A Multiclasse Car-Following Rule Based On The LWR Model

Traffic And Granular Flow '07

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Multiclasse in the LWR model

- Extensive researches have been conducted to introduce traffic heterogeneity in the LWR model
  (Wong and Wong, 2002) (Zhang and Jin, 2002) (Zhu et al, 2003) (Chanut and Buisson, 2003) (Benzoni-Gavage Colombo, 2003) (Logghe, 2003) (Chanut, 2005)
- These researches are based on a traffic decomposition into homogeneous classes represented continuously
- Derived numerical schemes are very diffusive

Scope of the research

- Introduce traffic heterogeneity into a Lagrangian formulation of the LWR model rather than the traditional Eulerian one
- Derive an efficient numerical scheme that is exact (and then with no diffusion) under little restrictive assumptions

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Outline

- Lagrangian resolution of the homogeneous LWR model
- Extension to heterogeneous flow
- Numerical examples

The homogeneous LWR model

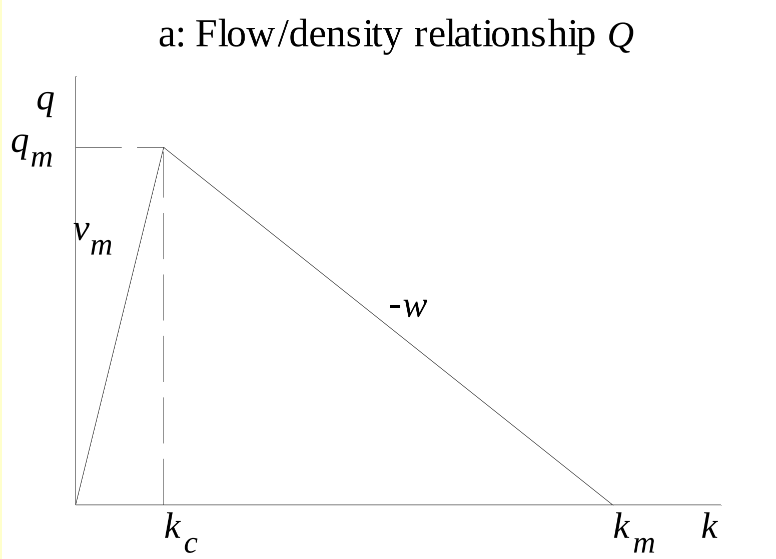
Variables: k, density; v, speed; q=kv, flow

Conservation equation:

$$\partial_t k + \partial_x kv = 0$$

Fundamental diagram (FD):

$$v = V(k) \quad \text{or} \quad q = kV(k) = Q(k)$$



The model can be synthesized as a scalar hyperbolic equation:

$$\partial_t k + \partial_x Q(k) = 0$$

The Lagrangian coordinates

- $N(x,t)$ represents the cumulative number of vehicles that cross location x by time t
- $k=-\partial_x N$ and $q=\partial_t N$
- The coordinates (x,t) can be changed to (N,t)
- The conservation equation becomes:

$$\partial_t s + \partial_N v = 0 \qquad \text{with } s=1/k \text{ (spacing)}$$

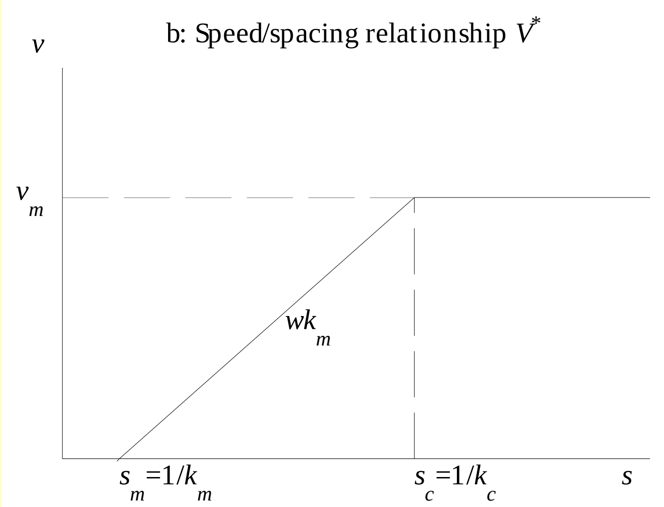
The homogeneous LWR model in Lagrangian coordinates

