The Weinberg Angle and Possible New Physics Beyond the Standard Model

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Outline

• Testing non-perturbative QCD

• Testing the Neutral Current Couplings at low energy

• The NuTeV anomaly

• Resolution of the NuTeV anomaly
  – CSV in parton distribution functions
  – a new EMC effect
  – strange quark asymmetry
Non-perturbative QCD
Testing Non-Perturbative QCD

• Strangeness contribution is a vacuum polarization effect, analogous to Lamb shift in QED

\[ g_e = 2 \left( 1 + \frac{\alpha}{2\pi} - 0.328 \frac{\alpha^2}{\pi^2} + \ldots \right) \]

• It is a fundamental test of non-perturbative QCD
Strange Quarks in the Proton

There have been a number of major steps forward recently, both theory and experiment:

- Calculation of $G_{E,M}^s (Q^2)$:
  - Indirect: JLab-Adelaide
  - Direct: Kentucky

- Experimental determination of $G_{E,M}^s (Q^2)$
  - G0 and Happex
  - Mainz PVA4 and Bates

- Strangeness sigma commutator
Magnetic Moments within QCD


\[ p = \frac{2}{3} u^p - \frac{1}{3} d^p + O_N \]

\[ n = -\frac{1}{3} u^p + \frac{2}{3} d^p + O_N \]

\[ 2p + n = u^p + 3 O_N \]

(and \( p + 2n = d^p + 3 O_N \))

\[ \Sigma^+ = \frac{2}{3} u^\Sigma - \frac{1}{3} s^\Sigma + O_\Sigma \]

\[ \Sigma^- = -\frac{1}{3} u^\Sigma - \frac{1}{3} s^\Sigma + O_\Sigma \]

\[ \Sigma^+ - \Sigma^- = u^\Sigma \]

HENCE: \[ O_N = \frac{1}{3} \left[ 2p + n - \left( \frac{u^p}{u^\Sigma} \right) (\Sigma^+ - \Sigma^-) \right] \]

Just these ratios from Lattice QCD

\[ O_N = \frac{1}{3} \left[ n + 2p - \left( \frac{u^n}{u^\Sigma} \right) (\Xi^0 - \Xi^-) \right] \]
First Accurate Determination of $G_M^s$ from QCD

Highly non-trivial that intersection lies on constraint line!

Yields: $G_M^s = -0.046 \pm 0.019 \, \mu_N$

Leinweber et al., PRL 94 (2005) 212001
# State of the Art Magnetic Moments

<table>
<thead>
<tr>
<th></th>
<th>QQCD</th>
<th>Valence</th>
<th>Full QCD</th>
<th>Expt.</th>
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<td>2.69 (16)</td>
<td>2.94 (15)</td>
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<tr>
<td>(n)</td>
<td>-1.72 (10)</td>
<td>-1.83 (10)</td>
<td>-1.91 (10)</td>
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<tr>
<td>(\Sigma^+)</td>
<td>2.37 (11)</td>
<td>2.61 (10)</td>
<td>2.52 (10)</td>
<td>2.46 (10)</td>
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<tr>
<td>(\Sigma^-)</td>
<td>-0.95 (05)</td>
<td>-1.08 (05)</td>
<td>-1.17 (05)</td>
<td>-1.16 (03)</td>
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<td>(\Lambda)</td>
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<td>-0.61 (03)</td>
<td>-0.63 (03)</td>
<td>-0.613 (4)</td>
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<td>(\Xi^0)</td>
<td>-1.16 (04)</td>
<td>-1.26 (04)</td>
<td>-1.28 (04)</td>
<td>-1.25 (01)</td>
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<tr>
<td>(\Xi^-)</td>
<td>-0.65 (02)</td>
<td>-0.68 (02)</td>
<td>-0.70 (02)</td>
<td>-0.651 (03)</td>
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<tr>
<td>(u^p)</td>
<td>1.66 (08)</td>
<td>1.85 (07)</td>
<td>1.85 (07)</td>
<td>1.81 (06)</td>
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<tr>
<td>(u^\Xi)</td>
<td>-0.51 (04)</td>
<td>-0.58 (04)</td>
<td>-0.58 (04)</td>
<td>-0.60 (01)</td>
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</table>
Direct Calculation of $G_M^s(Q^2)$ – K.-F. Liu et al.

Strangeness Magnetic Form Factors with 3 Quark Masses ($m_n = 0.6, 0.7, 0.8$ GeV); T. Doi et al. (χQCD) arXiv:0903.3232

$G_M^s(Q^2 = 0) = -0.017(25)(07) \mu_N$

c.f. $-0.046 \pm 0.019$ (Leinweber et al.)

N.B. Result of Doi et al. would increase by factor ~1.8 when light quark mass takes physical value with $m_s$ fixed

Global Analysis of PVES Data

- Proton not all that strange
- New data not yet included at 0.23 and 0.6 GeV$^2$ (PVA4, G0, HAPPEX III – data taken this year)

The Weak Neutral Current
Radiative Corrections Test of Weak Neutral Current

Not so long ago….

