

Enhanced Tensor Models

Mixing trees and planar maps

Thibault Delepoue

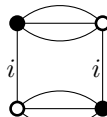
Université Paris-Sud

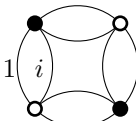
Tensor

$\mathbf{T}_{n^1 n^2 n^3 n^4}$ a rank 4 tensor of size N .

▶ 1 quadratic trace invariant , $\mathbf{T}_{\bar{n}\delta\bar{n}\bar{m}}\bar{\mathbf{T}}_{\bar{m}} = \mathbf{T} \cdot_{1234} \bar{\mathbf{T}}$.

▶ 7 quartic invariants :

▶ 4 melonic invariants \mathcal{B}_i^{mel}  $\text{Tr}_i(\mathbf{T} \cdot_{jkl} \bar{\mathbf{T}}) \cdot_i (\mathbf{T} \cdot_{jkl} \bar{\mathbf{T}})$

▶ 3 "matrix-type" \mathcal{B}_{1i}^{mat}  $\text{Tr}_{1i}(\mathbf{T} \cdot_{jk} \bar{\mathbf{T}}) \cdot_{1i} (\mathbf{T} \cdot_{jk} \bar{\mathbf{T}})$.

Quartic tensor model

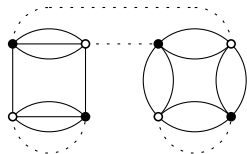
$$Z = \int e^{-N^3 S} d\mathbf{T} d\bar{\mathbf{T}}$$

$$S = \text{Diagram 1} + \sum_{i=1}^4 \lambda_{mel} \text{Diagram 2} + \sum_{i=2}^4 \lambda_{mat} \text{Diagram 3}$$

The equation defines the action S as a sum of three types of diagrams:

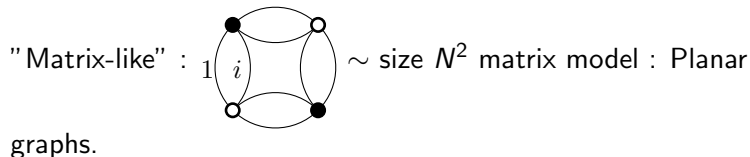
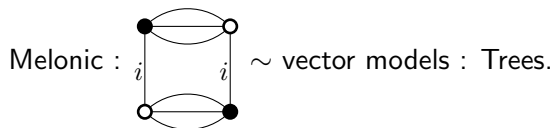
- A diagram with two vertices (one black, one white) and four edges connecting them in a complete graph K_2 .
- A diagram with four vertices (two black, two white) and four edges forming a square with two diagonal edges. The two white vertices are labeled i .
- A diagram with four vertices (two black, two white) and six edges forming a complete graph K_4 . The two white vertices are labeled i .

Feynman Expansion :



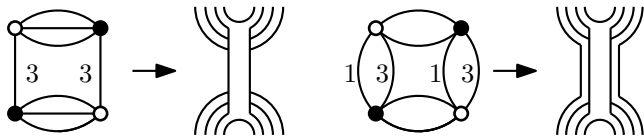
$$A(\text{Graph}) = \lambda_{mel}^{|\mathcal{B}^{mel}|} \lambda_{mat}^{|\mathcal{B}^{mat}|} N^{|\text{faces}| - 3|\text{bubbles}|}$$

Expectations for large N behaviour

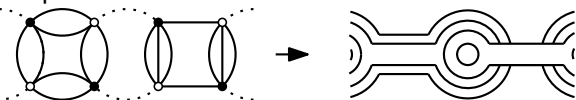


Intermediate field representation

- ▶ Graph \rightarrow Map
- ▶ Bubble \rightarrow Edge





- ▶ Loop \rightarrow Vertex



$$A(\text{Map}) = \lambda_{mel}^{|E^{mel}|} \lambda_{mat}^{|E^{mat}|} N^{|faces| - 3|edges|}$$

Face counting

$$\lambda_{mat} = \lambda_{mel} = \lambda \quad A \sim N^{|Faces| - 3|Edges|} = N^\omega$$

- ▶ Adding melon leaf  : +3faces +1 edge. $\Delta\omega = 0$
- ▶ Adding matrix leaf  : +2faces +1 edge. $\Delta\omega = -1$
- ▶ Adding loop edge : $\Delta face \leq 2, \Delta\omega \leq -1$

Leading order :

Trees of melon edges, no matrix edges.

Enhancing

Solution : $\lambda_{mat} = N\lambda_{mel}$

Enhanced model (Bonzom, Delepouve, Rivasseau 2015)

$$S = \text{Diagram 1} + \lambda \left(\sum_{i=1}^4 \text{Diagram 2}_i + N \sum_{i=2}^4 \text{Diagram 3}_i \right)$$

The diagram on the left shows two vertices, one black and one white, connected by three curved edges. The first term in the parentheses is a sum over $i=1$ to 4 of a square diagram with vertices i (top-left and bottom-right are black, top-right and bottom-left are white) and four curved edges. The second term is N times a sum over $i=2$ to 4 of a square diagram with vertices i (top-left and bottom-right are black, top-right and bottom-left are white) and four curved edges, with an additional curved edge connecting the top and bottom vertices on the right side.

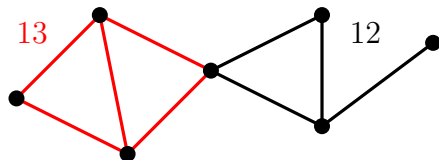
$$A(Map) = \lambda^{|\text{Edges}|} N^{|\text{Faces}|-3} |\text{Edges}|^{melon-2} |\text{Edges}|^{matrix}$$

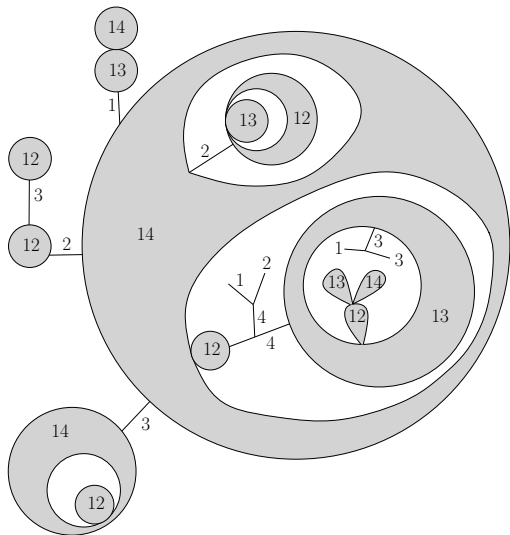
$$A(\text{Map}) \sim N^{|\text{Faces}| - 3|\text{Edges}|^{\text{melon}} - 2|\text{Edges}|^{\text{matrix}}}$$

- ▶ Adding matrix leaf  : $+2\text{faces} + 1 \text{ edge}$. $\Delta\omega = 0$
- ▶ Adding loop edge : $\Delta\text{face} \leq 2$, $\Delta\omega \leq 0$

Leading order graph

- ▶ Planar
- ▶ Melonic tree submaps
- ▶ Planar “matrix like” submaps
- ▶ Planar submaps of different colours can touch one another at only one vertex : **cactus** structure.





Thank you !