The FD can be expressed as :

$$v = V(1/s) = V^*(s)$$

The model reduces to
a scalar hyperbolic equation:

$$\partial_t s + \partial_N V^*(s) = 0$$



Let $X(n,t)$ be the inverse of $N(x,t)$

-> X is obtained by solving for x in $n=N(x,t)$

-> X represents the trajectory of vehicle n

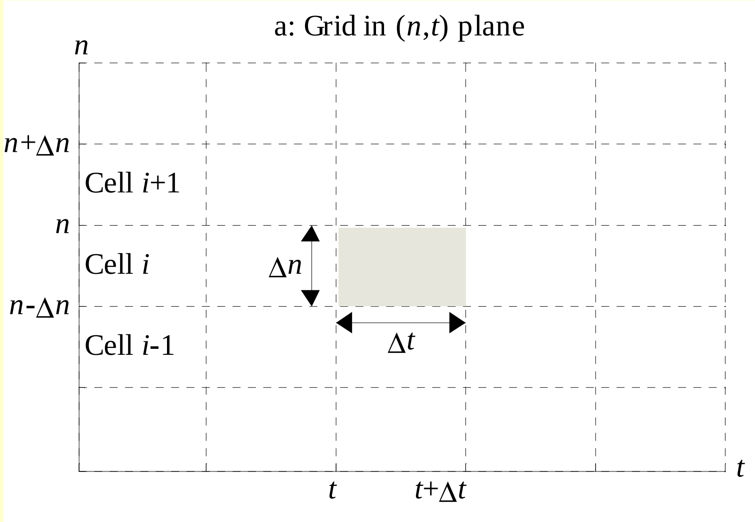
The model can also be expressed as:

$$\frac{\partial X}{\partial t} = V^*\left(-\frac{\partial X}{\partial N}\right)$$

Hamilton-Jacobi equation

Numerical resolution using the Godunov scheme

As the flux function V^* is increasing the Godunov scheme reduces to the upwind method:



$$\frac{X(n,t+\Delta t)-X(n,t)}{\Delta t}=V^*\left(-\frac{X(n,t)-X(n-\Delta n,t)}{\Delta n}\right)$$

When the FD is triangular:

$$X(n,t+\Delta t)=\min\begin{pmatrix}X(n,t)+v_m\Delta t,\\ (1-\alpha)X(n,t)+\alpha X(n-\Delta n,t)-w\Delta t\end{pmatrix}$$

$$\text{with}\quad \alpha=wk_m\Delta t/\Delta n$$

α should be lower than 1 (CFL condition)

Numerical resolution using the variational principle (Daganzo, 2005)

The model solution in *X* satisfies a least-cost path problem (Daganzo, 2006) (Leclercq et al, 2007):

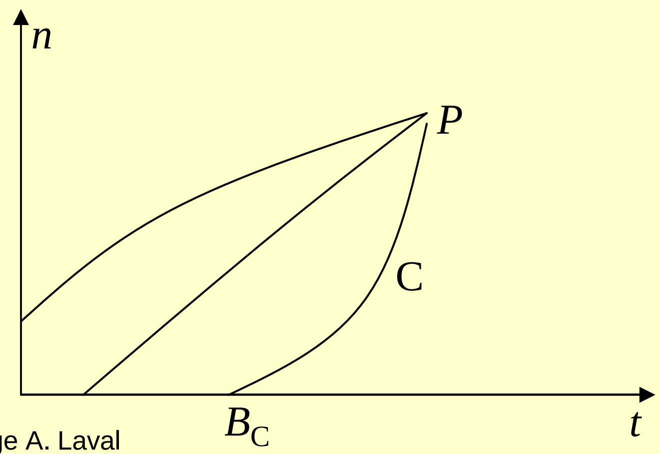
$$X_P = \min\left(B_C + \Delta(C) : \forall C \in V \cap S_P\right), \text{ where}$$