Success of Strangeness Search Leads Naturally to Measurement of $\sin^2\theta_W$ Using PVES

• Proton target

$$A^{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \left[ -\frac{G_F Q^2}{\pi \alpha \sqrt{2}} \right] \frac{\varepsilon G_{E}^{PV} G_{M}^{PZ}}{\varepsilon (G_{E}^{PV})^2 + \tau (G_{M}^{PV})^2} + \frac{\tau G_{M}^{PV} G_{M}^{PZ}}{2(1 - 4 \sin^2 \theta_W) \varepsilon' G_{M}^{PV} G_{A}^{P}}$$

Neutral-weak form factors

Axial form factor

Assume charge symmetry:

$$4G_{E,M}^{PZ} = (1 - 4 \sin^2 \theta_W) G_{E,M}^{PV} - G_{E,M}^{\text{str}} - G_{E,M}^{s}$$

Proton weak charge

Strangeness

(tree level)

Use data to constrain the parameters of the electroweak theory
Use Global Fit to Extract Slope at 0° and $Q^2 = 0$

$$\bar{A}_{LR}^p = A_z / (-G_F Q^2 / 4\pi \alpha \sqrt{2}) = Q_{weak}^p + Q^2 B(Q^2)$$

1σ bound from global fit to all PVES data (as of 2007)

Dotted line indicates effect of using theoretical input for axial terms

Q_{weak} experiment - underway at JLab

(R.D. Young et al., PRL 99, 122003 (2007))
Major progress on $C_{1q}$ couplings

Dramatic improvement in knowledge of weak couplings!

$Q_{\text{weak}} = 2C_{1u} + C_{1d}$

$L_{\text{eff}} \sim C_{1q} \bar{e} \gamma^\mu \gamma_5 e \bar{q} \gamma_\mu q$

95% Factor of 5 increase in precision of Standard Model test
Raises Mass of New Z’ to 0.9 TeV – from 0.4 TeV

\[ \delta C_{1u} \sim \cos \theta_h \]
\[ \delta C_{1d} \sim \sin \theta_h \]

New physics scale >0.9 TeV! (from 0.4 TeV)
Future $Q_{\text{weak}}$ at JLab – if in Agreement with SM
IF in accord with Standard Model...

Qweak constrains new physics to beyond 2 TeV

future Qweak with PVES
Atomic only

95% CL
Or... Discovery

Assume Qweak takes central value of current measurements

\[ 1.5 < \frac{\Lambda}{g} < 2.5 \text{ TeV} \]

If LHC finds new Z', Qweak will help determine nature of interaction
The NuTeV anomaly
NuTeV Anomaly


Fermilab press conference, Nov. 7, 2001:

“We looked at $\sin^2 \theta_W$, ” said Sam Zeller. The predicted value was 0.2227. The value we found was 0.2277…. might not sound like much, but the room full of physicists fell silent when we first revealed the result.”

“$3 \sigma$ discrepancy ) 99.75% probability $\nu$ are not like other particles…. only 1 in 400 chance that our measurement is consistent with prediction ,” MacFarland said.
Paschos-Wolfenstein Ratio

NuTeV measured (approximately) P-W ratio:

\[
R^{PW} = \frac{\sigma (\nu \text{ Fe} \rightarrow \nu \text{ X}) - \sigma (\bar{\nu} \text{ Fe} \rightarrow \bar{\nu} \text{ X})}{\sigma (\nu \text{ Fe} \rightarrow \mu^- \text{ X}) - \sigma (\bar{\nu} \text{ Fe} \rightarrow \mu^+ \text{ X})} \quad \text{NC} = \quad \frac{\text{ratio}}{\text{ratio}}
\]

\[
= \frac{1}{2} - \sin^2 \theta_W
\]

\[
\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.2277 \pm 0.0013 \pm 0.0009
\]

other methods

\[
c.f. \text{ Standard Model} = 0.2227 \pm 0.0004
\]

(c.f. 1978: 0.230 ± 0.015)
Traditionally there is NO label “p” on PDF’s!

It's assumed that charge symmetry is exact.

Good at < 1% : e.g. \( \frac{m_n - m_p}{m_p} \sim 0.1\% \)

That is: \( u \equiv u^p = d^n \)

\( d \equiv d^p = u^n \) etc.

