Flowing in Tensorial Field Theories

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Outline

1. Tensorial Field Theories
2. Renormalization Group: The quartic melonic models
   - Perturbative renormalization
   - Asymptotic freedom
3. Non-perturbative renormalization Group
   - Functional renormalization group
   - FRG for TFTs

Tensorial Field Theories

Definition

- The rank-d tensor is understood as a field over the discrete group $\mathbb{Z}^d$:

$$T, \bar{T} : (\mathbb{Z}^D)^d \to \mathbb{C}$$

- The fundamental theory is defined by the partition function:

$$Z[\bar{J}, J] = \int d\mu_C(\bar{T}, T) \exp\left(-S_{int}[\bar{T}, T] + \sum_{\vec{\mathbf{p}} \in (\mathbb{Z}^D)^d} \bar{J}_{\vec{\mathbf{p}}} T_{\vec{\mathbf{p}}} + \bar{T}_{\vec{\mathbf{p}}} J_{\vec{\mathbf{p}}} \right)$$

where

- The Gaussian measure $d\mu_C(\bar{T}, T)$ is defined as:

$$\int d\mu_C(\bar{T}, T) \bar{T}_{\vec{\mathbf{p}}} T_{\vec{\mathbf{p}}'} = \delta_{\vec{\mathbf{p}}\vec{\mathbf{p}}'} \int_{1/\Lambda^2}^{\infty} d\alpha e^{-\alpha(\mathbf{p}^2+m^2)} \delta\left(\sum_{i=1}^{d} \mathbf{p}_i \right)$$

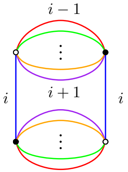
- $\Rightarrow$ Introduce a scale over the Feynman graphs
- The factor $\delta\left(\sum_{i=1}^{d} \mathbf{p}_i \right)$ called closure constraint can be viewed as a flatness constraint of the dual triangle, and is a physical input.

Tensorial Field Theories

Definition

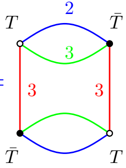
- The interaction part S_{int} is a sum of connected tensorial bubbles of valences higher than 2.
- We only consider the quartic melonic actions:

$$S_{int} = \lambda \sum_{i=1}^{d}$$



where we recall that (for $d = 3$):

$$\sum_{\vec{n},\vec{p}} T_{n_1 n_2 n_3} \bar{T}_{n_1 n_2 p_3} T_{p_1 p_2 p_3} \bar{T}_{p_1 p_2 n_3} =$$

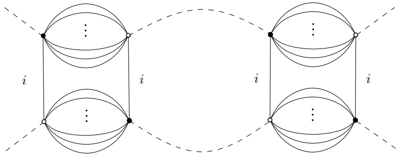


Renormalization group

The quartic melonic models

Illustration: Computation of the one-loop 4-points function.

- Involves diagrams of the form:



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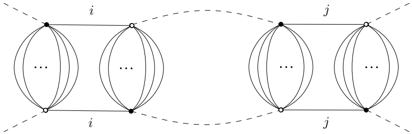

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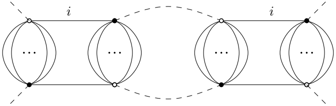
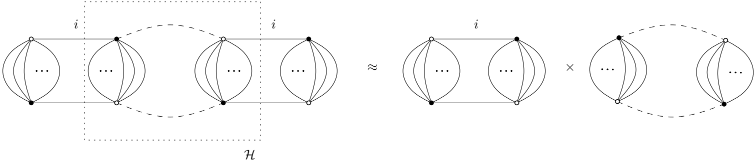
The leading order graphs (in Λ !) are the melonic contractions:



$$\sim \int_{1/\Lambda^2}^{\infty} d\alpha_1 d\alpha_2 \sum_{p_j, j\neq i} \prod_{i=1}^{d} e^{-(\alpha_1+\alpha_2)p_i^2} \delta\left(\sum_{i=1}^{d} p_i\right)$$

- Question : This expression can be approximated by a quartic tensorial invariant in the sector $\alpha_1, \alpha_2 \to 0$?
- The answer is YES for this graph:



- Such a graph is said to be tracial (=contractible and connected)
- All the melonic graphs are tracial
- All the divergences in the quartic models are in the melonic sector for $d \leq 6$

Renormalization make sense for this model !

Renormalization group

beta function and asymptotic freedom

The scale invariance of the model under the change of cut-off : $\Lambda \to \Lambda + d\Lambda$ imply:

$$\Lambda \frac{d\lambda}{d\Lambda} = -\lambda^2 \frac{4\pi^{(d-2)/2}(d+1)}{(d-1)^{3/2}} \Lambda^{d-6}$$

- Suggest to attribute a **canonical dimension** $6-d$ to the coupling λ
- In accordance with standard renormalization, this suggest that interaction is **just-renormalizable** for $d=6$ (Supported by rigorous approaches)
- In addition, for $d=6$, the model is asymptotically free !

Functional renormalization group

Scalar field theory

- We consider the fundamental action:

$$S[\phi] = \int dV_x \left\{ \frac{1}{2}(-\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \mathcal{V}(\phi) \right\}$$

- Introduce a deformed one-parameter family of free energies:

$$W_k[J] = \ln \left\{ \int d\phi \exp\left(- S[\phi] - \Delta S_k[\phi] + \int dV_x J\phi \right) \right\}$$

$$\Delta S_k[\phi] = \frac{1}{2} \int \frac{dV_p}{(2\pi)^d} R_k(p^2)\phi(p)\phi(-p)$$

- $R_k(p)$ discriminates between low/high momentum modes:

$$R_k(p^2) = \begin{cases} k^2 & p^2 \ll k^2 \\ 0 & p^2 \gg k^2 \end{cases} \qquad R_k(p^2) \simeq \begin{cases} \Lambda^2 & k^2 \simeq \Lambda^2 \\ 0 & k^2 \simeq 0 \end{cases}$$

  - High momentum modes are integrated out
  - Low momentum modes are suppressed by large mass term

- Effective average action $\Gamma_k[M]$ ($\equiv$ modified Legendre transform of $W_k[J]$):

$$\Gamma_k[\phi] := \int dV_x\, J\phi - W_k[J] - \Delta S_k[\phi]$$

$$\phi = \frac{\delta W_k}{\delta J}$$

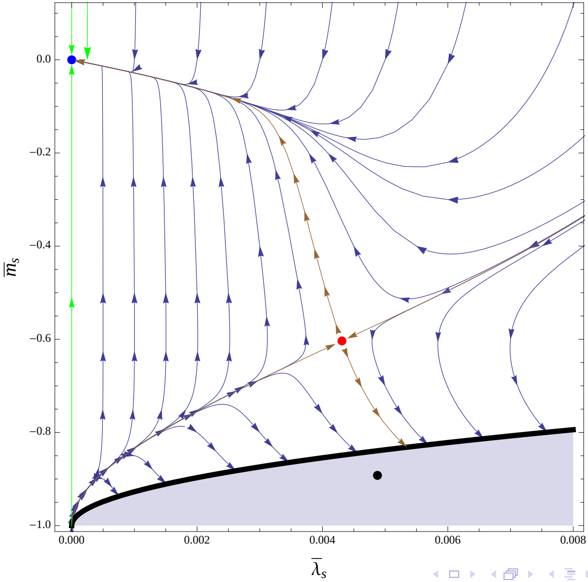
- k-dependence captured by the RG ("Wetterich") equation:

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}\int \frac{dV_p}{(2\pi)^d} k\partial_k R_k \left(\frac{\delta^2 \Gamma_k}{\delta\phi^2} + R_k\right)^{-1}\Bigg|_{p,-p}$$

- The same strategy can be applied to TFTs !

Functional renormalization group

Application to an Abelian rank-6 model



Thank you!