V: set of all valid paths

S_P: set of all path from the boundary condition to P

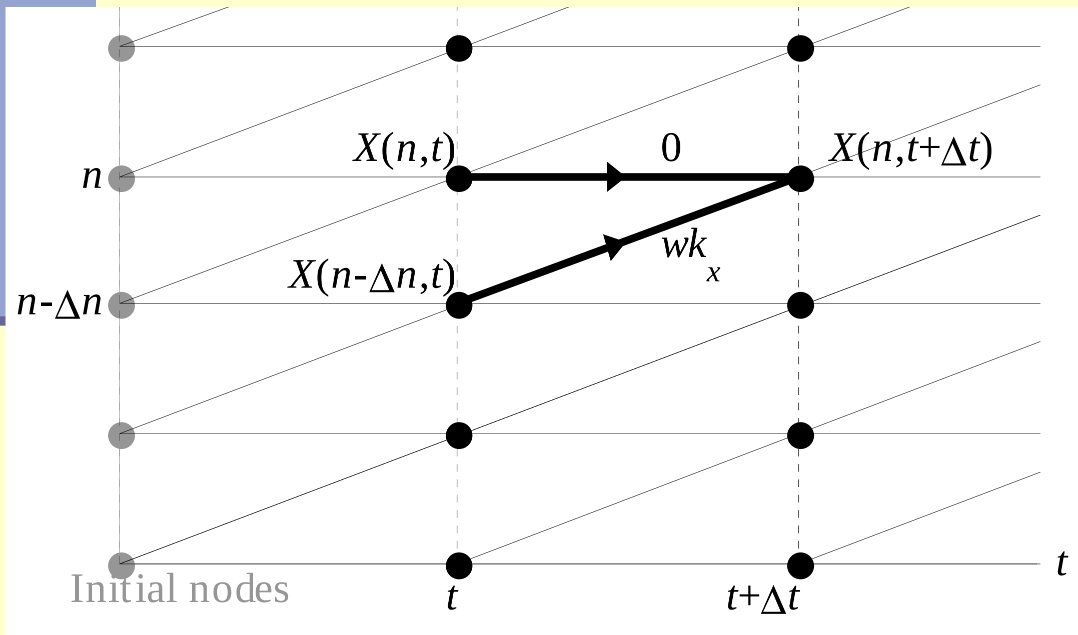
B_C: X value at the beginning of the path C

$\Delta(C)$: cost of path C



Numerical resolution using the variational principle (2)

- When the FD is triangular, only two paths has to be considered
  - Free-flow path (slope: 0 ; cost: v_m)
  - Congested path (slope: wk_x ; cost: $-w$)
- When discretizing the initial conditions in Δn units and setting $\Delta t = \Delta n/(wkx)$, the model solutions can be deduced from the following network:



$$X\left(n,t+\Delta t\right)=\min\begin{pmatrix} X\left(n,t\right)+v_m\Delta t \\ X\left(n-\Delta n,t\right)-w\Delta t \end{pmatrix}$$

Remarks

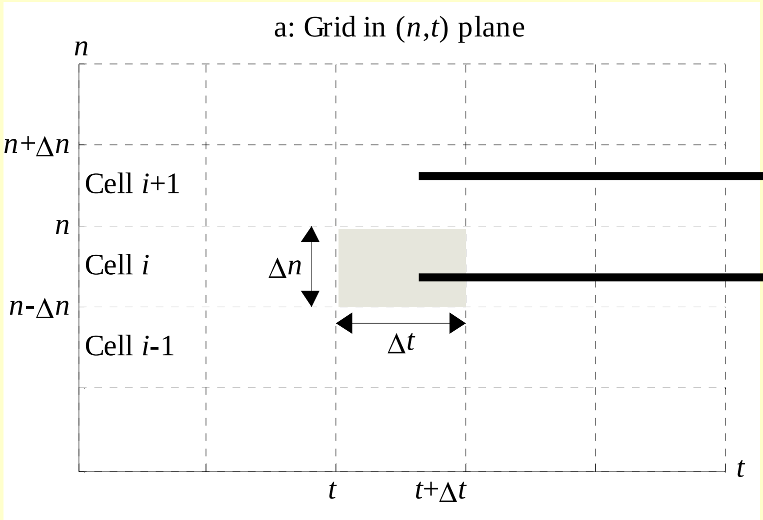
- The variational scheme is exact provided that the initial data is linear between two consecutive initial nodes