Hence:

\[
F_2^n = \frac{4}{9} x ( d(x) + \bar{d}(x) ) + \frac{1}{9} ( u(x) + \bar{u}(x) )
\]

up-quark in \( n \)    down-quark in \( n \)
Correction to Paschos-Wolfenstein from CSV

- General form of the correction is:

\[ \Delta R_{PW} \simeq \left( 1 - \frac{7}{3} s_W^2 \right) \frac{\langle x_A u_A^\bar{A} - x_A d_A^\bar{A} - x_A s_A^\bar{A} \rangle}{\langle x_A u_A^\bar{A} + x_A d_A^\bar{A} \rangle} \]

- \( u_A = u^p + u^n \); \( d_A = d^p + d^n \) and hence

\[ u_A - d_A = (u^p - d^n) - (d^p - u^n) \equiv \delta u - \delta d \]

- N.B. In general the corrections are C-odd and so involve only valence distributions: \( q^- = q - \bar{q} \)

Davidson et al., hep-ph/0112302
Estimates of Charge Symmetry Violation*

- Origin of effect is \( m_d \neq m_u \)

- Unambiguously predicted: \( \delta d \nu - \delta u \nu > 0 \)

- Biggest % effect is for minority quarks, i.e. \( \delta d \nu \)

- Same physics that gives: \( d \nu / u \nu \) small as \( x \rightarrow 1 \) and: \( g^p_1 \) and \( g^n_1 > 0 \) at large \( x \)

i.e. mass difference of quark pair spectators to hard scattering

* Sather, Phys Lett B274 (1992) 433;
Rodionov et al., Mod Phys Lett A9 (1994) 1799

Non-Perturbative Structure of Nucleon

To calculate PDFs need to evaluate non-perturbative matrix elements

Using either: i) lattice QCD or ii) Model

i) Lattice QCD can only calculate low moments of $u^p - d^p$

quite a lot has been learnt....

BUT nothing yet about CSV
Formally, using OPE (A_+ = 0 gauge) *:

\[ q( x, Q_0^2 ) = \frac{1}{4} \pi \int_{-1}^{1} dz \exp[-i M x z] <p| \psi_+^+(z;00-z) \psi_+(0) |p> \]

Insert complete set of states:

\[ \sum_n \int d^3 p_n |n> <n| = 1 \]

and do \( \int dz \) using translational invariance:

\[ q( x, Q_0^2 ) = \sum_n \int d^3 p_n |<n| \psi_+(0) |p>|^2 \delta \left( M (1 - x) - p_+^n \right) \]

with \( p_+^n = (m_n^2 + p_n^2)^{1/2} + p_z > 0 \)

* Q_0^2 is the scale at which nucleon momentum is carried by predominantly valence quarks: below 1 GeV^2
Di-quark Spectator States Dominate Valence

For s-wave valence quarks, most likely three-momentum is zero:

\[ \delta( M (1 - x) - m_n ) \] determines \( x \) where \( q( x, Q^2_0 ) \) is maximum

i.e. \( x_{\text{peak}} = ( M - m_n ) / M \) and hence lowest \( m_n \) \( \rightarrow \) large \(-x\) behaviour

Natural choice is two-quark state

\[ m_2 / M = 2/3 \text{ (CQM)}; \]
\[ = 3/4 \text{ MIT bag} \]
\[ x_{\text{peak}} \sim 1/4 \text{ to } 1/3 \]

If \( m_2 \downarrow \): \( x_{\text{peak}} \) moves to right
Effect of “Hyperfine” Interaction

\[ \Delta - N \text{ mass splitting } \) \ S=1 \ “di-quark” \ mass \ is \ 0.2 \ GeV \ greater \ S=0 \]

SU(6) wavefunction for proton:

remove d-quark: ONLY S=1 left

c.f. remove u-quark: 50% S=0 and 50% S=1

\[ \begin{align*}
&\text{•} \quad \text{u}(x) \ \text{dominates over} \ d(x) \ \text{for} \ x > 0.3 \\
\text{Hence*:} &
\quad \text{•} \quad \text{u}^\uparrow \ \text{dominates over} \ \text{u}^\downarrow \ \text{at large} \ x \\
&\quad \text{and hence:} \ g^p_1(x) > 0 \ \text{at large} \ x \\
&\quad \text{•} \quad \text{Similarly} \ g^n_1(x) > 0 \ \text{at large} \ x
\end{align*} \]

*Close & Thomas: 1988
More Modern (Confining) NJL Calculations

Cloet et al.,
($\mu = 0.4$ GeV)
Application to Charge Symmetry Violation

- $d$ in $p$: $uu$ left
- $u$ in $n$: $dd$ left
- Hence $m_2$ lower by about 4 MeV for $d$ in $p$ than $u$ in $n$
- Hence $d^p > u^p$ at large $x$.

