 13 times more sensitive than PLANCK.
- **Angular resolution of** $\approx 20 - 40'$ Lower than PLANCK ; therefore, one does not necessarily need a large mirror deployed in space.
- **Excellent control of systematic errors.** (These should not exceed the intrinsic random detector noise.)
- **Full sky coverage.** In particular important for mapping the low-ℓ modes of the re-ionization bump.
- **Sufficiently broad frequency coverage to remove galactic foreground components (synchrotron radiation, bremsstrahlung radiation, spinning dust (?), polarized dust emission.**