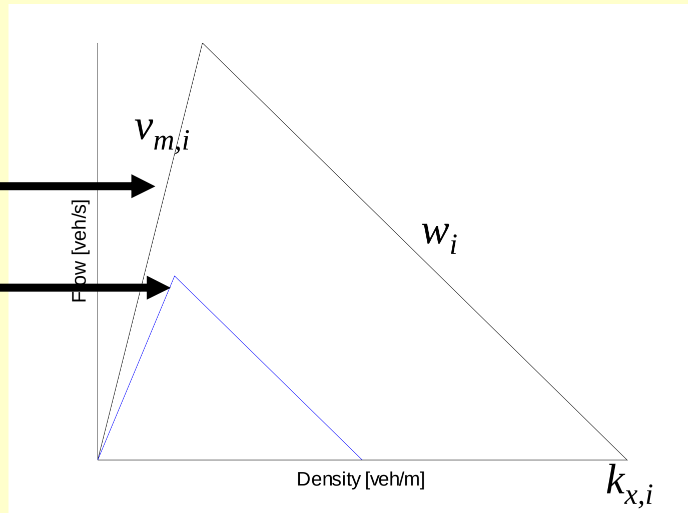
- When the FD is triangular, the Godunov scheme with α=1 and the variational one are equivalent

Multiclasse in Lagrangian Framework

Principle

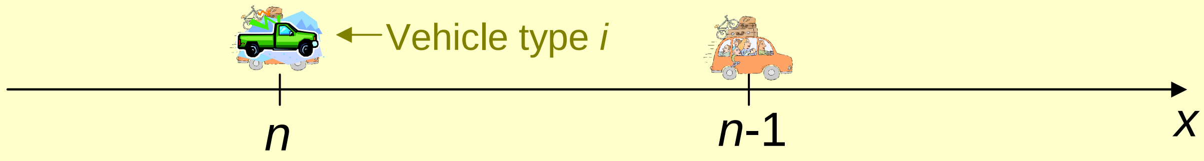
- Define a specific FD for each Lagrangian cell (Δn=1)
- Each vehicle type *i* is defined by three parameters:
  - The free-flow speed $v_{m,i}$
  - The jam density $k_{x,i}$ (≈inverse of vehicle size)
  - The wave-speed w_i





Numerical resolution using the Godunov scheme

- The Godunov scheme is adapted to heterogeneous problems (Lebacque, 1996)



$$X(n,t+\Delta t)=\min\left(X(n,t)+v_m\Delta t,(1-\alpha)X(n,t)+\alpha X(n-1,t)-w\Delta t\right)$$

with $\alpha = wk_m\Delta t$

becomes:

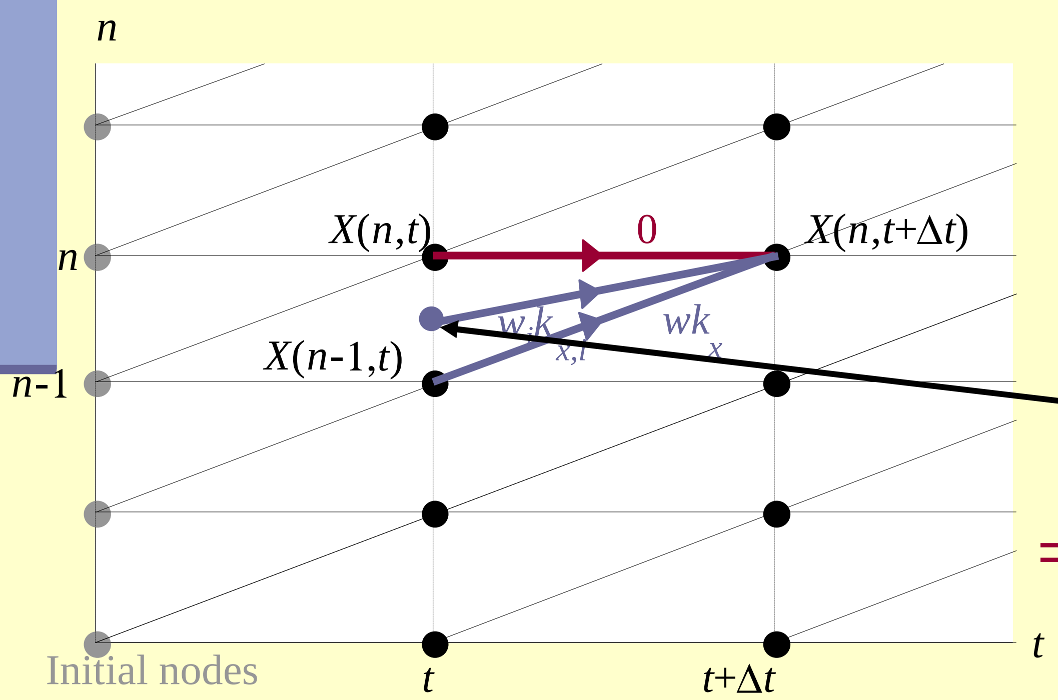
$$X(n,t+\Delta t)=\min\left(X(n,t)+v_{m,i}\Delta t,(1-\alpha_i)X(n,t)+\alpha_i X(n-1,t)-w_i\Delta t\right)$$