From: Rodionov et al., Mod Phys Lett A9 (1994) 1799
FIG. 5: The phenomenological valence quark CSV function from Ref. [23], corresponding to best fit value $\kappa = -0.2$ defined in Eq. (35). Solid curve: $x\delta d_\nu$; dashed curve: $x\delta u_\nu$. 
Model Calculations Reduce NuTeV by $1\sigma$

Two original ('92 and '93) calculations agree very (too?) well with each other and with recent approximation based on phenomenological PDFs

Includes effect of NuTeV acceptance

( Zeller et al., hep-ex/0203004)

TABLE II: CSV corrections to determination of $\sin^2 \theta_W$ in neutrino scattering. $PW$ is the contribution to the Paschos-Wolfenstein ratio, $Nu$ is the result weighted by the NuTeV functional. $\Delta U$ is the total contribution from $\delta u_\nu$, $\Delta D$ is the contribution from $\delta d_\nu$ and $Tot$ is the total CSV correction.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta U_{PW}$</th>
<th>$\Delta D_{PW}$</th>
<th>$Tot_{PW}$</th>
<th>$\Delta U_{Nu}$</th>
<th>$\Delta D_{Nu}$</th>
<th>$Tot_{Nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rodionov</td>
<td>-.0010</td>
<td>.0011</td>
<td>-.0020</td>
<td>-.00065</td>
<td>-.00081</td>
<td>-.0015</td>
</tr>
<tr>
<td>Sather</td>
<td>-.00078</td>
<td>.0013</td>
<td>-.0021</td>
<td>-.00060</td>
<td>-.0011</td>
<td>-.0017</td>
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<tr>
<td>analytic</td>
<td>-.0008</td>
<td>.0014</td>
<td>-.0022</td>
<td>-.0006</td>
<td>-.0012</td>
<td>-.0017</td>
</tr>
</tbody>
</table>

An additional source of CSV

- In addition to the u-d mass difference, MRST (Eur Phys J C39 (2005) 155) and Glück et al (PRL 95 (2005) 022002) suggested that “QED splitting”:

\[
\bar{p}_{qq}(\frac{x}{y})
\]

- which is obviously larger for u than d quarks, would be an additional source of CSV. Assume zero at some low scale and then evolve – so CSV from this source grows with \(Q^2\)

- Effect on NuTeV is exactly as for regular CSV and magnitude but grows logarithmically with \(Q^2\)

- For NuTeV it gives: \(\Delta R^{QED} = -0.0011\) to which we assign 100% error
Isovector EMC Effect
The EMC Effect: Nuclear PDFs

- Observation stunned and electrified the HEP and Nuclear communities 20 years ago
- Nearly 1,000 papers have been generated....
- Medium modifies the momentum distribution of the quarks!

![Graph showing EMC effect]

Attempt to Understand this based on QMC

- **Two major, recent papers:**

- **Built on earlier work on QMC: e.g.**

- **Major review of applications of QMC to many nuclear systems:**
g_1(A) – “Polarized EMC Effect”

- Calculations described here show larger effect for polarized structure than unpolarized: mean scalar field modifies lower components of the confined quark’s Dirac wave function.
- Spin-dependent parton distribution functions for nuclei unmeasured.

( Cloet, Bentz, AWT, PRL 95 (2005) 0502302 )
Recent Calculations for Finite Nuclei

Spin dependent EMC effect TWICE as large as unpolarized

NuTeV Reassessed

- New realization concerning EMC effect:
  - isovector force in nucleus (like Fe) with $N \neq Z$
  - effects ALL $u$ and $d$ quarks in the nucleus
  - subtracting structure functions of extra neutrons is not enough
  - *there is a shift of momentum from all $u$ to all $d$ quarks*

- This has same sign as charge symmetry violation associated with $m_u \neq m_d$

- Sign and magnitude of both effects exhibit little model dependence

Iso-vector EMC Effect

Means that excess neutrons in Fe shift momentum from all u- to all d-quarks and subtracting their direct contribution does not remove this effect.

This has implications for the NuTeV anomaly.
Correction to Paschos-Wolfenstein from $\rho_p - \rho_n$

$$\Delta R_{PW} \simeq \left(1 - \frac{7}{3}s_W^2\right) \frac{\langle x_A u_A^- - x_A d_A^- - x_A s_A^- \rangle}{\langle x_A u_A^- + x_A d_A^- \rangle}$$

- Excess of neutrons means d-quarks feel more repulsion than u-quarks
- Hence shift of momentum from all u to all d in the nucleus!
- Negative change in $\Delta R_{PW}$ and hence $\sin^2 \theta_W \uparrow$
- Isovector force controlled by $\rho_p - \rho_n$ and symmetry energy of nuclear matter – both well known!
- N.B. $\rho^0$ mean field included in QHD and QMC and earlier work with Bentz but no-one thought of this!!

Finally need to address strange quark asymmetry
Strange Quark Asymmetry

• Required in principle by chiral symmetry (s and \( \bar{s} \) have different chiral behaviour*)

• Experimental constraint primarily through opposite sign di-muon production with neutrinos (CCFR & NuTeV)

