Enhanced Tensor Models

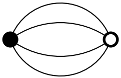
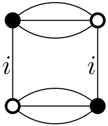
Mixing trees and planar maps

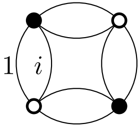
Thibault Delepouve

Université Paris-Sud

Tensor

$\mathbf{T}_{n^1 n^2 n^3 n^4}$ a rank 4 tensor of size N.

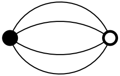
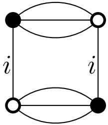
- 1 quadratic trace invariant , $\mathbf{T}_{\vec{n}}\delta_{\vec{n}\vec{m}}\bar{\mathbf{T}}_{\vec{m}} = \mathbf{T} \cdot_{1234} \bar{\mathbf{T}}$.
- 7 quartic invariants :
  - 4 melonic invariants $\mathcal{B}_i^{mel}$ 

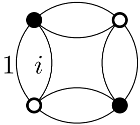
$\mathrm{Tr}_i(\mathbf{T} \cdot_{jkl} \bar{\mathbf{T}}) \cdot_i (\mathbf{T} \cdot_{jkl} \bar{\mathbf{T}})$
  - 3 "matrix-type" $\mathcal{B}_{1i}^{mat}$ 

$\mathrm{Tr}_{1i}(\mathbf{T} \cdot_{jk} \bar{\mathbf{T}}) \cdot_{1i} (\mathbf{T} \cdot_{jk} \bar{\mathbf{T}})$.

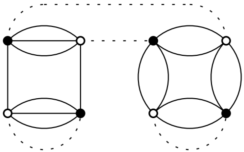
Quartic tensor model

$$Z = \int e^{-N^3 S} d\mathbf{T} d\bar{\mathbf{T}}$$

$S =$  $+ \sum_{i=1}^{4} \lambda_{mel}$ 

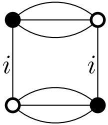
$+ \sum_{i=2}^{4} \lambda_{mat}$ 

Feynman Expansion :

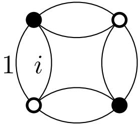


$$A(Graph) = \lambda_{mel}^{|\mathcal{B}^{mel}|} \lambda_{mat}^{|\mathcal{B}^{mat}|} \; N^{|faces| - 3|bubbles|}$$

Expectations for large N behaviour

Melonic : 

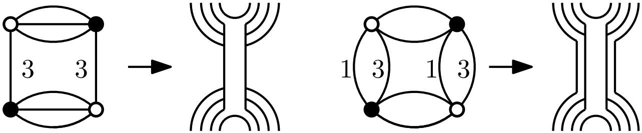
$\sim$ vector models : Trees.

"Matrix-like" : 

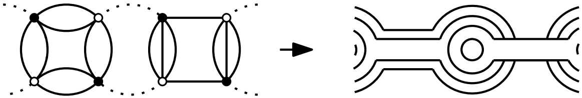
$\sim$ size N^2 matrix model : Planar graphs.

Intermediate field representation

- Graph → Map
- Bubble → Edge



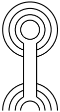
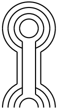
- Loop → Vertex



$$A(Map) = \lambda_{mel}^{|E^{mel}|} \lambda_{mat}^{|E^{mat}|} \; N^{|faces| - 3|edges|}$$

Face counting

$$\lambda_{mat} = \lambda_{mel} = \lambda \qquad A \sim N^{|Faces|-3|Edges|} = N^{\omega}$$

- Adding melon leaf  : $+3$faces $+1$ edge. $\Delta\omega = 0$
- Adding matrix leaf  : $+2$faces $+1$ edge. $\Delta\omega = -1$
- Adding loop edge : $\Delta face \leq 2$, $\Delta\omega \leq -1$

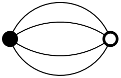
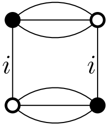
Leading order :

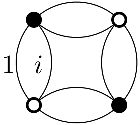
Trees of melon edges, no matrix edges.

Enhancing

Solution : $\lambda_{mat} = N\lambda_{mel}$

Enhanced model (Bonzom, Delepouve, Rivasseau 2015)

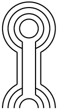
$S =$  $+ \lambda \Bigg(\sum_{i=1}^{4}$ 

$+ N \sum_{i=2}^{4}$ 

$\Bigg)$

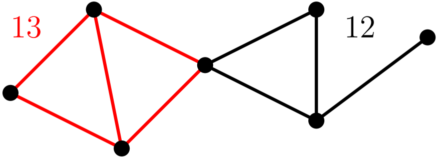
$$A(Map) = \lambda^{|Edges|} \; N^{|Faces| - 3|Edges|^{melon} - 2|Edges|^{matrix}}$$

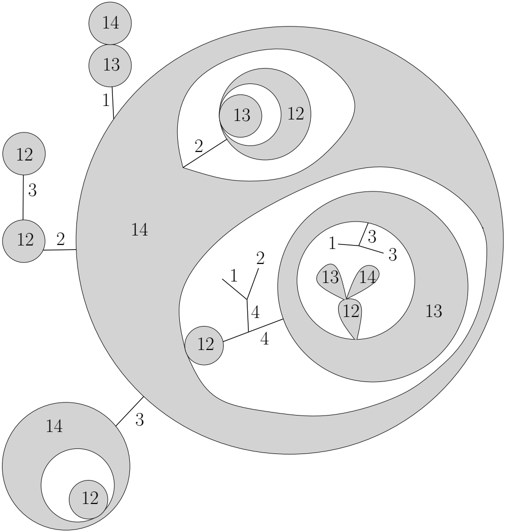
$$A(Map) \sim \; N^{|Faces| - 3|Edges|^{melon} - 2|Edges|^{matrix}}$$

- Adding matrix leaf  : $+2$faces $+1$ edge. $\Delta\omega = 0$
- Adding loop edge : $\Delta face \leq 2$, $\Delta\omega \leq 0$

Leading order graph

- Planar
- Melonic tree submaps
- Planar “matrix like” submaps
- Planar submaps of different colours can touch one another at only one vertex : **cactus** structure.





Thank you !