with $\alpha_i = w_i k_{m,i}\Delta t$

CFL condition: $\max(\alpha_i)\leq 1$

Numerical resolution using the variational principle

- Free-flow path: only the cost ($v_{m,i}$) is modified
- Congested path: the slope ($w_i k_{x,i}$) and the cost ($-w_i$) are modified



Slope modifications change the structure of the network

This point is not on the network grid
=> the value of *X* is unknown

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X value estimation – solution 1

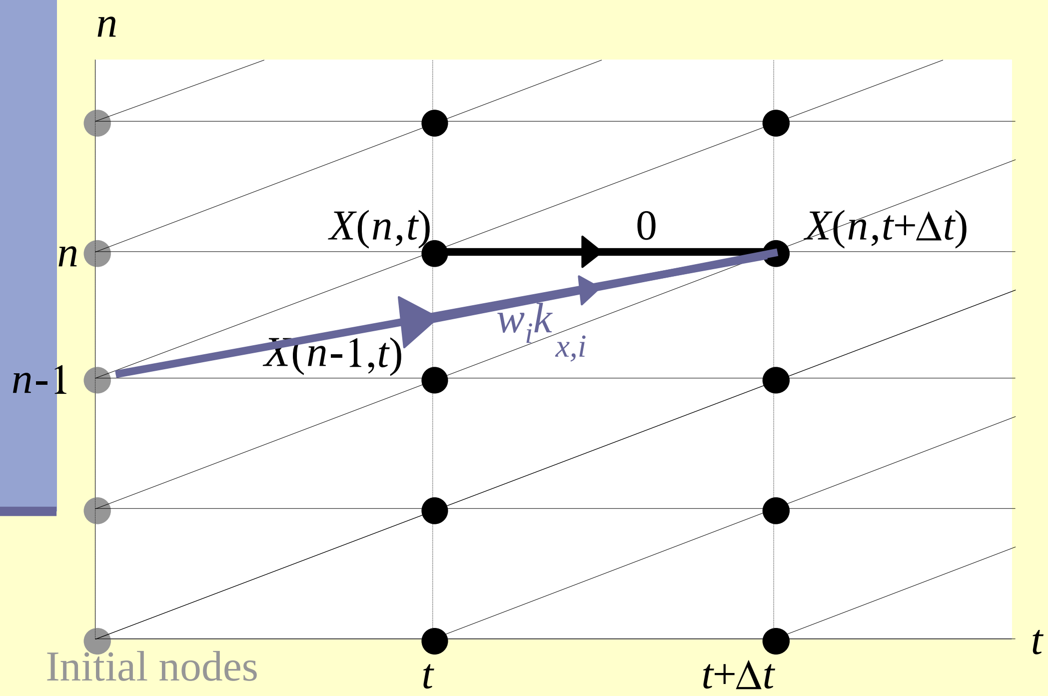
- Assume that the spacing is uniform between *n* and *n*-1 at time *t* and estimate the *X* value:

$$X(t) = (1-\alpha_i) X(n,t) + \alpha_i X(n-1,t) \quad \text{with} \quad \alpha_i = w_i k_{m,i} \Delta t$$

- The numerical scheme is then equivalent to the Godunov one
- The numerical scheme does not remain exact

X value estimation – solution 2

- Store the *X* values at previous time steps and look for the time when the congested path join a network node



This is possible if for each vehicle:

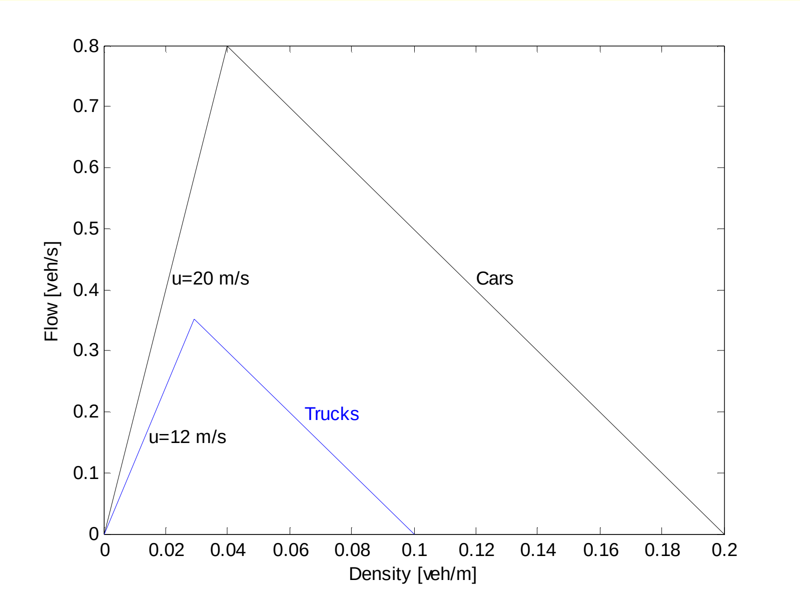
- w_i is the same
- the ratio $k_{x,i}/\max(k_{x,i})$ is an integer

Under these assumptions, the numerical scheme is exact

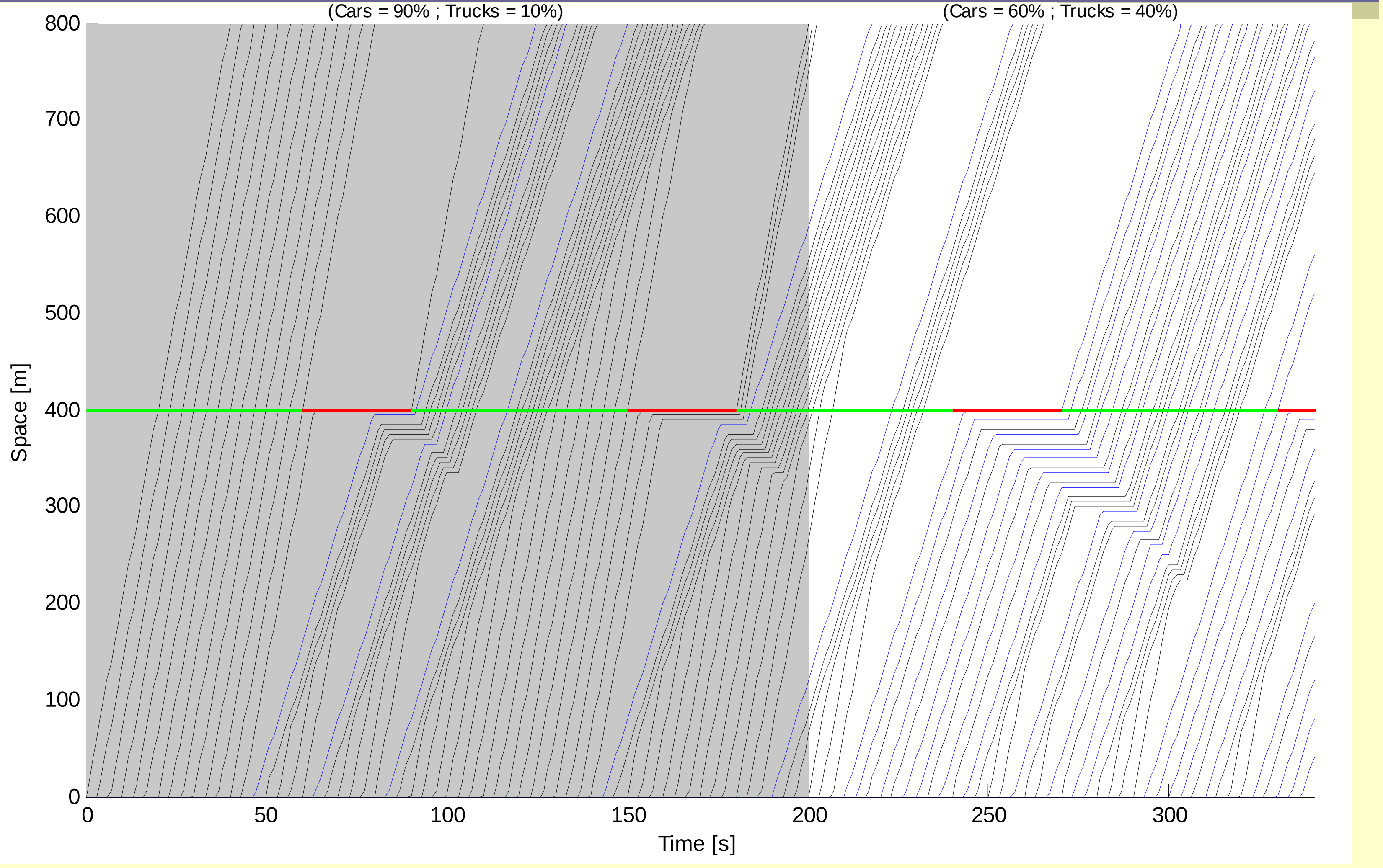
Numerical examples

Case of study

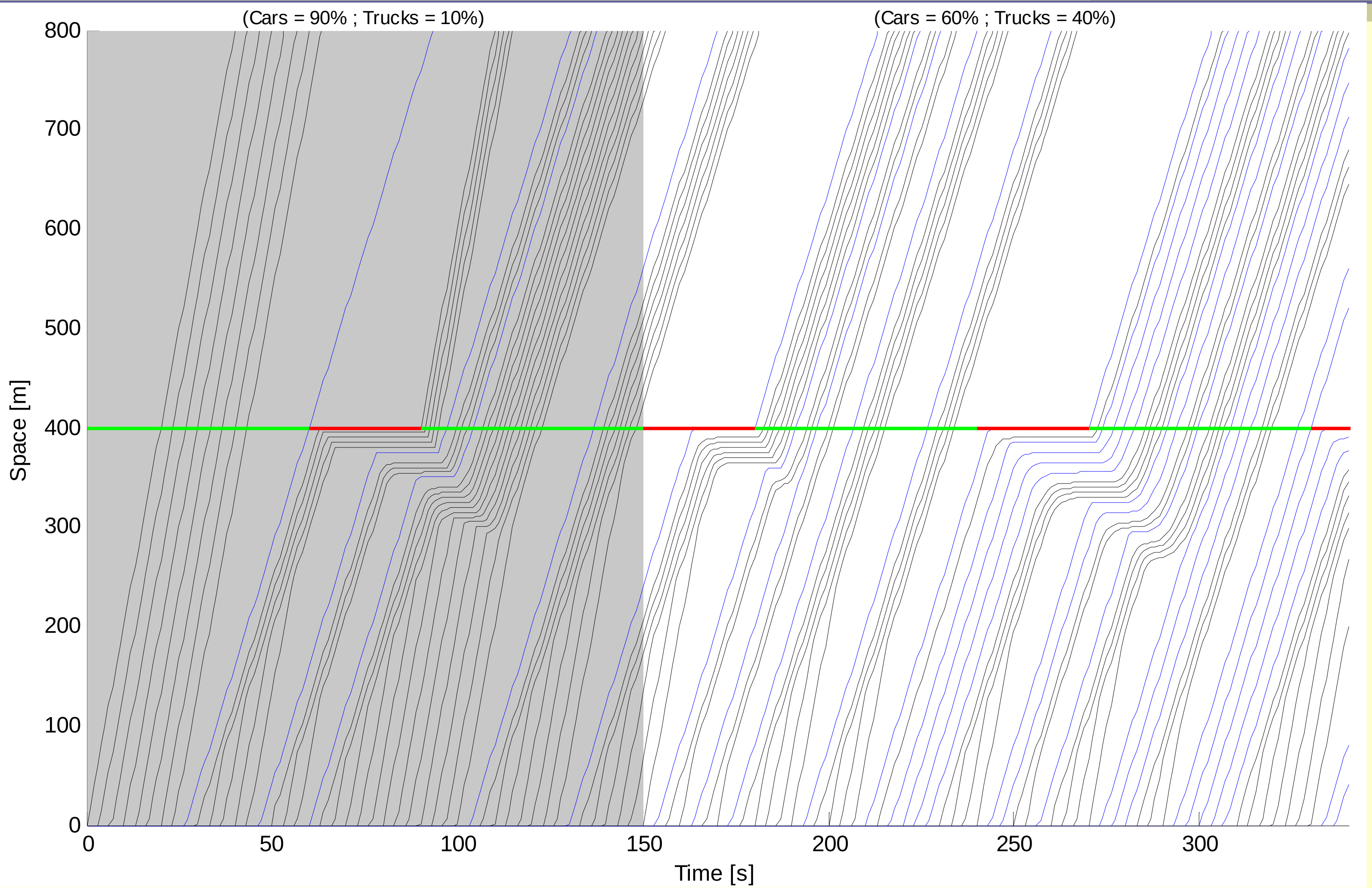
- One-lane road
- Cars and trucks with specific FDs
- A constant flow rate at the entry (1080 veh/h)
- Composition:
  - 0-150s: 90% cars, 10% trucks
  - 150s-350s: 60% cars, 40% trucks
- A traffic signal (cycle=90s, green time=60s)



Variational scheme

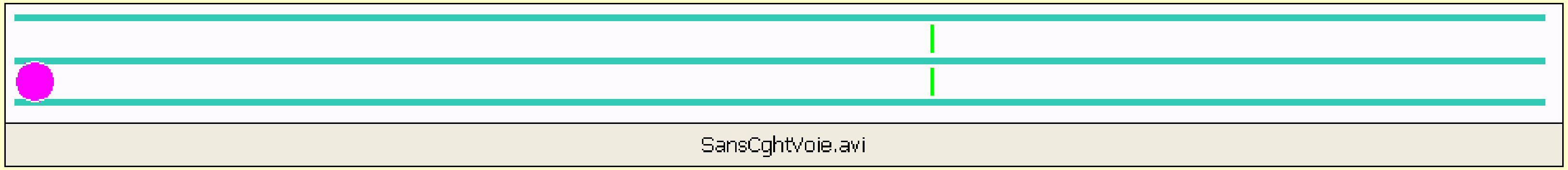


Godunov scheme

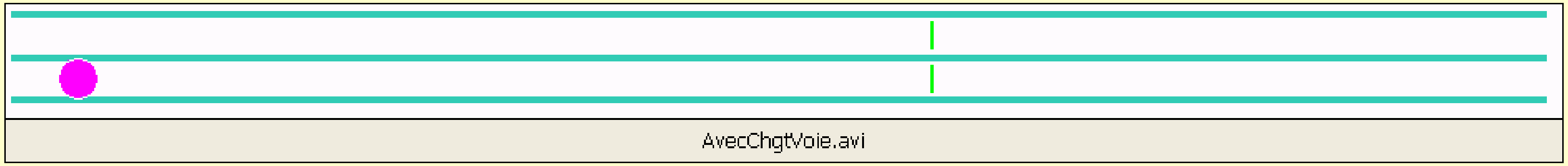


Simulation

Without lane-changing:



Coupled with a lane-changing model:



Conclusion

- A multiclasse car-following rule that is:
  - Parsimonious and easy to calibrate (FD by vehicle type)
  - Exact under little restrictive assumptions (triangular FD, constant *w*, integer ratio between jam densities)
  - Fully compatible with existing extension of the LWR model and especially lane-changing one (Laval and Leclercq, 2007)

Thank you for your attention