<table>
<thead>
<tr>
<th></th>
<th>( \langle x s^- \rangle )</th>
<th>( \Delta R^s )</th>
<th>( \Delta R^{total} )</th>
<th>( \sin^2 \theta_W \pm \text{syst.} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mason et al. [8]</td>
<td>0.00196 ± 0.00143</td>
<td>-0.0018 ± 0.0013</td>
<td>-0.0063 ± 0.0018</td>
<td>0.2214 ± 0.0020</td>
</tr>
<tr>
<td>NNPDF [9]</td>
<td>0.0005 ± 0.0086</td>
<td>-0.0005 ± 0.0078</td>
<td>-0.0050 ± 0.0079</td>
<td>0.2227 ± large</td>
</tr>
<tr>
<td>Alekhin et al. [31]</td>
<td>0.0013 ± 0.0009 ± 0.0002</td>
<td>-0.0012 ± 0.0008 ± 0.0002</td>
<td>-0.0057 ± 0.0015</td>
<td>0.2220 ± 0.0017</td>
</tr>
<tr>
<td>MSTW [32]</td>
<td>0.0016±0.0011 -0.0009</td>
<td>-0.0014 -0.0010 +0.0008</td>
<td>-0.0059 ± 0.0015</td>
<td>0.2218 ± 0.0018</td>
</tr>
<tr>
<td>CTEQ [33]</td>
<td>0.0018±0.0016 -0.0004</td>
<td>-0.0016 -0.0014 +0.0004</td>
<td>-0.0061 +0.0019 -0.0013</td>
<td>0.2216±0.0021 -0.0016</td>
</tr>
<tr>
<td>This work (Eq. (10))</td>
<td>0.0 ± 0.0020</td>
<td>0.0 ± 0.0018</td>
<td>-0.0045 ± 0.0022</td>
<td>0.2232 ± 0.0024</td>
</tr>
</tbody>
</table>

Summary of Corrections to NuTeV Analysis

• Isovector EMC effect: \( \Delta R^{\rho_0} = -0.0019 \pm 0.0006 \)
  – using NuTeV functional

• CSV: \( \Delta R^{CSV} = -0.0026 \pm 0.0011 \)
  – again using NuTeV functional

• Strangeness: \( \Delta R^s = -0.0011 \pm 0.0014 \)
  – this is largest uncertainty (systematic error); desperate need for an accurate determination of \( s^-(x) \), e.g. semi-inclusive DIS?

• Final result: \( \sin^2 \theta_W = 0.2221 \pm 0.0013 \text{(stat)} \pm 0.0020 \text{(syst)} \)
  – c.f. Standard Model: \( \sin^2 \theta_W = 0.2227 \pm 0.0004 \)

Bentz et al., arXiv: 0908.3198
Separate Neutrino and Anti-neutrino Ratios

• Biggest criticism of this explanation has been that NuTeV actually measured $R^\nu$ and $R^{\bar{\nu}}$, separately:
  Claim we should compare directly with these.

• Have done this:

\[
\begin{align*}
\delta R^\nu &= \frac{2 \left( 3 g_{Lu}^2 + g_{Ru}^2 \right) \left< x_A u_A^- - x_A d_A^- \right>}{\left< 3 x_A u_A + 3 x_A d_A + x_A \bar{u}_A + x_A \bar{d}_A + 6 x_A s_A \right>} \\
\delta R^{\bar{\nu}} &= \frac{-2 \left( 3 g_{Rd}^2 + g_{Ld}^2 \right) \left< x_A u_A^- - x_A d_A^- \right>}{\left< x_A u_A + x_A d_A + 3 x_A \bar{u}_A + 3 x_A \bar{d}_A + 6 x_A \bar{s}_A \right>}
\end{align*}
\]

• Then $R^\nu$ moves from $0.3916 \pm 0.0013$ c.f. $0.3950$ in the Standard Model to $0.3933 \pm 0.0015$;

\[ R^{\bar{\nu}} \text{ moves from } 0.4050 \pm 0.0027 \text{ to } 0.4034 \pm 0.0028, \text{ c.f. } 0.4066 \text{ in SM} \]

• This is tremendous improvement:
  $\chi^2$ changes from 7.2 to 2.6 for the two ratios!

Bentz et al., arXiv: 0908.3198
The Standard Model works... again

Bentz et al., arXiv: 0908.3198
Summary

• Standard Model has again survived major tests:
  − strange quarks as analog of Lamb shift in QED
  − weak charge of the proton

• $Q_{\text{weak}}$ has considerable potential for a further advance

• The outstanding discrepancy with Standard Model predictions for $Z^0$ was NuTeV anomaly
  − this is resolved by CSV and newly discovered isovector correction to nuclear structure functions

• Parity Violating DIS is an ideal way to test both effects

• Major remaining uncertainty is $s(x) - \bar{s}(x)$ ....
Chisq for Standard Model =48/45; PVES gives reduction by 0.7

