3D gluings of octahedra

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Joint work with V.Bonzom

RGP2016 - IHP

1 – Colored triangulations and edge-colored graphs

2 – Bijection with hypermaps

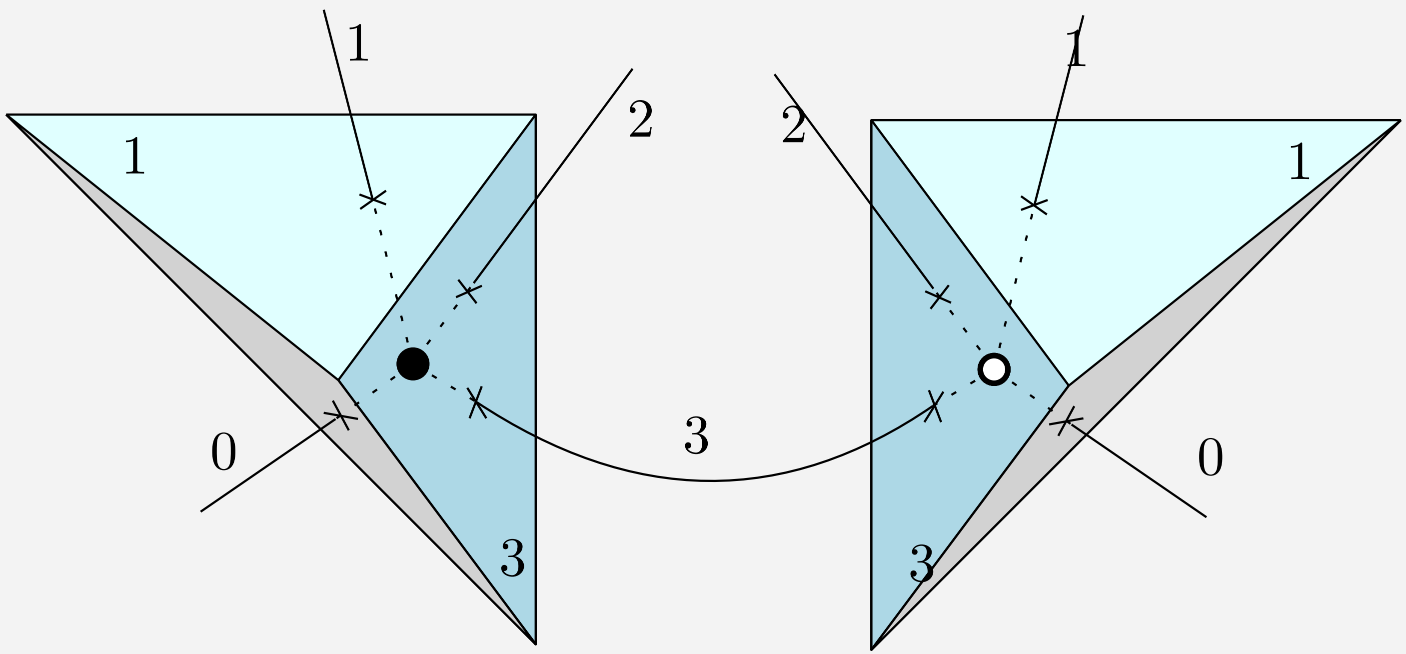
3 – Maximal gluings of octahedra

4 – Conclusions

1 – Colored triangulations and edge-colored graphs

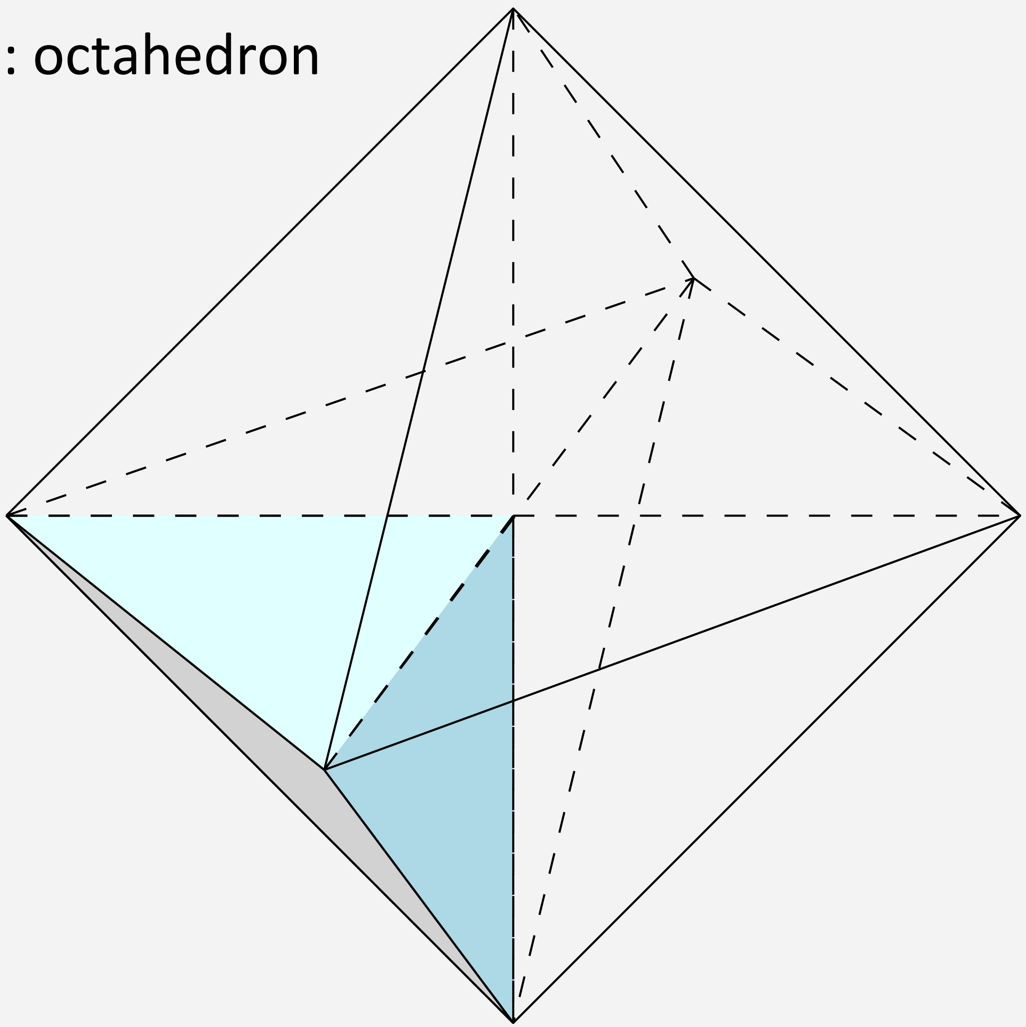
Tetrahedra are represented by 4-valent vertices

The contraction of a color-*i* edge encodes the gluing of two color-*i* faces in the *unique* possible way

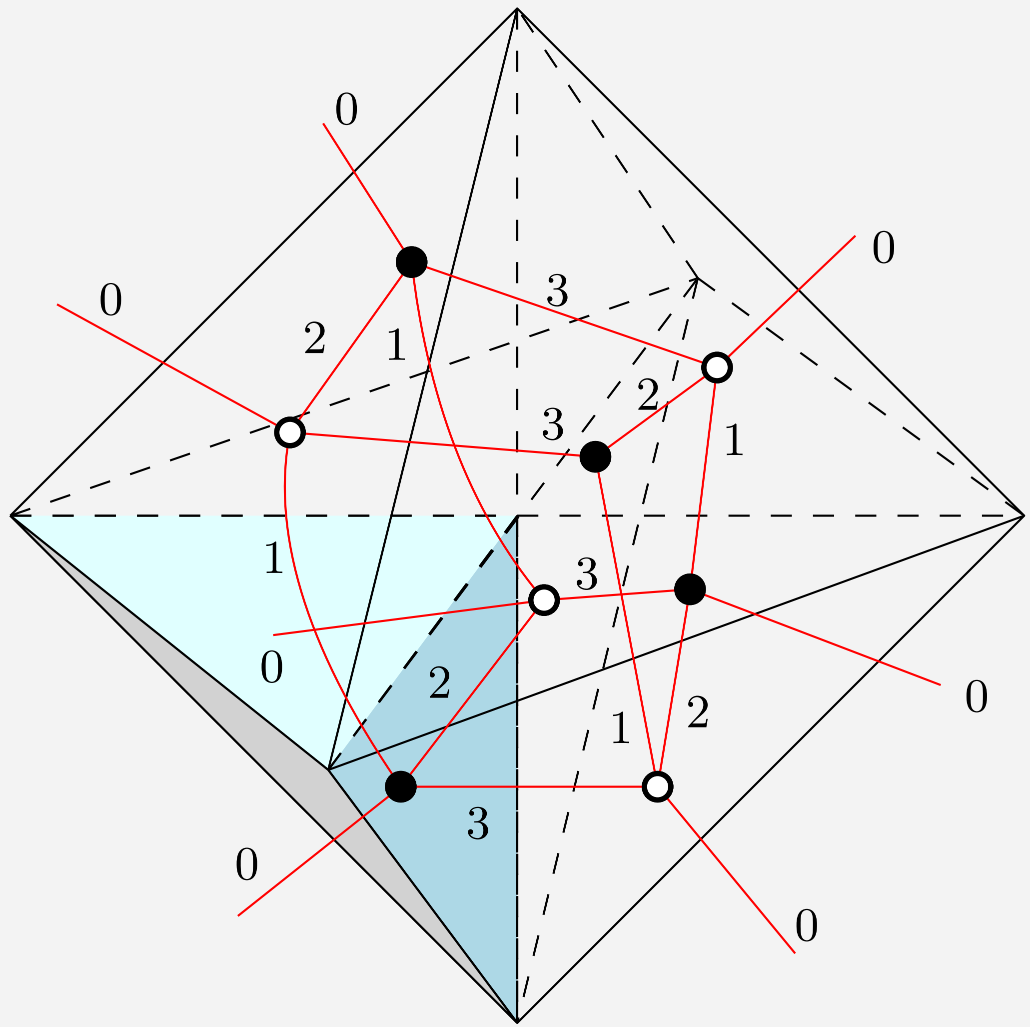


1 – Colored triangulations and edge-colored graphs

Building block : octahedron



1 – Colored triangulations and edge-colored graphs



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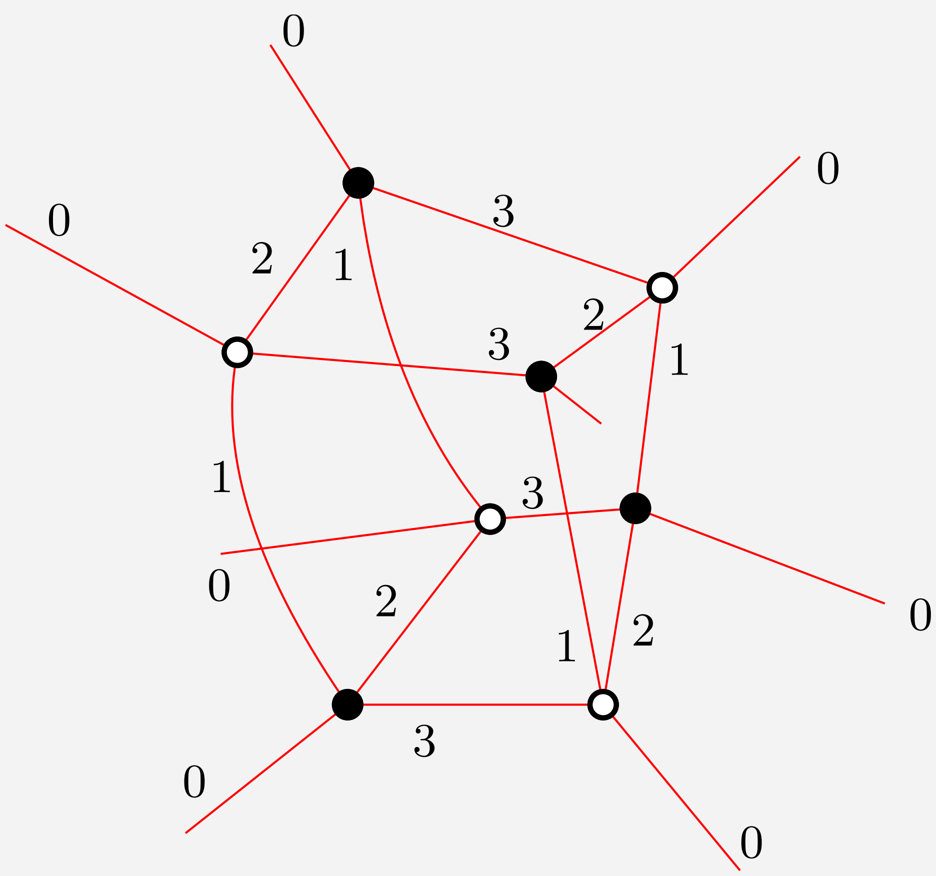

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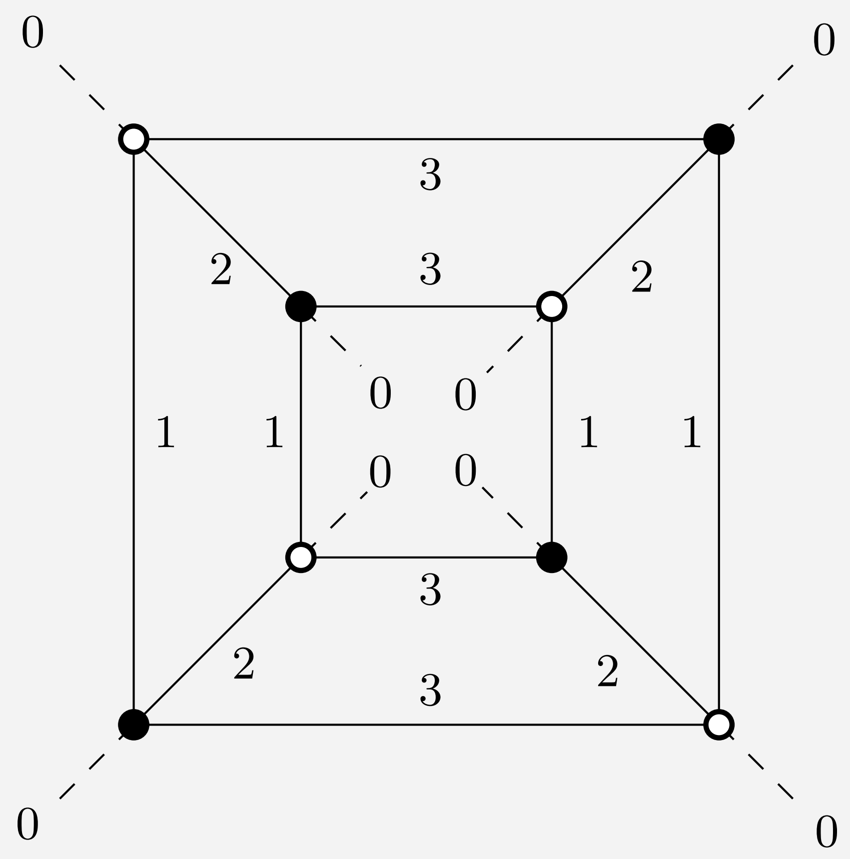
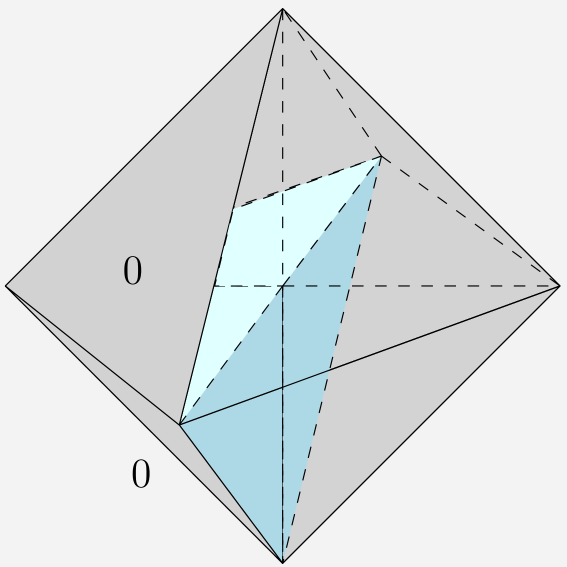
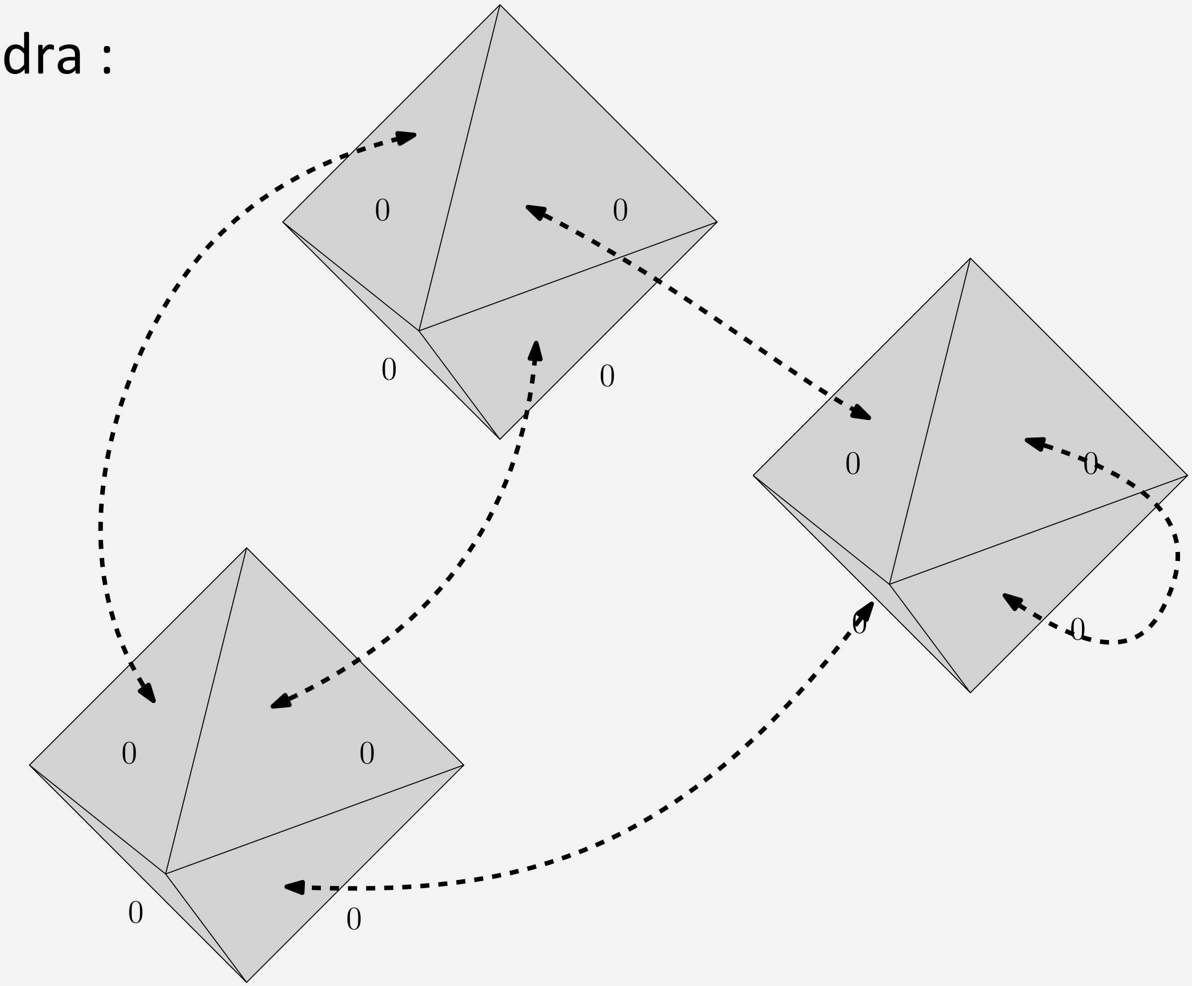




Building block in the dual picture

1 – Colored triangulations and edge-colored graphs

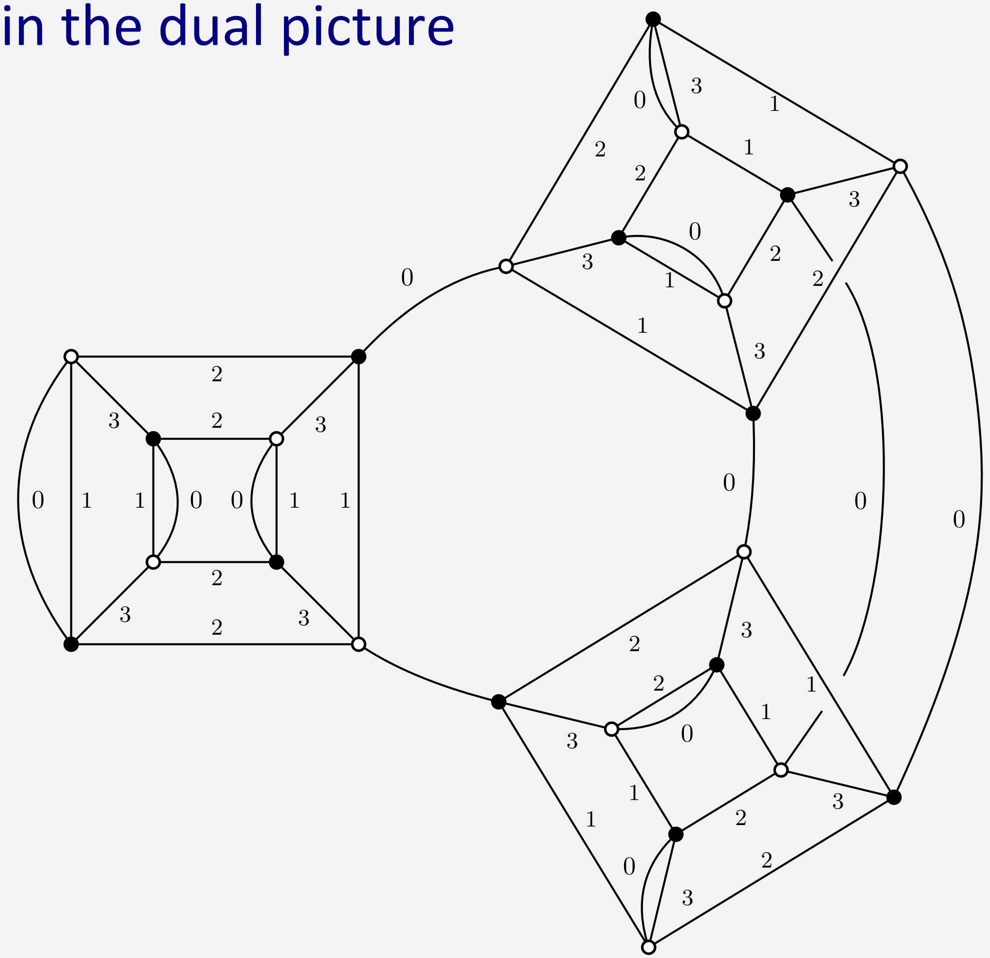
Gluings of octahedra :



Generalize 8-angulations in 3D

1 – Colored triangulations and edge-colored graphs

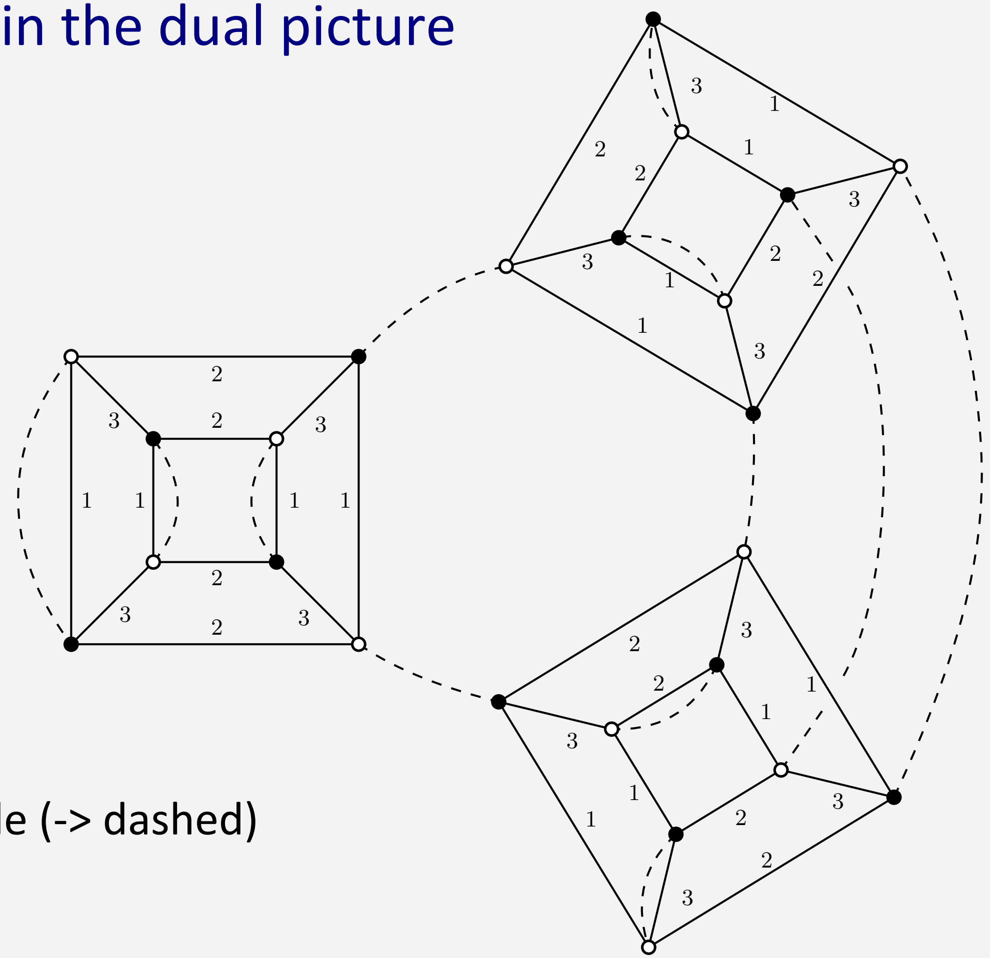
Gluings of octahedra in the dual picture



Generalize 8-angulations in 3D

1 – Colored triangulations and edge-colored graphs

Gluings of octahedra in the dual picture



Color 0 plays a special role (-> dashed)

Generalize 8-angulations in 3D

1 – Colored triangulations and edge-colored graphs

Any gluings of octahedra?

→ Those with maximal number of edges at fixed number of octahedra

→ Graphs that maximize the number of two-colored cycles at fixed number of octahedra.

Why?

→ Large N limit of size N and rank 4 random tensor models

We are interested in the critical behavior of their generating function.

1 – Colored triangulations and edge-colored graphs

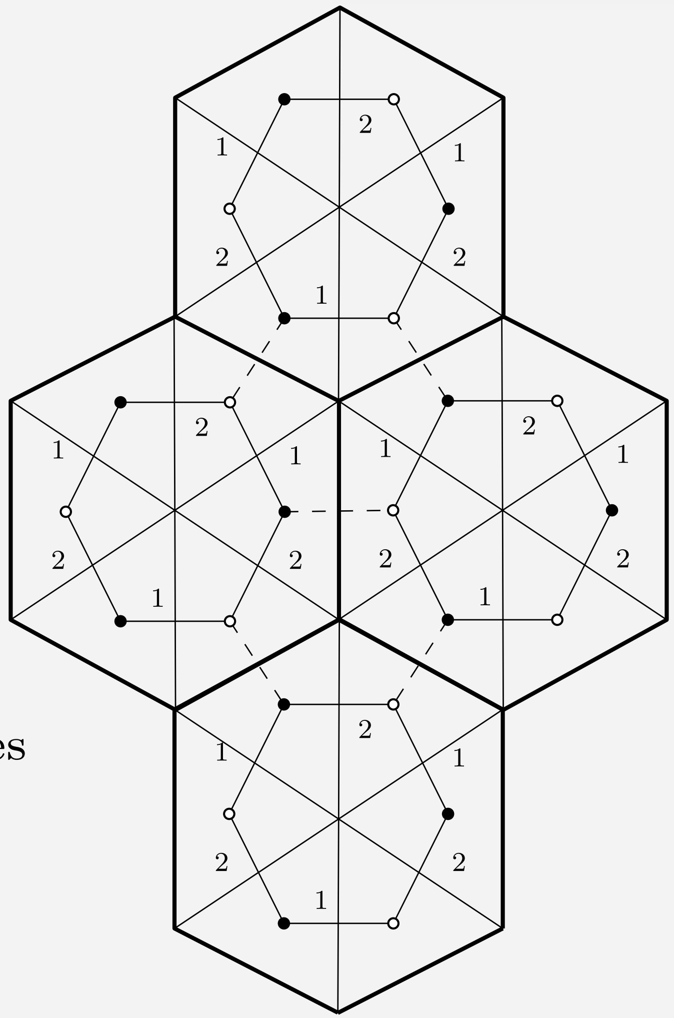
Example 1 : p-angulation in 2D

Maximize the number of vertices at fixed number of p-angles

$$2N_{\rm edges} = pN_{\rm p-angles}$$

→ $$2-2g = N_{\rm vertices} - \frac{p-2}{2}N_{\rm p-angles}$$

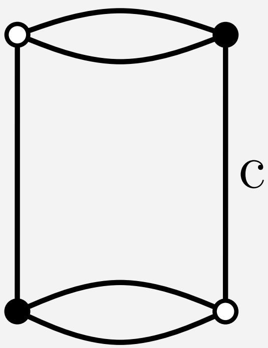
→ Selects planar p-angulations



hexangulation, locally

1 – Colored triangulations and edge-colored graphs

Example 2 : Quartic "melonic" building blocks in 3D

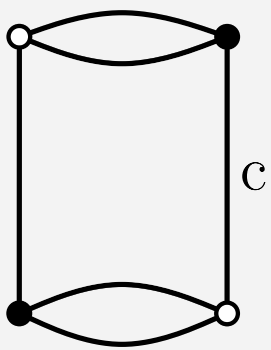


$c = 1,2,3$

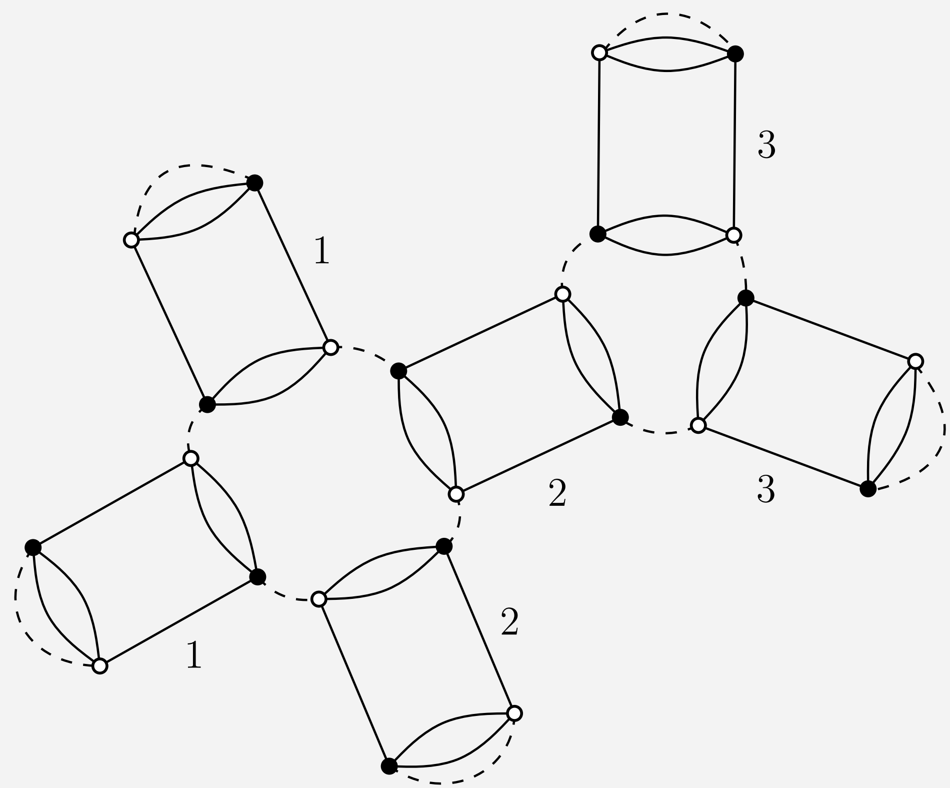
1 – Colored triangulations and edge-colored graphs

Example 2 : Quartic "melonic" building blocks in 3D

→ Maximizing graphs = "*melonic*", *e.g :*

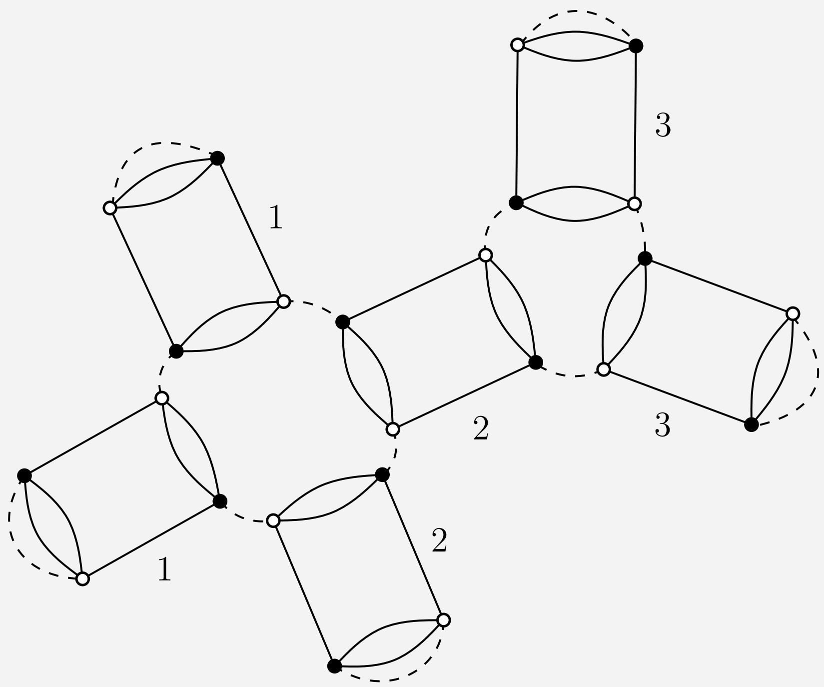


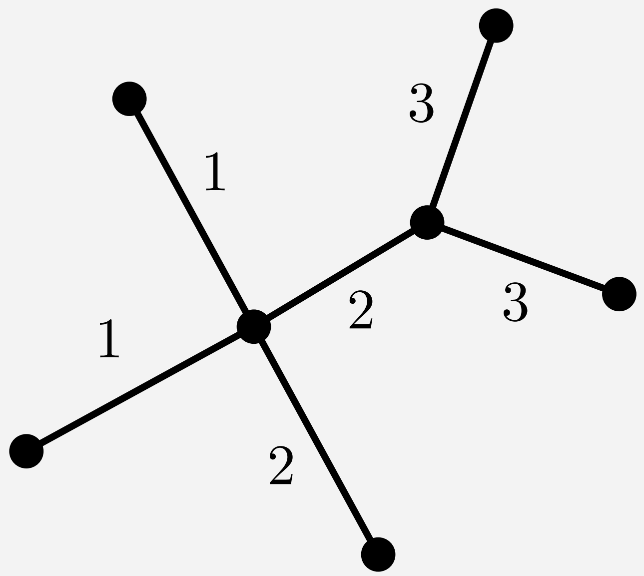
$$c = 1,2,3$$



1 – Colored triangulations and edge-colored graphs

Maximizing graphs <-> **plane trees** with colored edges





2 – Bijection with hypermaps

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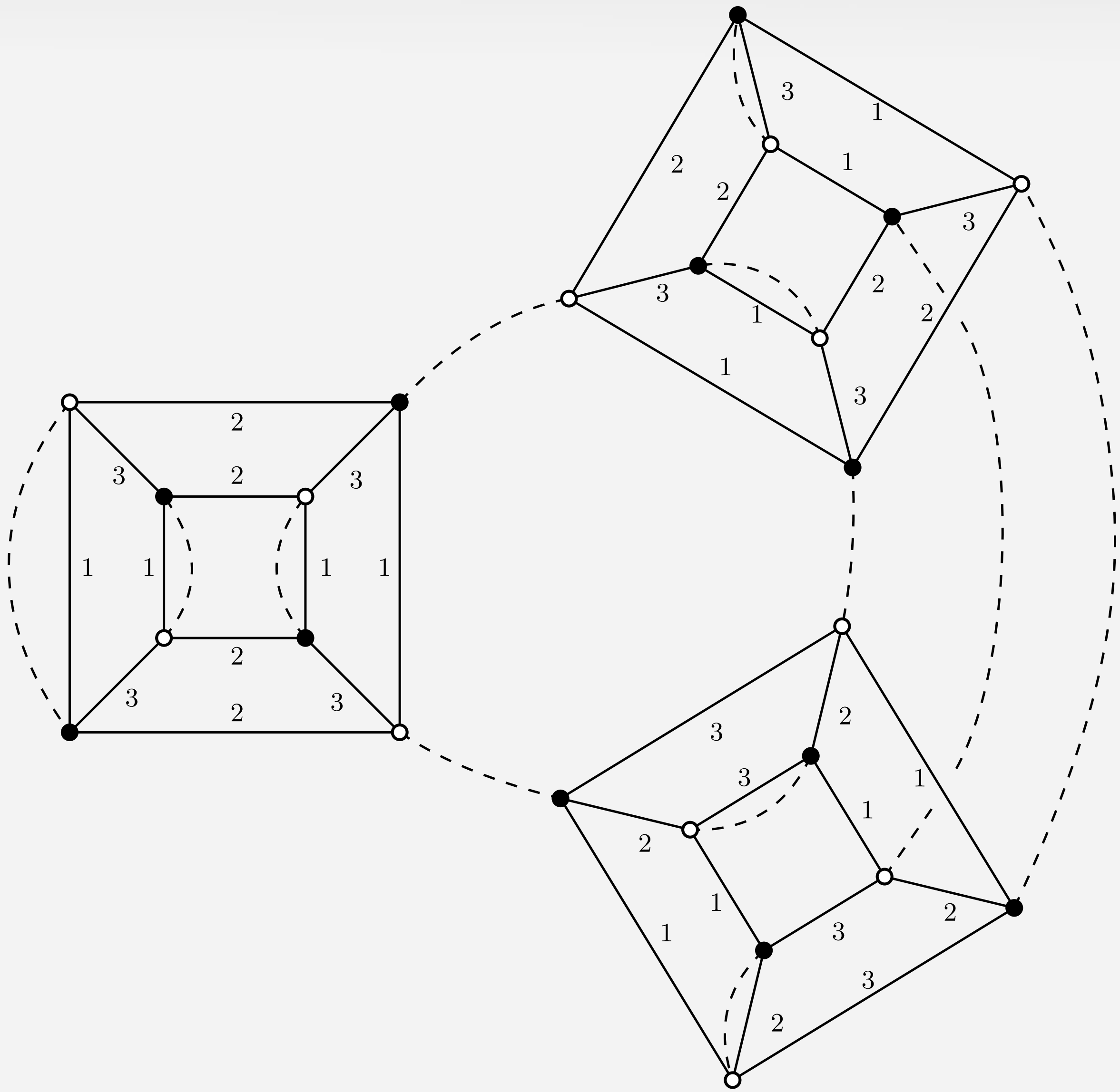
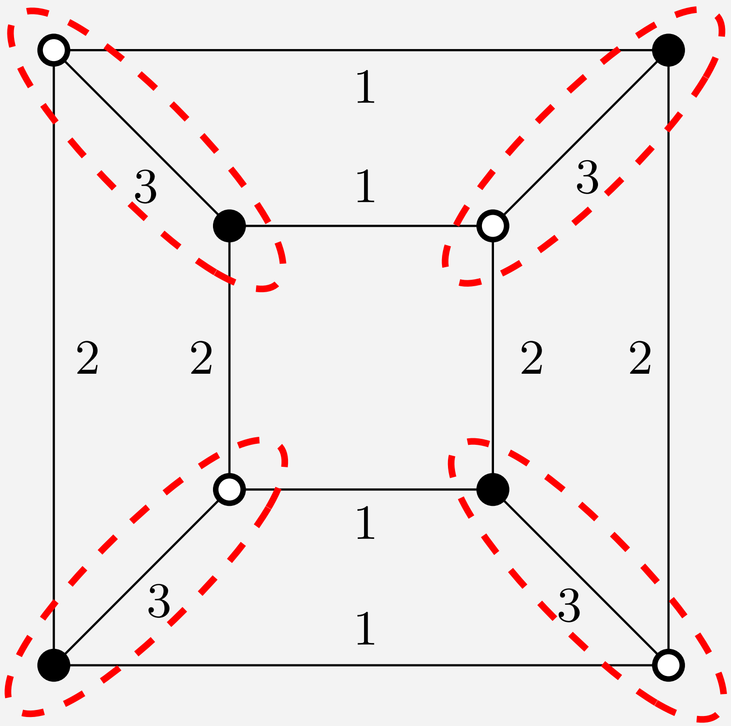
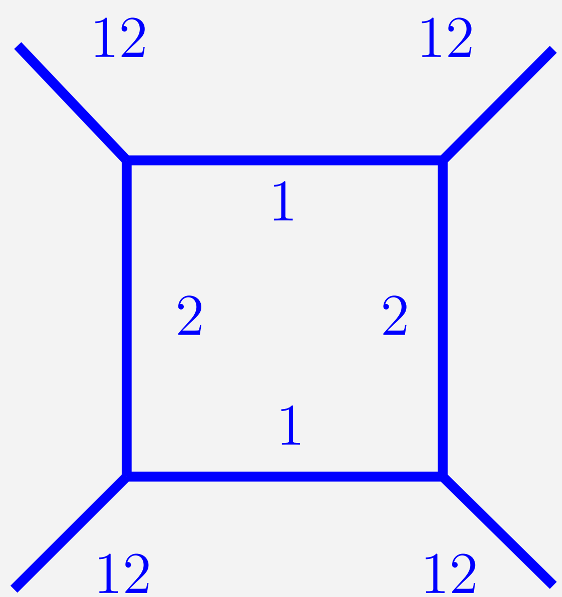


2 – Bijection with hypermaps

Building block :

Color 3 edge = half an edge, colored with colors 1 and 2





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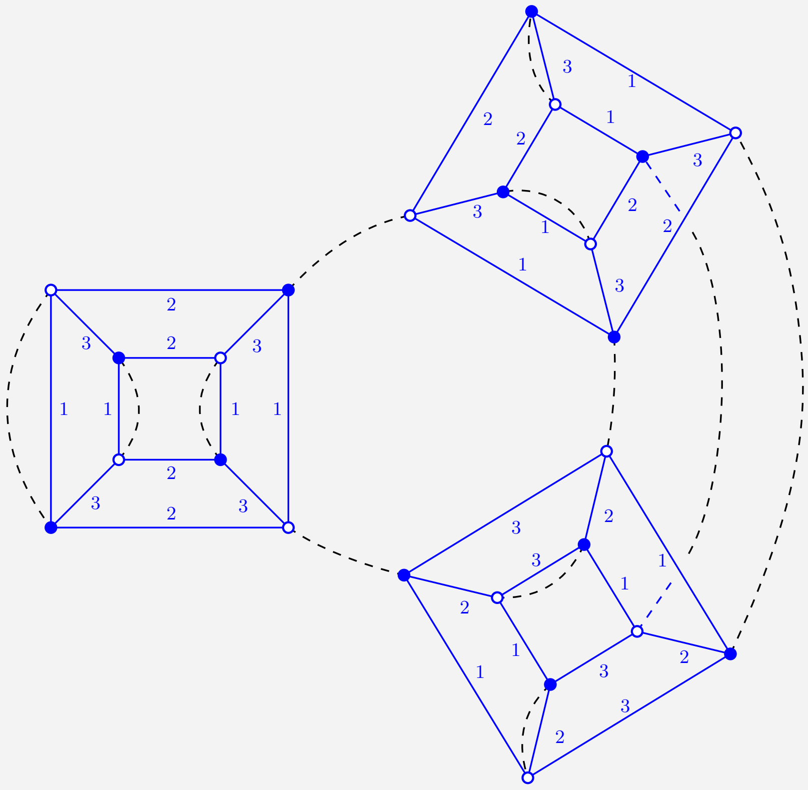
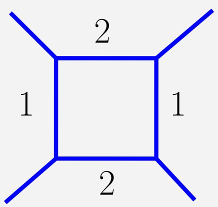
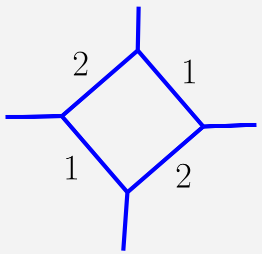
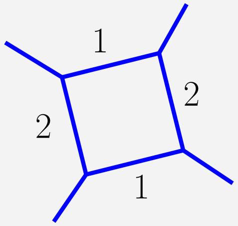
Building block :





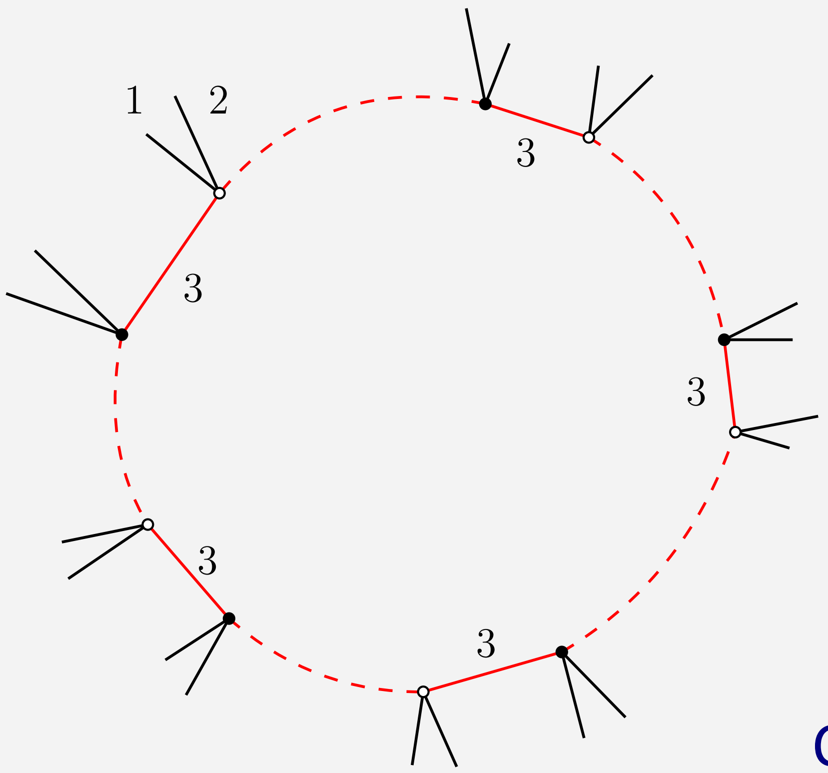


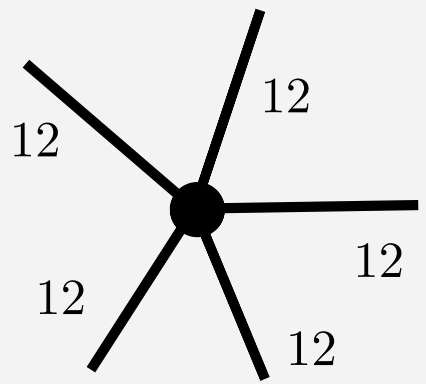


2 – Bijection with hypermaps

Glue building blocks together?

⇔ Cycles that alternate edges of color 0 and 3





Color 3 edge = half an edge around a black vertex

2 – Bijection with hypermaps

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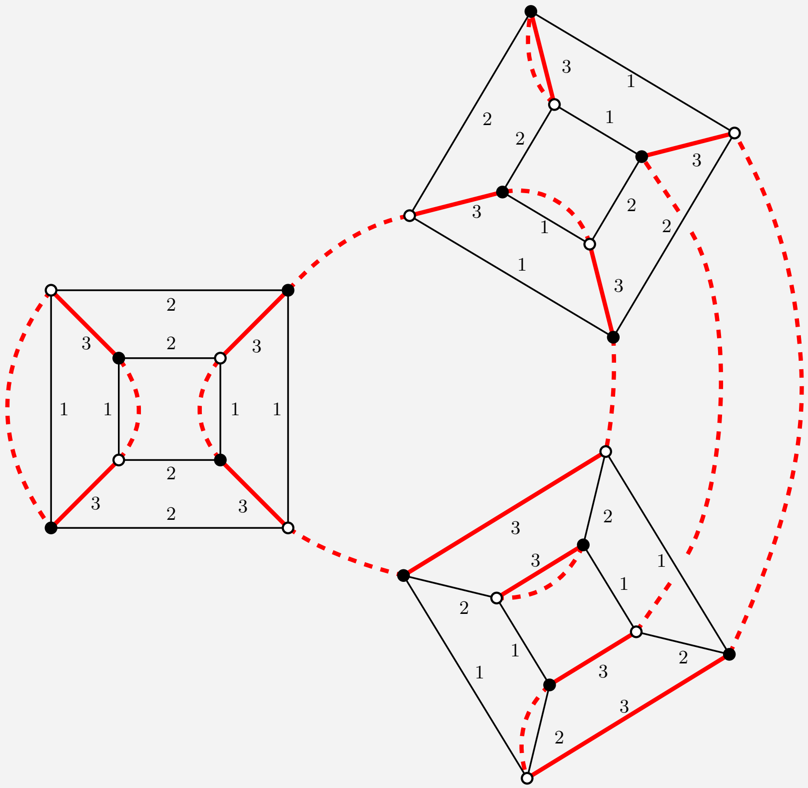
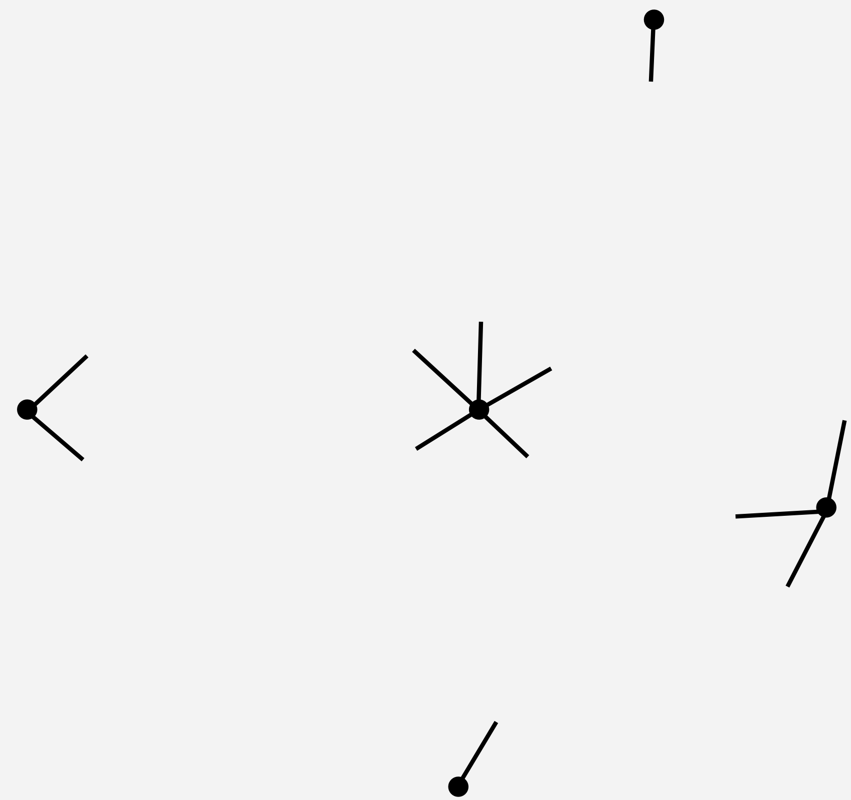
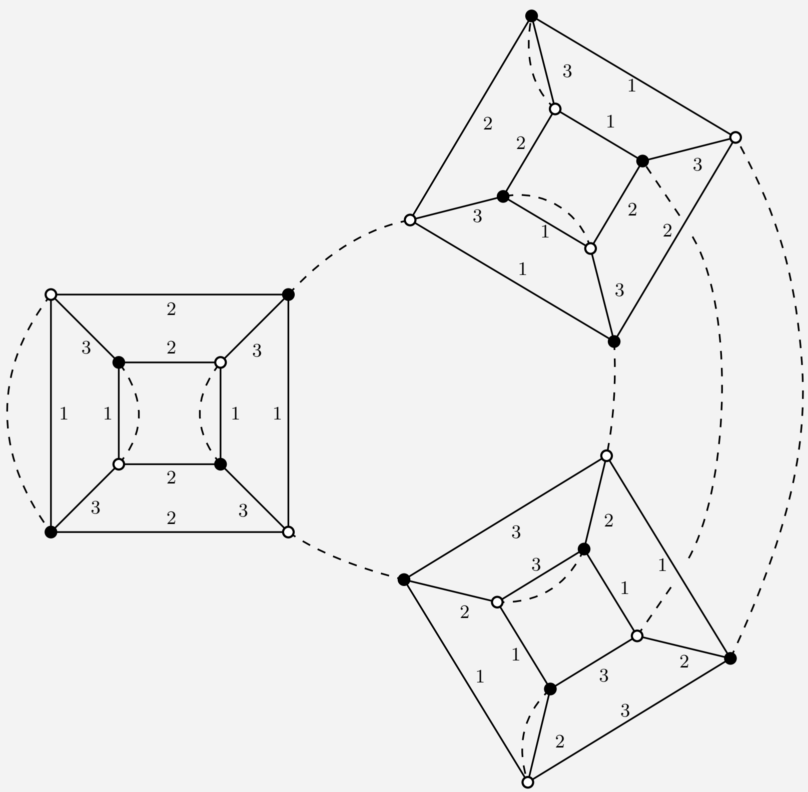
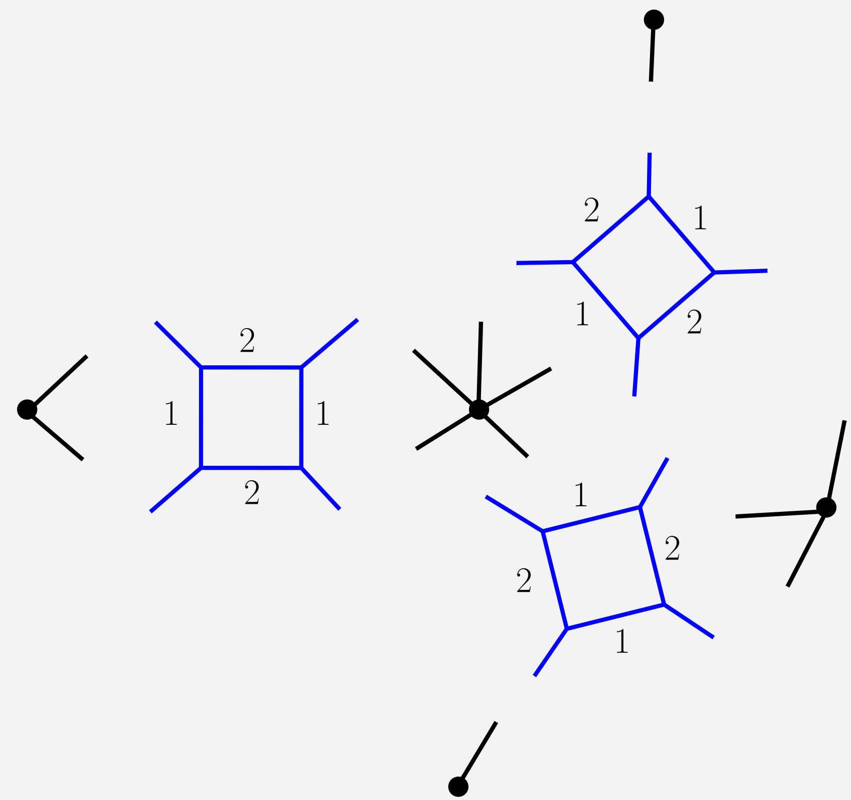




2 – Bijection with hypermaps

Color 3 edge = two half edges : *one* around a blue sector, *one* around a black vertex.

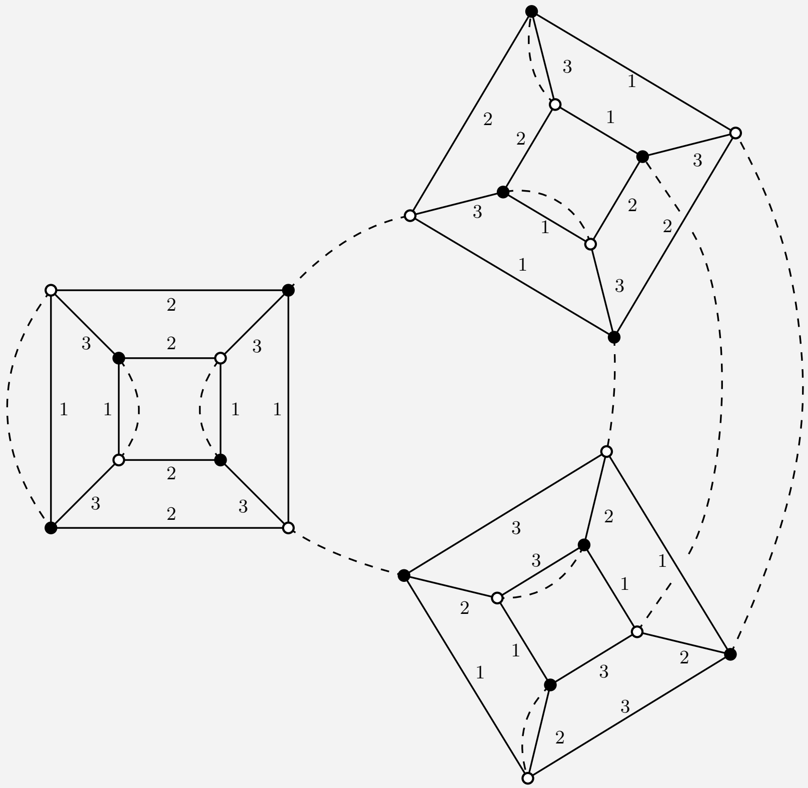


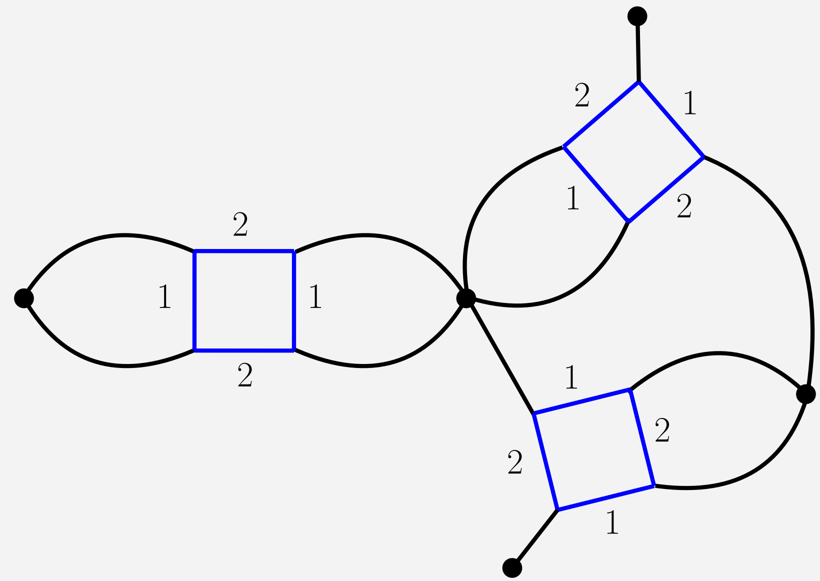


2 – Bijection with hypermaps

Color 3 edge = two half edges : *one* around a blue sector, *one* around a black vertex.

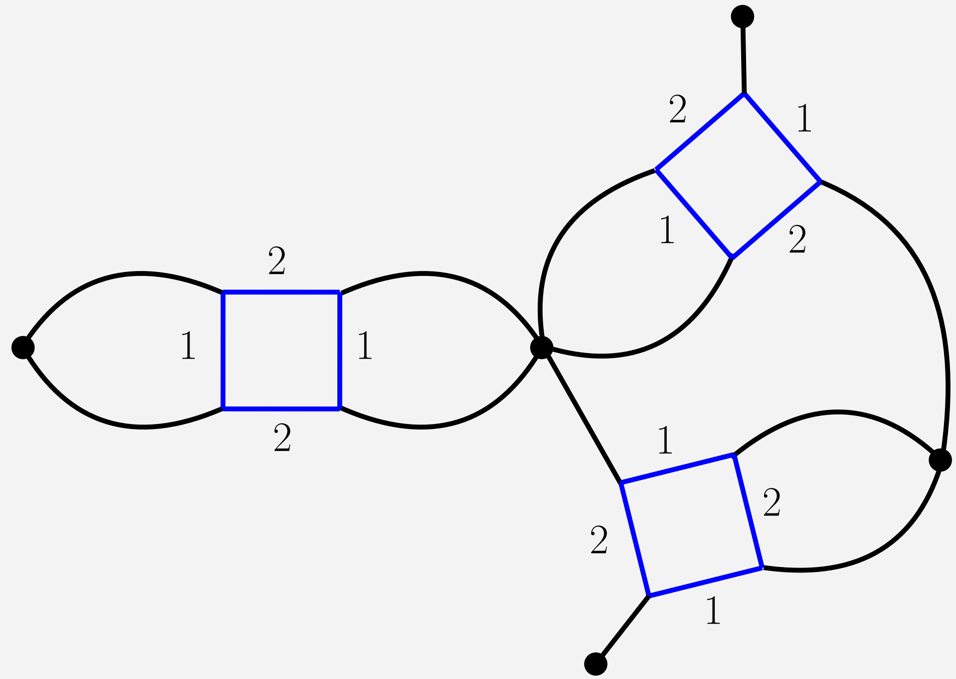
→ contract them to form an edge!





2 – Bijection with hypermaps

Edge in triangulation
<-> Two-colored cycle in graph
<-> Face around combinatorial map of single color



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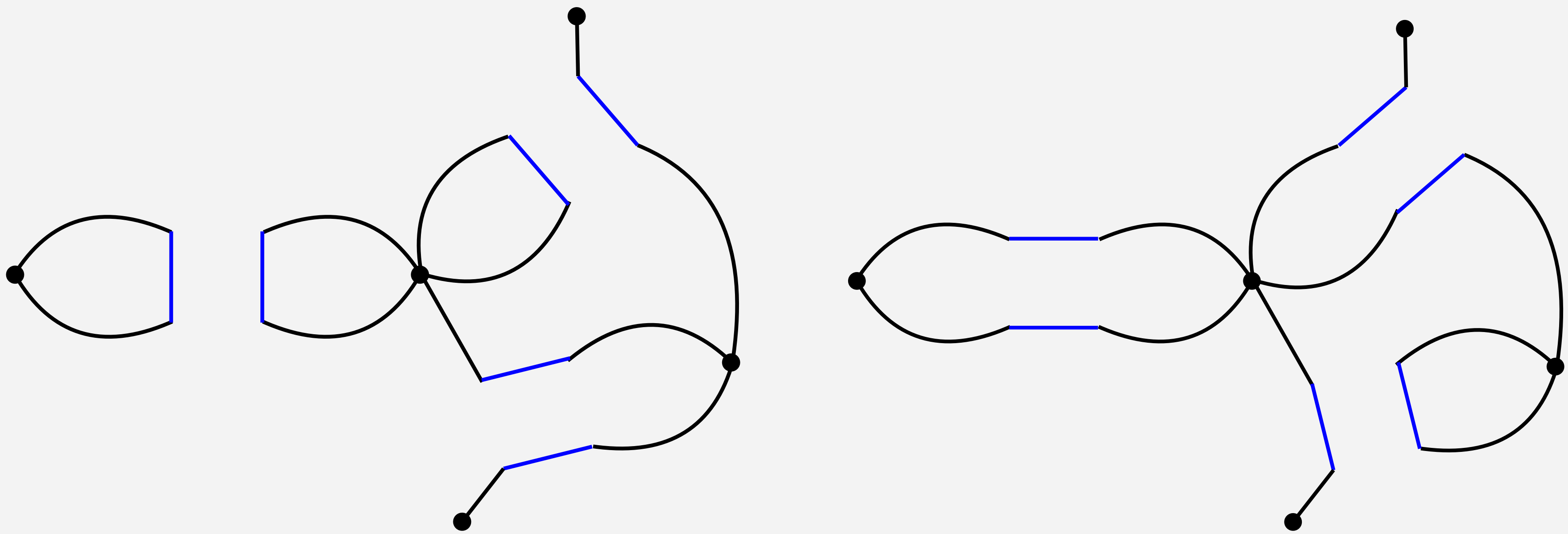

Color 1 : 5 faces

Color 2 : 3 faces

Color 3 : 5 faces

3 – Maximal gluings of octahedra

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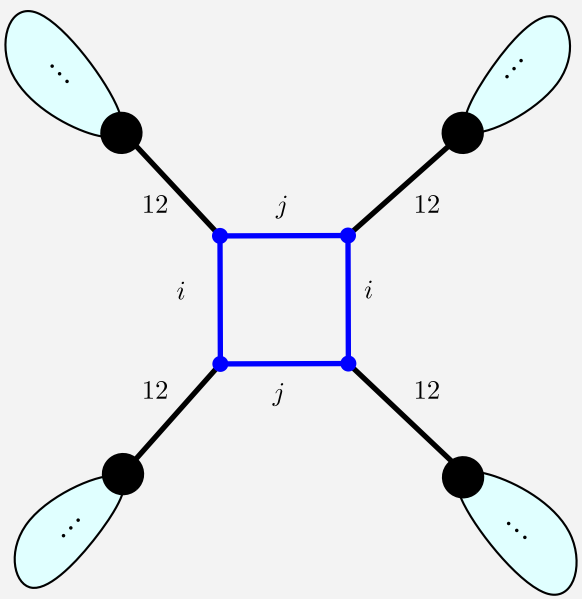
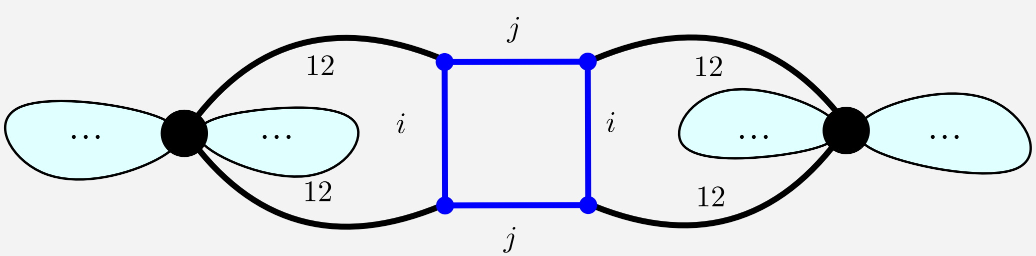
Maximizing maps :

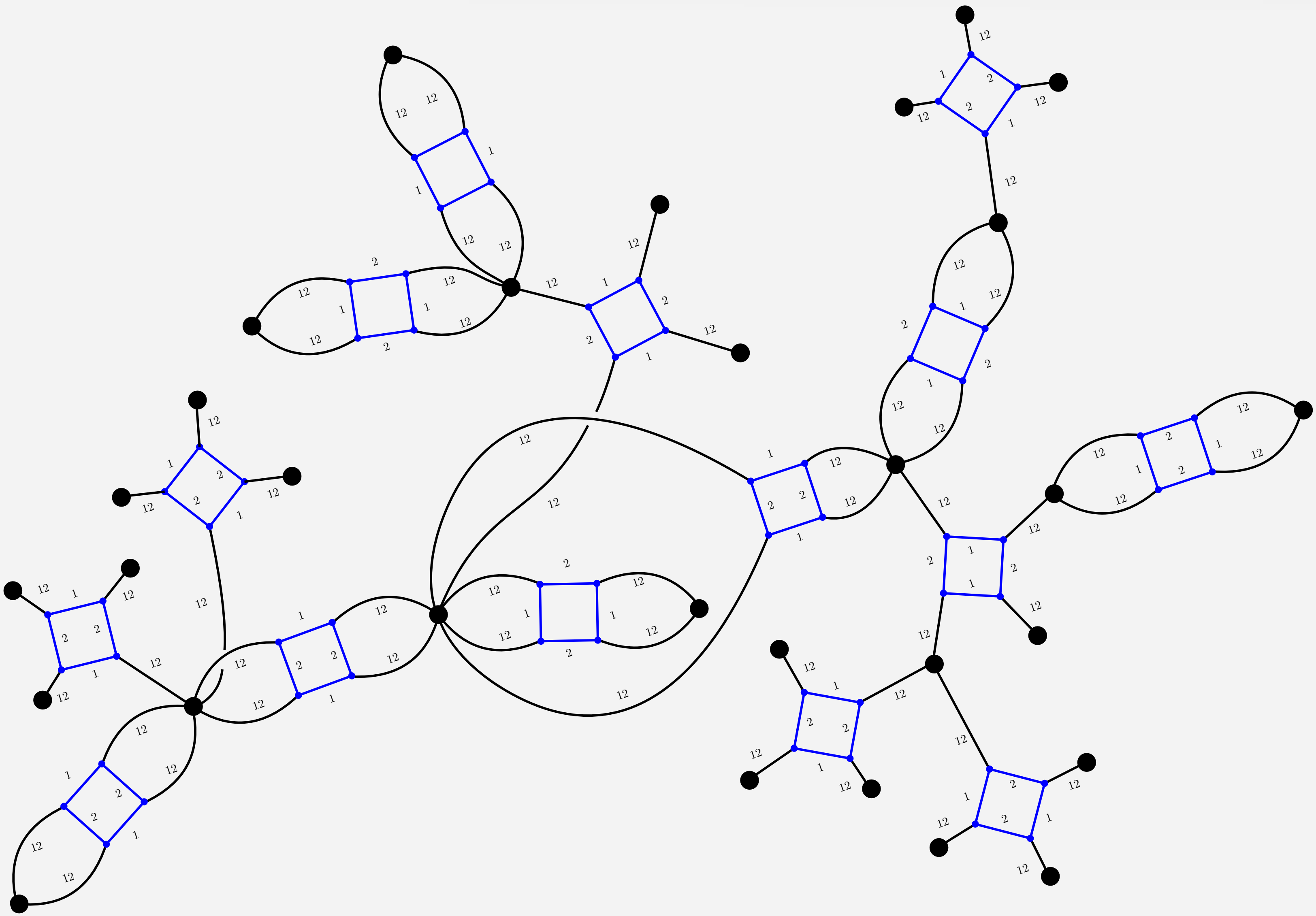
→ Planar

→ Each blue sector locally s.t.





3 – Maximal gluings of octahedra



3 – Maximal gluings of octahedra

Maximal triangulations verify

$$N_{\text{edges}} = 3 + 5N_{\text{octahedra}}$$

And 3D gluings of octahedra verify

$$3 + 5N_{\text{octahedra}} - N_{\text{edges}} \in \mathbb{N}$$

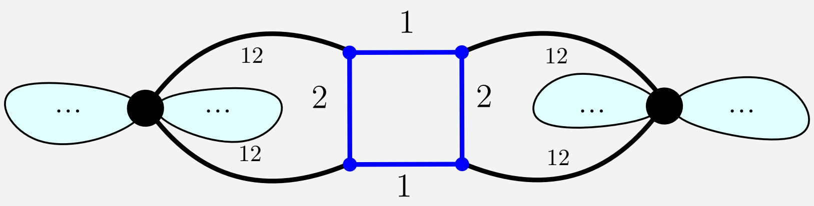
Compare with 2D gluings of p-gons

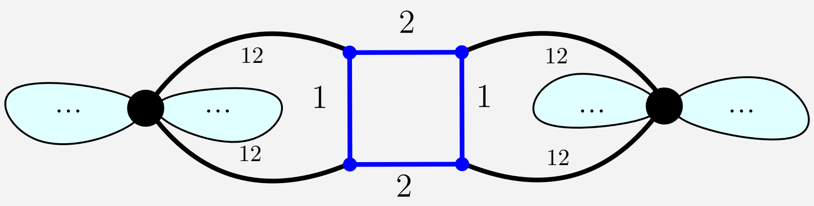
$$2 + \frac{p-2}{2} N_{p-\text{gons}} - N_{\text{vertices}} = 2g \in 2\mathbb{N}$$

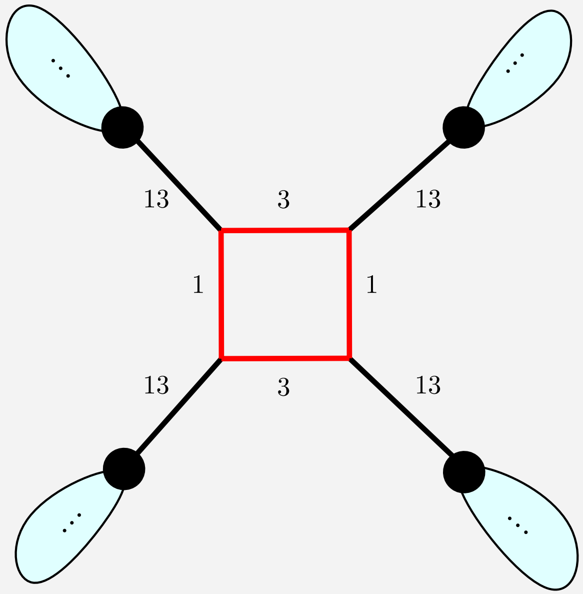
3 – Maximal gluings of octahedra

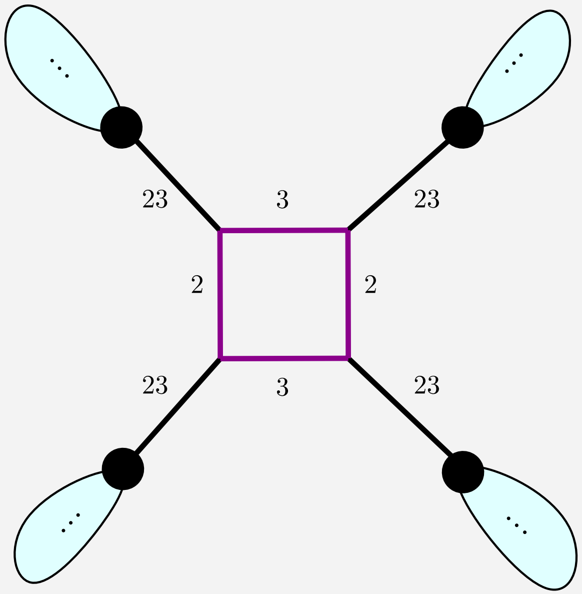
Maximal triangulations are in bijection with a family of trees.

3 – Maximal gluings of octahedra









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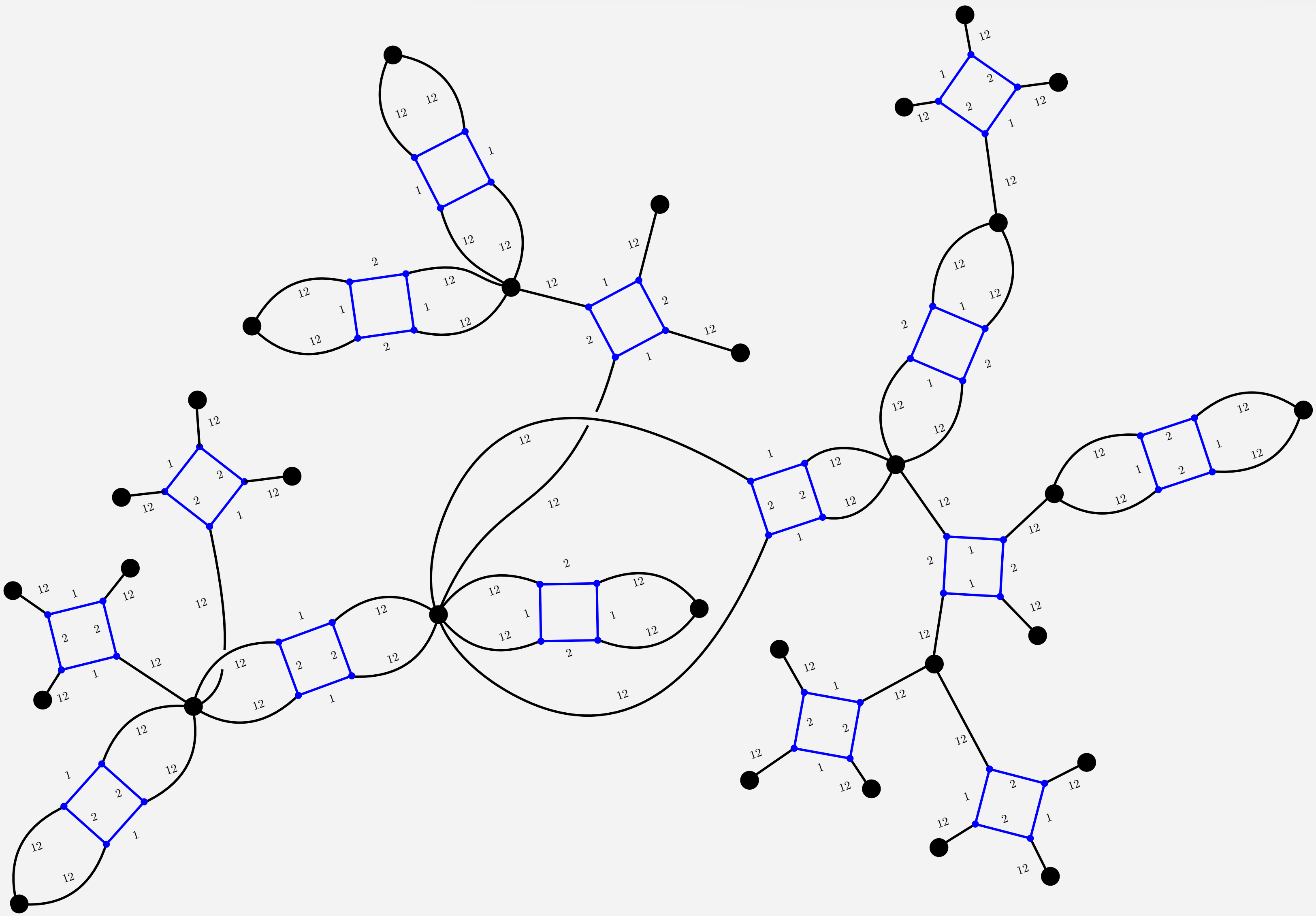

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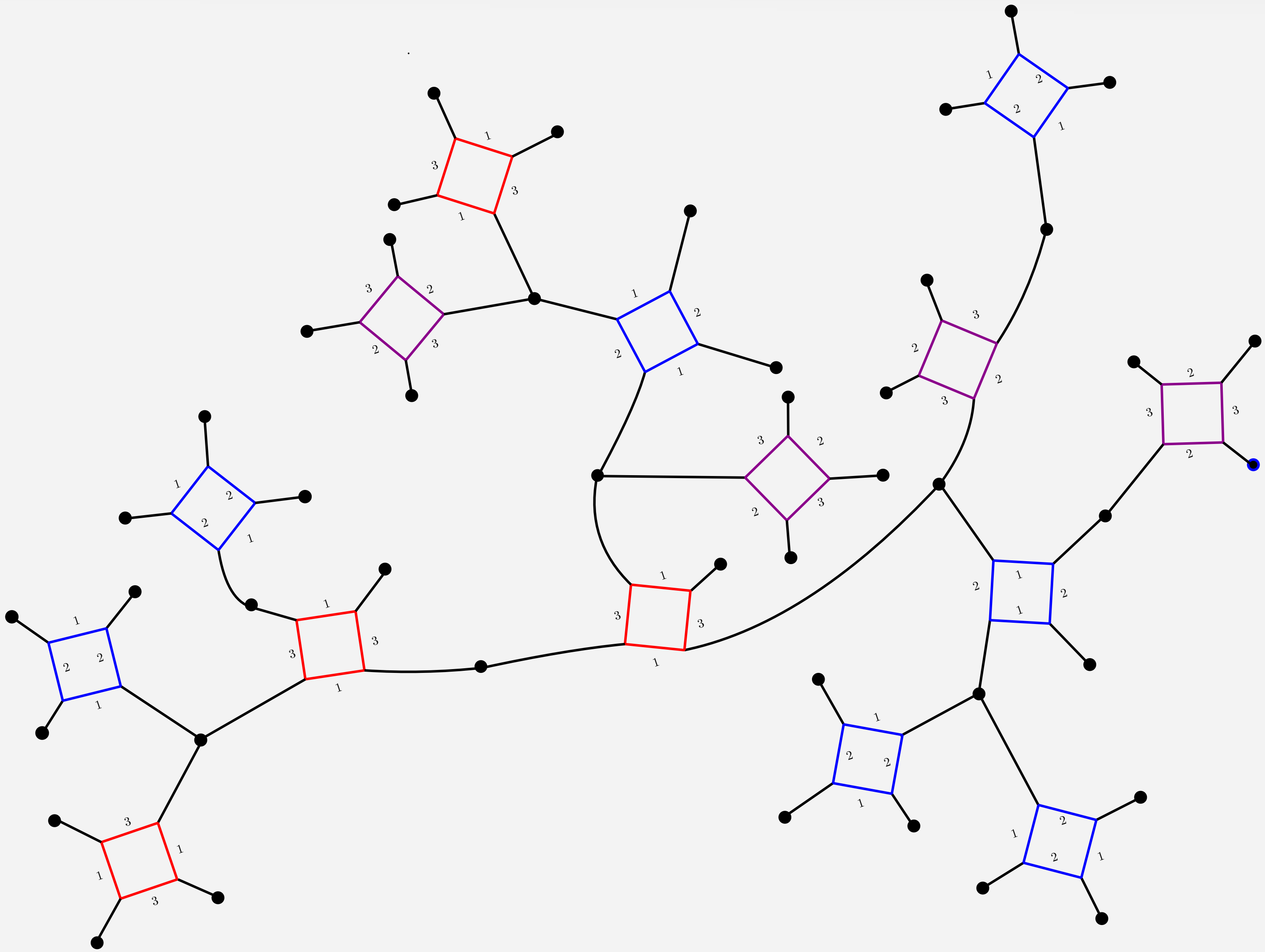

3 – Maximal gluings of octahedra

Their two-point function is such that :

(= generating function of maximal maps with one marked corner)

$$G(z) = 1 + 3zG(z)^4 \quad \rightarrow \quad G(z) = \frac{4}{3} - \sqrt{\frac{2048}{243}\left(\frac{9}{256} - z\right)} + \cdots$$

$$\rightarrow \quad z_c = \frac{9}{256} \qquad \gamma = \frac{1}{2}$$

Maximal triangulations are shown to have the topology of the 3-sphere.

Conclusions

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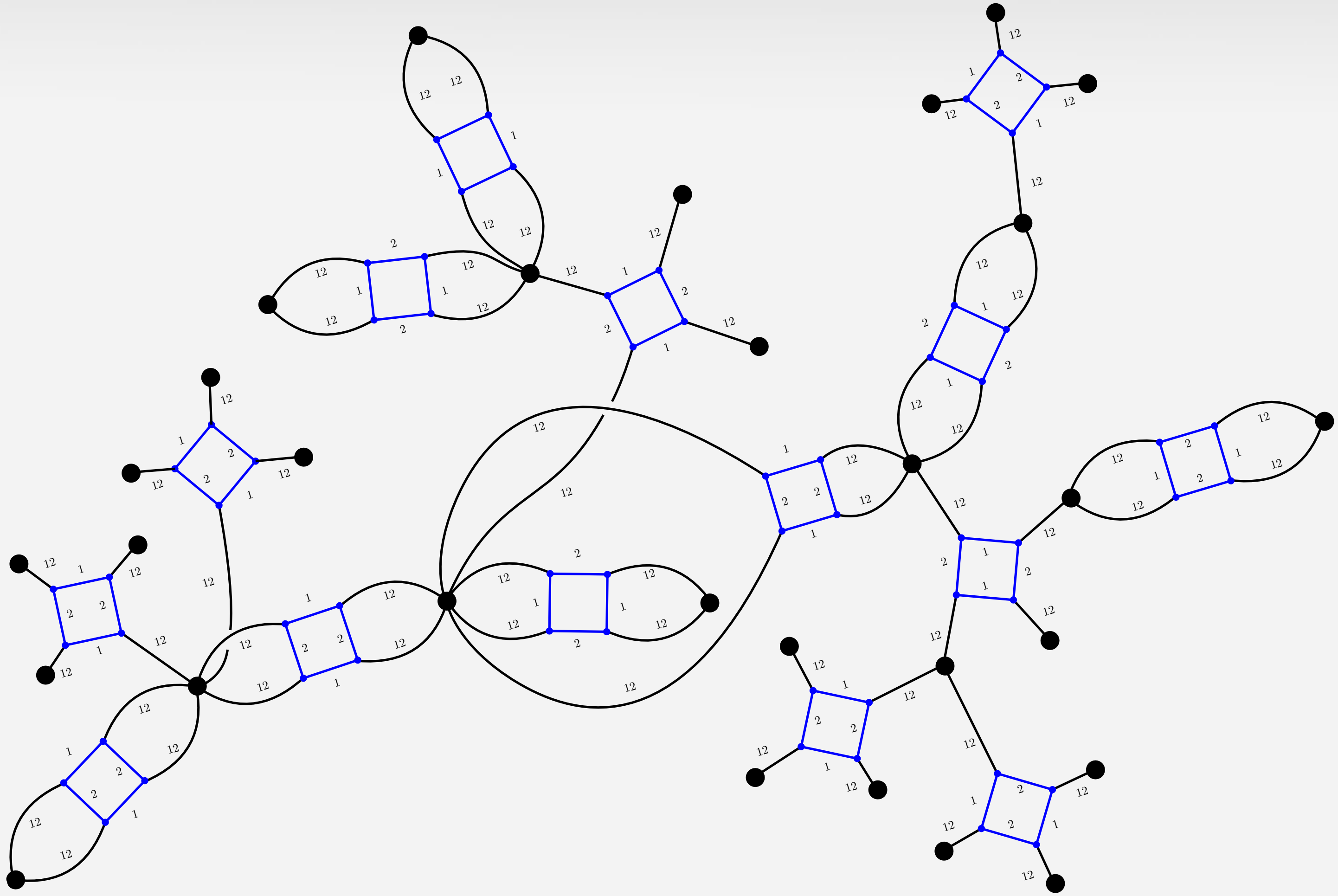
- Colored triangulations provide a good framework for combinatorics
- Bijection generalizes Tutte's bijection for 3D 8-angulations
- Precisely represent topologies by superposed hypermaps
- Identify and count maximal triangulations
- Are there building blocks that exhibit a "3D" critical behavior?



THANK YOU !!