<table>
<thead>
<tr>
<th>$Z'$</th>
<th>electroweak</th>
<th>CDF</th>
<th>LEP 2</th>
<th>$\theta_{ZZ'}^\text{min}$</th>
<th>$\theta_{ZZ'}^\text{max}$</th>
<th>$\chi^2_{\text{min}}$</th>
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<td>$Z_\chi$</td>
<td>1,141</td>
<td>892</td>
<td>673</td>
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<td>0.00006</td>
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<td>$Z_\psi$</td>
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<td>878</td>
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<td>$Z_{LR}$</td>
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</tr>
<tr>
<td>$Z_{\psi}$</td>
<td>(803)</td>
<td>(740)</td>
<td>(740)</td>
<td>-0.0094</td>
<td>0.0081</td>
<td>47.7</td>
</tr>
<tr>
<td>$Z_{SM}$</td>
<td>1,403</td>
<td>1,030</td>
<td>1,787</td>
<td>-0.0026</td>
<td>0.0006</td>
<td>47.2</td>
</tr>
</tbody>
</table>
Octet-baryon masses

- Leading-order expansion $O(1)$

\[
M_N = M_0 + 2(\alpha_M + \beta_M)m_q + 2\sigma_M(2m_q + m_s)
\]
\[
M_\Lambda = M_0 + (\alpha_M + 2\beta_M)m_q + \alpha_M m_s + 2\sigma_M(2m_q + m_s)
\]
\[
M_\Sigma = M_0 + \frac{1}{3}(5\alpha_M + 2\beta_M)m_q + \frac{1}{3}(\alpha_M + 4\beta_M)m_s + 2\sigma_M(2m_q + m_s)
\]
\[
M_\Xi = M_0 + \frac{1}{3}(\alpha_M + 4\beta_M)m_q + \frac{1}{3}(5\alpha_M + 2\beta_M)m_s + 2\sigma_M(2m_q + m_s)
\]

\[
m^2_\pi = 2Bm_q \quad m^2_K = B(m_q + m_s)
\]

\[m_q \to \frac{m^2_\pi}{2B}, \quad m_s \to \frac{2m^2_K - m^2_\pi}{2B} \quad \{\alpha, \beta, \sigma\} \to B\{\alpha', \beta', \sigma'\}\]

Fit using SU(3) expansions plus FRR loops ($\pi$, $\eta$ and $K$).
PACS-CS Data

(Aoki et al., arXiv:0807.1661[hep-lat])

Young & Thomas, arXiv:0901.3559 [nucl-th]
\[ \sigma = <N | (m_u + m_d) (\bar{u} u + \bar{d} d)/2 | N > \equiv m_q \frac{\partial M_N}{\partial m_q} 
= <N|[Q_5,[Q_5,H_{QCD}]]|N> \]

\[ \delta \sigma = 35 \Lambda - 23 + 9.6 - 3 + 0.8 + \ldots = 18 \text{ MeV (} \Lambda = 1\text{ GeV)} \]
Naïve Expansion Traditionally Used to Extract $\sigma$ Terms is Hopeless!

• Leading-order expansion $O(1)$

\[
M_N = M_0 + 2(\alpha_M + \beta_M)m_q + 2\sigma_M(2m_q + m_s) \\
M_\Lambda = M_0 + (\alpha_M + 2\beta_M)m_q + \alpha_M m_s + 2\sigma_M(2m_q + m_s) \\
M_\Sigma = M_0 + \frac{1}{3}(5\alpha_M + 2\beta_M)m_q + \frac{1}{3}(\alpha_M + 4\beta_M)m_s + 2\sigma_M(2m_q + m_s) \\
M_\Xi = M_0 + \frac{1}{3}(\alpha_M + 4\beta_M)m_q + \frac{1}{3}(5\alpha_M + 2\beta_M)m_s + 2\sigma_M(2m_q + m_s)
\]

Need $O(m_\pi^6)$ to get accurate light quark $\sigma$ term

While for strange condensate expansion is useless!

BUT through FRR have closed expression and can evaluate …. 
Octet Baryon Masses - LHPC Data

(Walker-Loud et al., arXiv:0806.4549)

Young & Thomas, arXiv:0901.3559 [nucl-th]

• Stress: This involves just 4 SU(3) parameters plus FRR mass, Λ, fit to lowest 8 data points

• There is a great deal of physics to be extracted from this fit
Summary of Results of Combined Fits
(of 2008 LHPC & PACS-CS data)

<table>
<thead>
<tr>
<th>$B$</th>
<th>Mass (GeV)</th>
<th>Expt.</th>
<th>$\bar{\sigma}_{Bl}$</th>
<th>$\bar{\sigma}_{Bs}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>0.945(24)(4)(3)</td>
<td>0.939</td>
<td>0.050(9)(1)(3)</td>
<td>0.033(16)(4)(2)</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>1.103(13)(9)(3)</td>
<td>1.116</td>
<td>0.028(4)(1)(2)</td>
<td>0.144(15)(10)(2)</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>1.182(11)(2)(6)</td>
<td>1.193</td>
<td>0.0212(27)(1)(17)</td>
<td>0.187(15)(3)(4)</td>
</tr>
<tr>
<td>$\Xi$</td>
<td>1.301(12)(9)(1)</td>
<td>1.318</td>
<td>0.0100(10)(0)(4)</td>
<td>0.244(15)(12)(2)</td>
</tr>
</tbody>
</table>

\[ \bar{\sigma}_{Bq} = \left( \frac{m_q}{M_B} \right) \frac{\partial M_B}{\partial m_q} \]

Of particular interest:
- $\sigma$ commutator well determined: $\sigma_{\pi N} = 47 (9) (1) (3) \text{ MeV}$
- and strangeness sigma commutator small
  - $m_s \frac{\partial M_N}{\partial m_s} = 31 (15) (4) (2) \text{ MeV}$
  - NOT several 100 MeV!

Profound Consequences for Dark Matter Searches
(and s-wave K condensation)
CMSSM Predictions for Dark Matter $\sigma$

In response to request by Ellis, Olive & Savage, who explored CMSSM

Cross section accurately fixed (e.g. “New model C”) c.f. using old relation to unknown $\pi N$ sigma commutator (“Old Model C”)

Giedt et al., arXiv: 0907.4177v1
PRL 103 (2009) 201802
CMSSM Predictions for Dark Matter $\sigma$

95% CL predictions for all Constrained Minimal Super-Symmetric Standard Model extensions consistent with astrophysical data

Cross sections 1-2 orders of magnitude smaller than before BUT very well determined and separated!

Giedt et al., arXiv: 0907.4177v1
PRL 103 (2009) 201802
Hadronic Uncertainties in the Elastic Scattering of Supersymmetric Dark Matter

John Ellis,¹,* Keith A. Olive,²,† and Christopher Savage²,‡

We find that the spin-independent cross section may vary by almost an order of magnitude for $48 \text{ MeV} < \Sigma_{\pi N} < 80 \text{ MeV}$, the ±2-σ range according to the uncertainties in Table I. This uncertainty is already impacting the interpretations of experimental searches for cold dark matter. Propagating the ±2-σ uncertainties in $\Delta_s^{(p)}$, the next most important parameter, we find a variation by a factor $\sim 2$ in the spin-dependent cross section. Since the spin-independent cross section may now be on the verge of detectability in certain models, and the uncertainty in the cross section is far greater, we appeal for a greater, dedicated effort to reduce the experimental uncertainty in the $\pi$-nucleon $\sigma$ term $\Sigma_{\pi N}$. This quantity is not just an object of curiosity for those interested in the structure of the nucleon and non-perturbative strong-interaction effects: it may also be key to understanding new physics beyond the Standard Model.

\[ \mathcal{L} = \alpha_2 i \bar{\chi} \gamma^\mu \gamma^5 \chi \bar{q} i \gamma^\mu \gamma^5 q + \alpha_3 i \bar{\chi} \chi \bar{d} q_i q_i \]
Proton contains a number of non-interacting quarks and gluons (partons), which carry fraction $x$ of the momentum of the target: $p = (xP; 0 0 xP)$

Define: PDF’s (number densities) $u(x)$, $d(x)$, $s(x)$ etc..

E.g. $x u(x) \, dx$ is the fraction of the momentum of the proton carried by up quarks with momentum between $(x, x + dx)$ in the infinite momentum frame

Then for $e$ (or $\mu$) DIS:

$$F_2^{ep}(x) = 2 \times F_1(x) = \frac{4}{9} x (u(x) + \bar{u}(x)) + \frac{1}{9} x (d(x) + \bar{d}(x))$$
Summary of Charged Current Cross Section

\[ \int_0^1 dy \, (1 - y)^2 = 1/3 \]

\[ \sigma_{cc}(\nu \, N=Z) \sim x \{ (u + d + 2s) + \frac{1}{3} (u + d + 2c) \} \]

\[ \sigma_{cc}(\bar{\nu} \, N=Z) \sim x \{ \frac{1}{3} (u + d + 2c) + (u + d + 2s) \} \]

and hence:

\[ \sigma_{cc} (\nu \, N=Z) - \sigma_{cc} (\bar{\nu} \, N=Z) = 2/3 \times \{ u - \bar{u} + d - \bar{d} \} + 2 \times \{ s - \bar{s} \} + \frac{2}{3} \times \{ c - \bar{c} \} \]

\[ = 2/3 \times ( u_\nu + d_\nu ) + \ldots \]

(Valence distributions: \[ \int dx \, u_\nu = 2 ; \int dx \, d_\nu = 1 \] )
Neutral Current Cross Section

<table>
<thead>
<tr>
<th>Z coupling</th>
<th>$g_L$</th>
<th>$g_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u, c, t$</td>
<td>$+ \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w$</td>
<td>$-\frac{2}{3} \sin^2 \theta_w$</td>
</tr>
<tr>
<td>$d, s, b$</td>
<td>$- \frac{1}{2} + \frac{1}{3} \sin^2 \theta_w$</td>
<td>$+\frac{1}{3} \sin^2 \theta_w$</td>
</tr>
</tbody>
</table>

In Cross Section:

$\nu q_L \sim 1; \nu q_R \sim \frac{1}{3}$

$\bar{\nu} q_L \sim \frac{1}{3}; \bar{\nu} q_R \sim 1$

Hence, for $N=Z$ nucleus:

Defining $g_L^2 = g_{Lu}^2 + g_{Ld}^2 = \frac{1}{2} - \sin^2 \theta_W + \frac{5}{9} \sin^4 \theta_W$

and $g_R^2 = g_{Ru}^2 + g_{Rd}^2 = \frac{5}{9} \sin^4 \theta_W$

\[ \sigma_{NC}(\nu A) \sim (g_L^2 + g_R^2/3) \times (u + d) + (g_R^2 + g_L^2/3) \times (\bar{u} + \bar{d}) \]

\[ \sigma_{NC}(\bar{\nu} A) \sim (g_L^2 + g_R^2/3) \times (\bar{u} + \bar{d}) + (g_R^2 + g_L^2/3) \times (u + d) \]
Finally: Paschos-Wolfenstein

\[ \sigma_{NC}(\nu A) - \sigma_{NC}(\bar{\nu} A) \sim \frac{2}{3} (g^2_L - g^2_R) \times (u_\nu + d_\nu) \]

c.f.

\[ \sigma_{CC}(\nu N=Z) - \sigma_{CC}(\bar{\nu} N=Z) \sim \frac{2}{3} \times (u_\nu + d_\nu) \]

and therefore ratio of NC to CC cross section differences is

\[ R_{PW} = g^2_L - g^2_R = \frac{1}{2} - \sin^2 \theta_W \]

Provided:  

i) Charge Symmetry  

ii) \( s(x) = \bar{s}(x) \)

iii) \( c(x) = \bar{c}(x) \)  

iv) No higher-twist effects (e.g. VMD shadowing)
Neutrino Scattering

In $\nu - q$ Breit frame:

$$q_L \quad \sigma \sim |d_{11}^{11}(\cos \theta^*)|^2 \sim (1 + \cos \theta^*)^2 / 4 \sim (1 - y)^2$$

Use covariant variables, $x$, $Q^2$ and $y = \nu / \varepsilon = p \cdot q / p \cdot k \quad \varepsilon (0,1)$
Sather (’92) : “Close and Thomas reproduced the strong deviation of the ratio \( d/u \) from 2 at large \( x \), which signals the breaking of SU(6) symmetry. A related approach employed here predicts the breaking of isospin (actually charge symmetry) albeit on a much smaller scale”

Consider \( n=2 \) only (i.e. valence PDFs) & set \( E_{n=2} \sim m_2 \):

\[
q_V(x, Q^2_0) = M \int d^3p \ P(p) \ \delta \left( \frac{p_z}{M} - \frac{m_2}{M} - x \right)
\]

And hence (e.g.):

\[
m_2 \rightarrow m_2 + \delta m_2
\]

\[
\delta q_V(x) = \delta m_2 / M \ d q_V / dx
\]

///’ly \( M \rightarrow M + \delta M \)

Now could use model OR phenomenological distributions…

OR….
For NuTeV it is (Essentially) Model Independent

\[ \delta D_V \equiv \int dx \times \delta d_V \]

\[ = - \frac{\delta m_2}{M} \int dx \times \frac{dd_V}{dx} + O(\delta M / M) \]

Integrate by parts:

\[ = - \frac{\delta m_2}{M} \int d_v (x) \, dx + x \, d_v \bigg|_0^1 \]

Unity – normalization
i.e. model independent

\[ \text{vanishes} \]
\[ \delta D_V = \delta \frac{M}{M} D_V + \delta \frac{m_2}{M} \sim 0.0046 \]

\[ \delta U_V = \delta \frac{M}{M} (U_V - 2) \sim -0.0020 \]

Small dependence on “bag / quark model” scale (\( Q^2_0 \)):

\[ D_V \sim 0.2 : \ U_V \sim 0.6 \quad \text{– i.e. 10\% & 30\% respectively} \]

Correction to Paschos-Wolfenstein is therefore:

\[ \Delta R^{PW} = 0.5 \left( g_L^2 - g_R^2 \right) \frac{\delta U_V - \delta D_V}{U_V + D_V} \sim -0.0020 \]

N.B. Ratio of non-singlet moments independent of \( Q^2 \) under NLO evolution
Recently Developed Covariant Model Built on the Same Physical Ideas

- Use NJL model ($\chi'$al symmetry)

- Ensure **confinement** through proper time regularization (following the Tübingen group)

- Self-consistently solve Faddeev Eqn. in mean scalar field

- This **solves chiral collapse problem** common for NJL (because of **scalar polarizability** again)

- Can **test against experiment**
  - e.g. spin-dependent EMC effect

- Also apply **same model** to NM, NQM and SQM – hence n-star
Covariant Quark Model for Nuclear Structure

• Basic Model:

• Applications to DIS:
  • Cloet, Bentz, Thomas, Phys. Rev. Lett. 95 (2005) 052302

• Applications to neutron stars – including SQM: