Holographic self-tuning of the cosmological constant

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work with Elias Kiritsis and Christos Charmousis, 1704.05075

Introduction: the CC and QFT

The Cosmological Constant (CC) problem arises as a clash between classical GR and QFT (in the modern effective FT sense).

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QFT: the source of semiclassical gravity becomes $\langle T_{\mu\nu} \rangle$.
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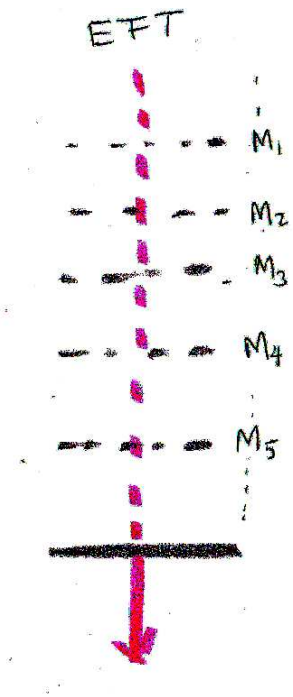
For curvatures $R \ll M$ the flat result gets small corrections:

$$\langle T_{\mu\nu} \rangle = \mathcal{E}_{vac} \, g_{\mu\nu} + O\left((R/M)^2\right) \quad \Rightarrow \quad \Lambda_{eff} = \Lambda_0 + 8\pi G_N \mathcal{E}_{vac}$$

$\Rightarrow$ Solution to Einstein eq. has curvature of order Λ_{eff}

The problem



In the SM there are many mass scales, and cosmological observations probe the largest distances, so all these massive modes are integrated out:

$$\mathcal{E}_{vac} = c_1 M_1^4 + c_2 M_2^4 + \ldots + c_n M_n^4$$

Moreover, any first order phase transition with latent heat Λ_h contributes generically Λ_h^4.

All these unrelated terms, *plus* the bare Λ_0 must sum up in such a way that the observed $G_N^{-1}\Lambda_{eff} \sim (10^{-3}\, eV)^4 \lll$ any of the scales M_i.

Possible way out

Modify gravity to disconnect vacuum energy from curvature: allow large $\mathcal{E}_{vac}$ but make it so it does not gravitate.

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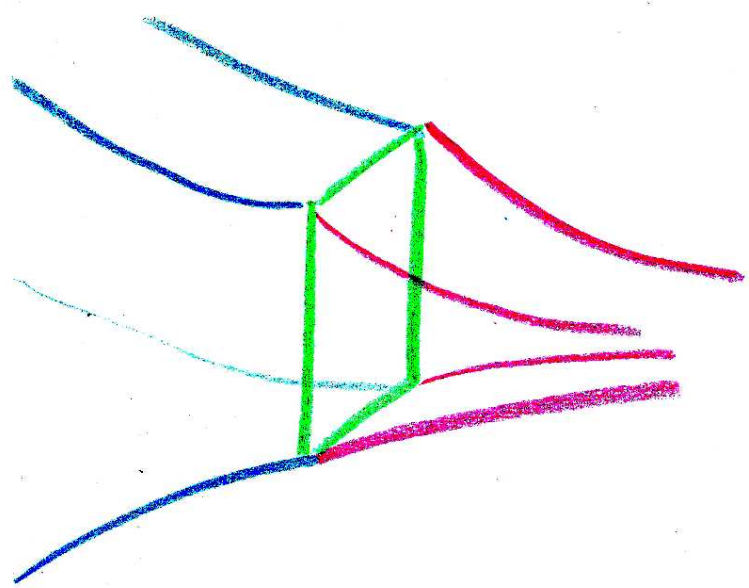
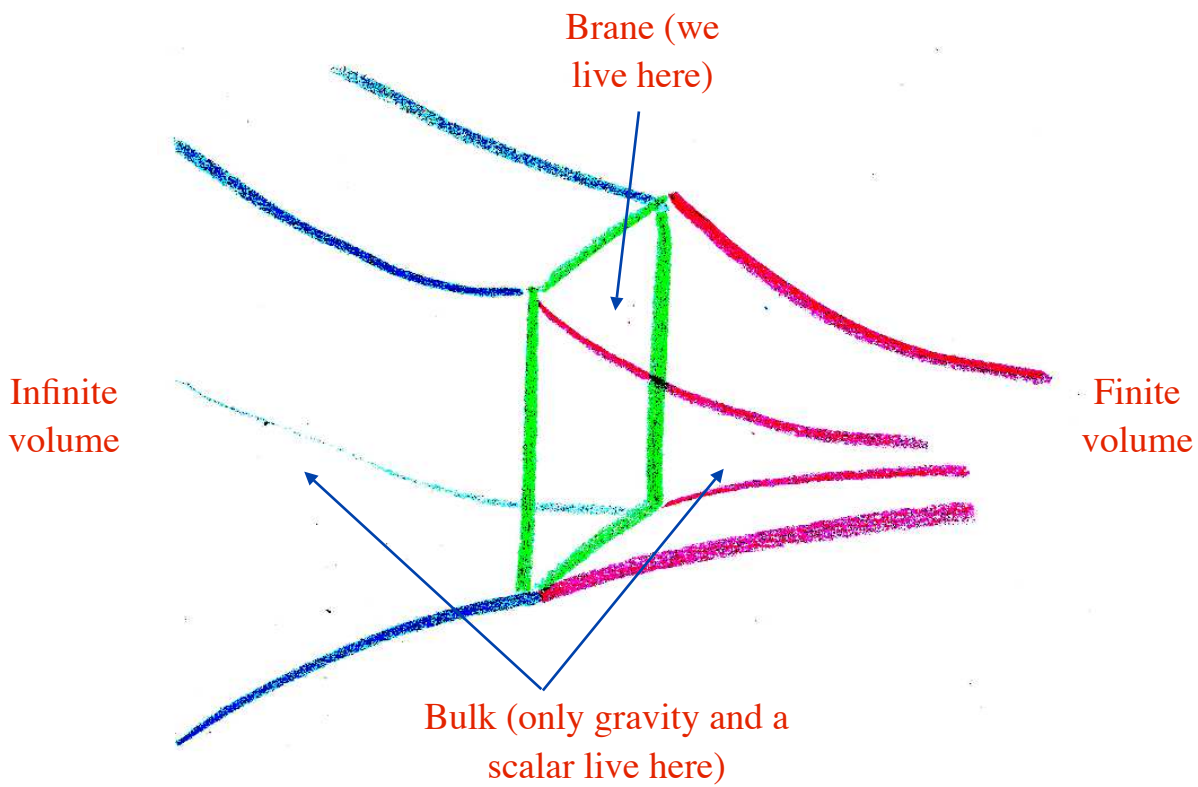
- Self-tuning: any mechanism which allows flat spcacetime solutions for generic values of $\mathcal{E}_{vac}$.
- Braneworld in extra dimension: $\mathcal{E}_{vac}$ curves the bulk, but not the brane.



Previous attempts: Arkani-Hamed *et al.* '00; Kachru,Schulz,Silverstein '00. They all either lead to bad singularities, or failure to reproduce 4d gravity, or need for fine-tuning. See also Charmousis, Gregory, Padilla '07

Content of this talk

- Self-tuning possible in the a general framework of a dilatonic, asymmetric braneworld with general 2-derivative induced terms.



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- Model based on holographic model building: dual of 4-dimensional, strongly coupled, non-gravitational fundamental theory.
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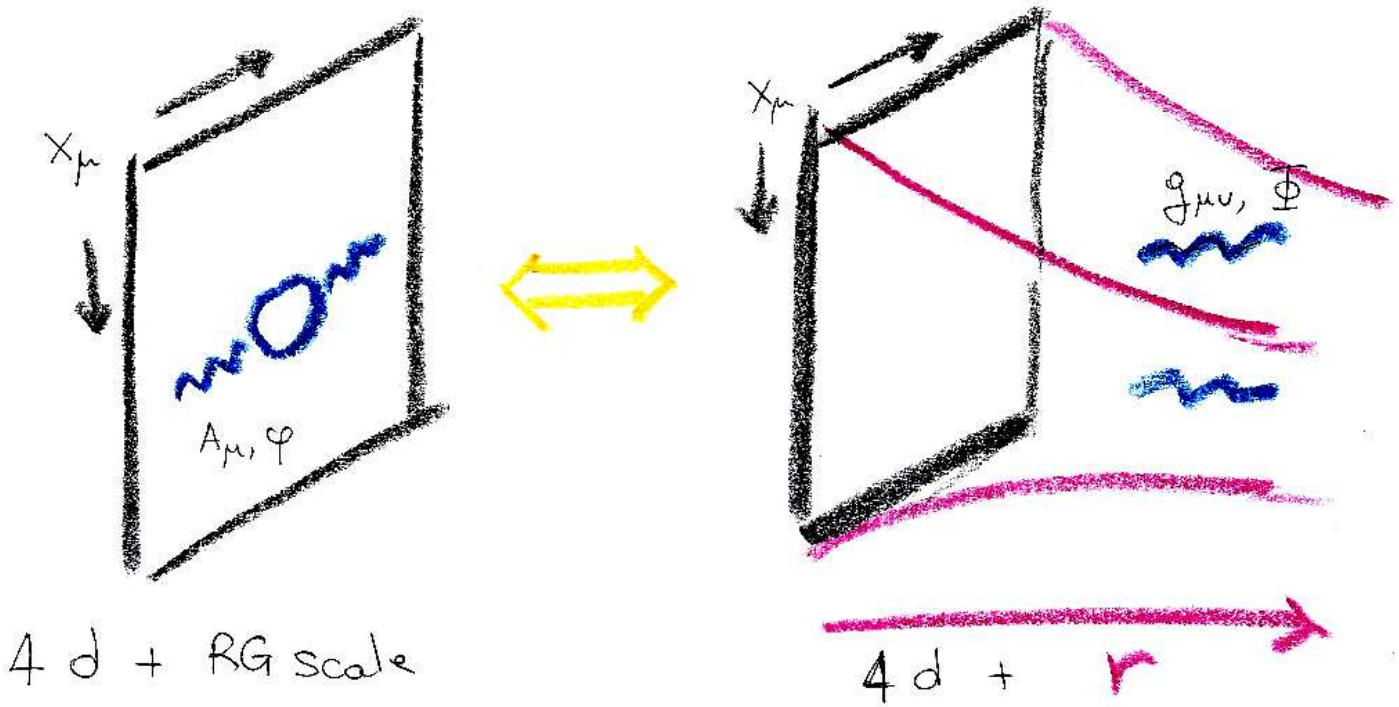
- Outline
  - AdS/CFT micro-review
  - Setup
  - Flat vacua: self-tuning
  - Tensor perturbations: emergent braneworld gravity
  - Scalar perturbations: stability
  - Perspectives

AdS/CFT detour

The AdS/CFT duality: conjecture that certain quantum field theories are equivalent to theories of gravity in higher dimensions Maldacena '98.



AdS/CFT detour

- Conformal field theory in d dimension $\Leftrightarrow$
  Anti de Sitter spacetime AdS_{d+1}

$$ds^2 = du^2 + e^{-2u/\ell}\eta_{\mu\nu}dx^\mu dx^\nu$$

- x^μ: QFT coordinates; u dual to energy scale $E \propto e^{-u/\ell}$.

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- x^μ: QFT coordinates; u dual to energy scale $E \propto e^{-u/\ell}$.
- bulk scalar field $\varphi(u) \Leftrightarrow$ running coupling $g(E)$. The corresponding holographic RG-flow geometry breaks conformal invariance (except at fixed points where $\dot{\varphi} = 0$).

$$ds^2 = du^2 + e^{A(u)}\eta_{\mu\nu}dx^\mu dx^\nu, \quad \varphi = \varphi(u).$$

$$E \propto e^{A(u)}$$

Setup

Consider a 4d QFT with a UV conformal fixed point, made out of:

1. A strongly coupled large-N CFT, deformed by a relevant operator;
2. The weakly coupled Standard Model fields;
3. Some heavy messangers with mass scale Λ, coupling the first two.

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semi-holographic description:

- Describe the strongly coupled large-N theory by a 5d gravity dual with the metric g_{ab} and some bulk scalar fields φ_i, dual to the operators that drive the CFT to the IR.
- The weakly coupled SM fields have a standard field-theoretical description, and they sit on a 4d defect in th 5d dual geometry.

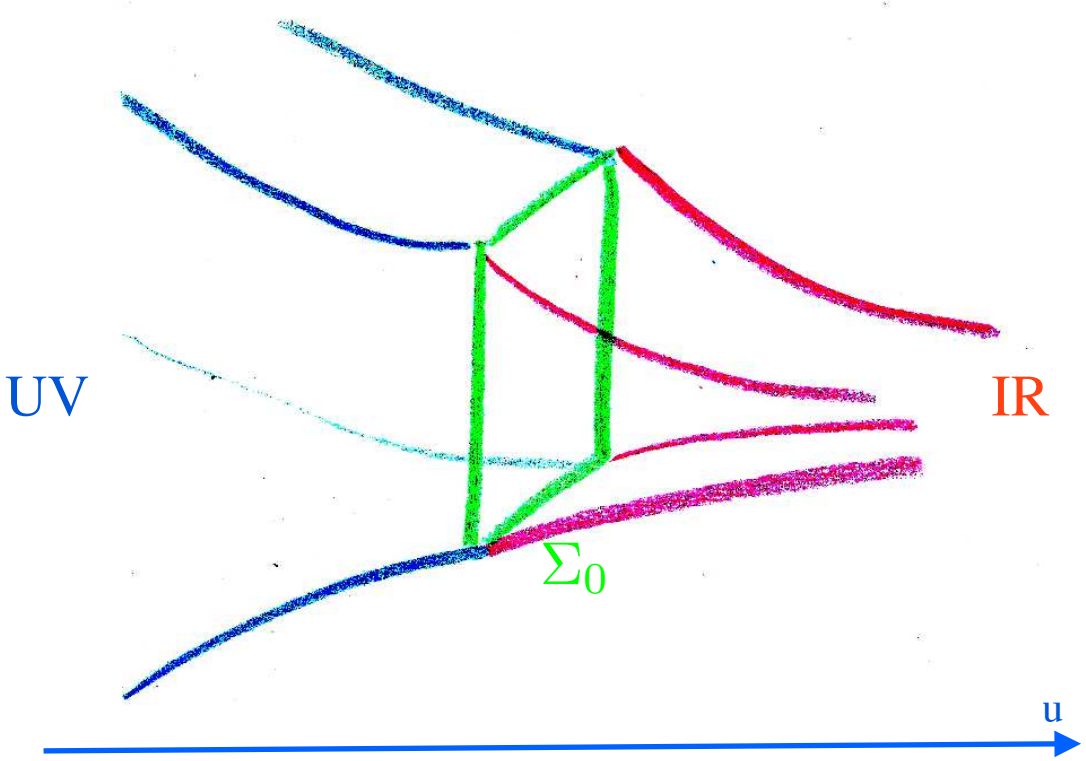
Semi-holographic setup

$$S = M^3 \int d^4x \int du \sqrt{-g} \left[R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - V(\varphi) \right]$$

5d Gravity dual of 4d CFT

$$+ \int_{\Sigma_0} d^4\sigma \sqrt{-\gamma} \mathcal{L}(\psi_i, H, W^a, \ldots, \varphi, \gamma_{\mu\nu}).$$

Weakly coupled 4d QFT



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- Action is the most general up to two derivates preserving 4d diffeos.

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Localized Effective action induced by quantum effects of weakly coupled QFT

(up to two derivative in the bulk fields)

We take this class of actions as the starting point and the definition of our model

The unknown functions appearing in the localized action can be taken as a phenomenologicalinput or motivated by weakly coupled calculation.

work in progress with E. Kiritsis and L. Witkowski

Field equations and matching conditions

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Einstein equations + Israel junction conditions ($[\ \ ] \equiv$ jump across Σ_0):

$$G_{ab} = \frac{1}{2} \partial_a \varphi \partial_b \varphi - \frac{1}{2} g_{ab} \left(\frac{1}{2} g^{cd} \partial_c \varphi \partial_d \varphi + V(\varphi) \right),$$

$$\left[\gamma_{\mu\nu} \right] = \left[\varphi \right] = 0; \quad \left[K_{\mu\nu} - \gamma_{\mu\nu} K \right] = \frac{1}{\sqrt{-\gamma}} \frac{\delta S_{\Sigma_0}}{\delta \gamma^{\mu\nu}}; \quad \left[n^a \partial_a \varphi \right] = -\frac{1}{\sqrt{-\gamma}} \frac{\delta S_{\Sigma_0}}{\delta \varphi}$$

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Self tuning if $\exists$ solutions with flat defect for generic $W_B \sim \Lambda^4$.

Bulk equations

$$S_5 = M^3 \int d^4x \int du \sqrt{-g} \left[R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - V(\varphi) \right]$$

Vacuum (Poincaré invariant) solutions:

$$ds^2 = du^2 + e^{2A(u)} \eta^{\mu\nu} dx_\mu dx_\nu, \qquad \varphi = \varphi(u)$$

$$6\ddot{A} + \dot{\varphi}^2 = 0, \qquad 12\dot{A}^2 - \frac{1}{2}\dot{\varphi}^2 + V(\varphi) = 0.$$

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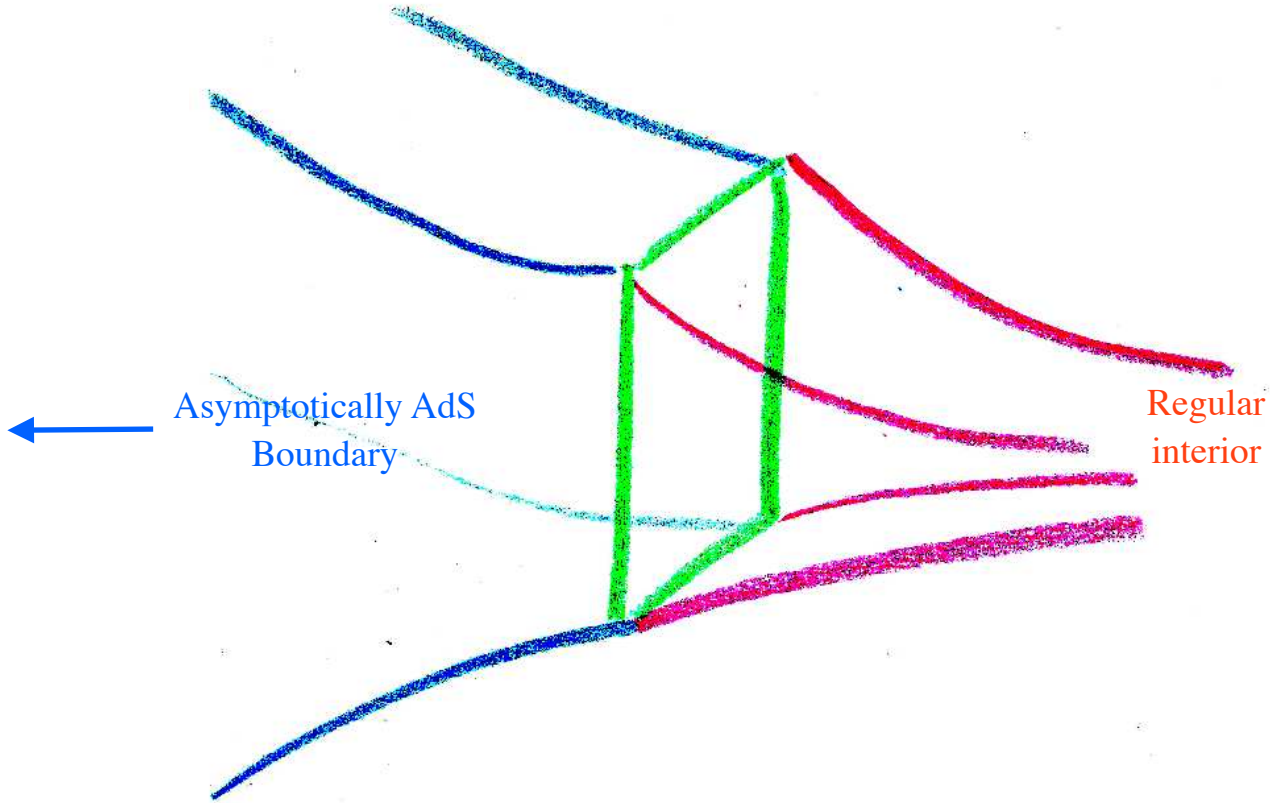
One has to *solve independently on each side* of the defect (at $u = u_0$), and glue the solutions using Israel junction conditions:

$$\left[A\right] = \left[\varphi\right] = 0; \quad \left[\dot{A}\right] = -\frac{1}{6} W_B(\varphi(u_0)); \quad \left[\dot{\varphi}\right] = \frac{dW_B}{d\varphi}(\varphi(u_0))$$

Vacuum Geometry



$A_{UV}(u), \varphi_{UV}(u)$ $\qquad$ $A_{IR}(u), \varphi_{IR}(u)$

$e^{A_{UV}} \to +\infty, \ \varphi_{UV} \to 0$

UV-AdS boundary

$e^{A_{IR}} \to 0, \ \varphi_{IR} \to \varphi_*$

Interior of IR-AdS space

Superpotential

Write Einstein's equations as first order flow equations, with an auxiliary scalar function $W(\varphi)$ $\quad (' = d/d\varphi)$:

$$\dot{A} = -\frac{1}{6} W(\varphi) \qquad \dot{\Phi} = W'(\varphi),$$

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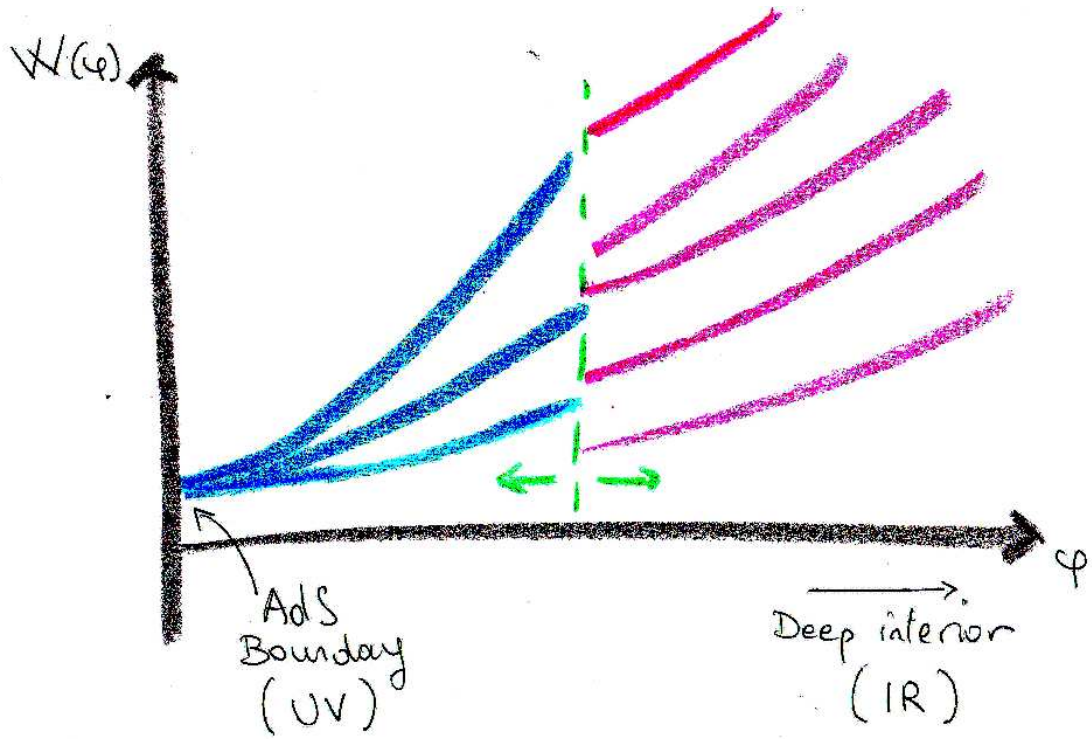
- Up to a rescaling of the scale factor, W completely determines the geometry.

$$W(\varphi) = \begin{cases} W^{UV}(\varphi) & \varphi < \varphi_0 \\ W^{IR}(\varphi) & \varphi > \varphi_0 \end{cases}$$

- On each side of the interface ($\varphi = \varphi_0$), W is determined by one integration consntant C.

Junction conditions for the superpotential



Junction conditions take a simple form:

$$W^{IR}(\varphi_0) - W^{UV}(\varphi_0) = W_B(\varphi_0), \qquad \frac{dW^{UV}}{d\varphi}(\varphi_0) - \frac{dW^{IR}}{d\varphi}(\varphi_0) = \frac{dW_B}{d\varphi}(\varphi_0)$$

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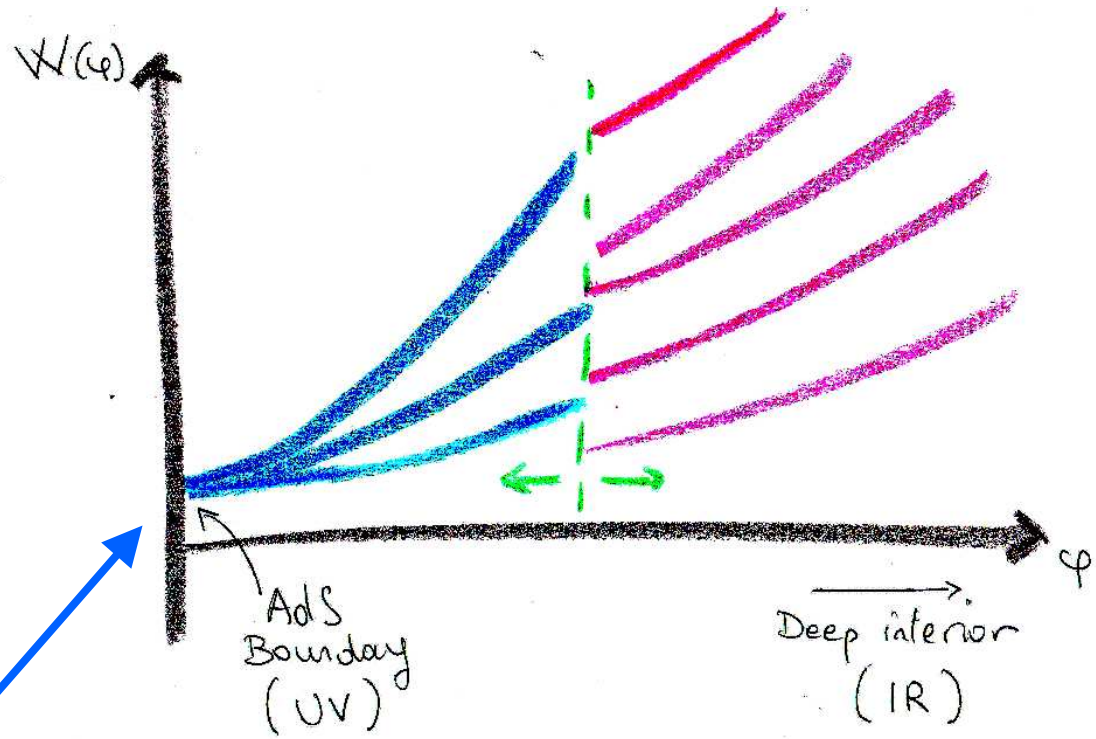

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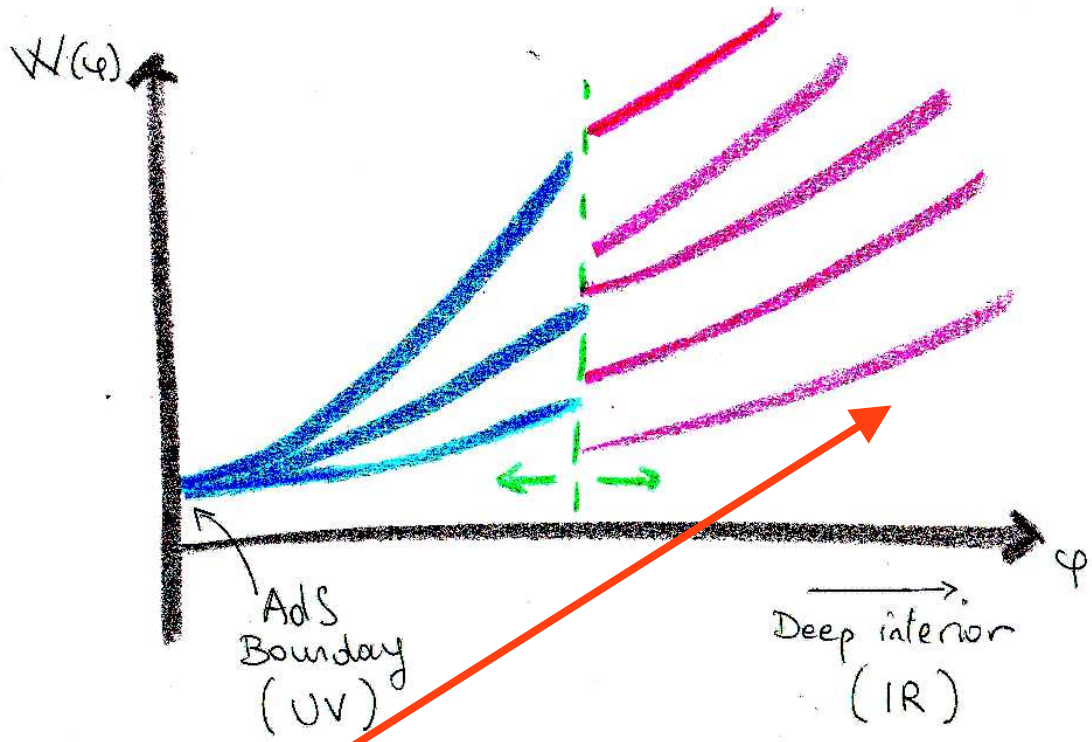

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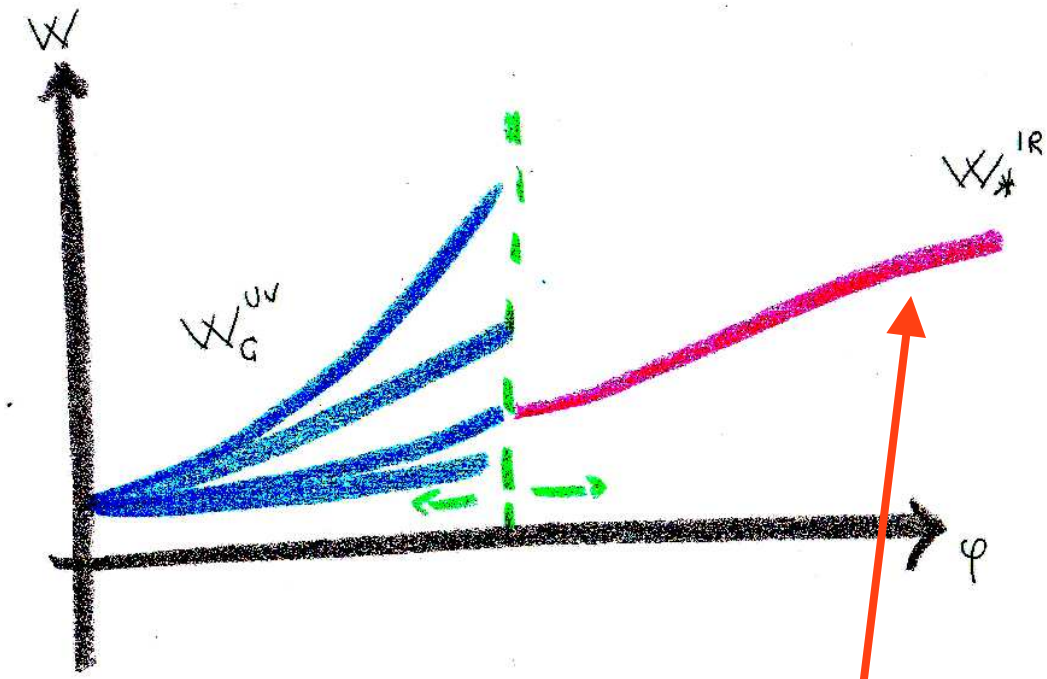
IR side: Only certain IRs are acceptable (e.g. IR AdS fixed point) This picks out a single solution W_*^{IR} and fixes $C_{IR} = C_*$

IR Selection



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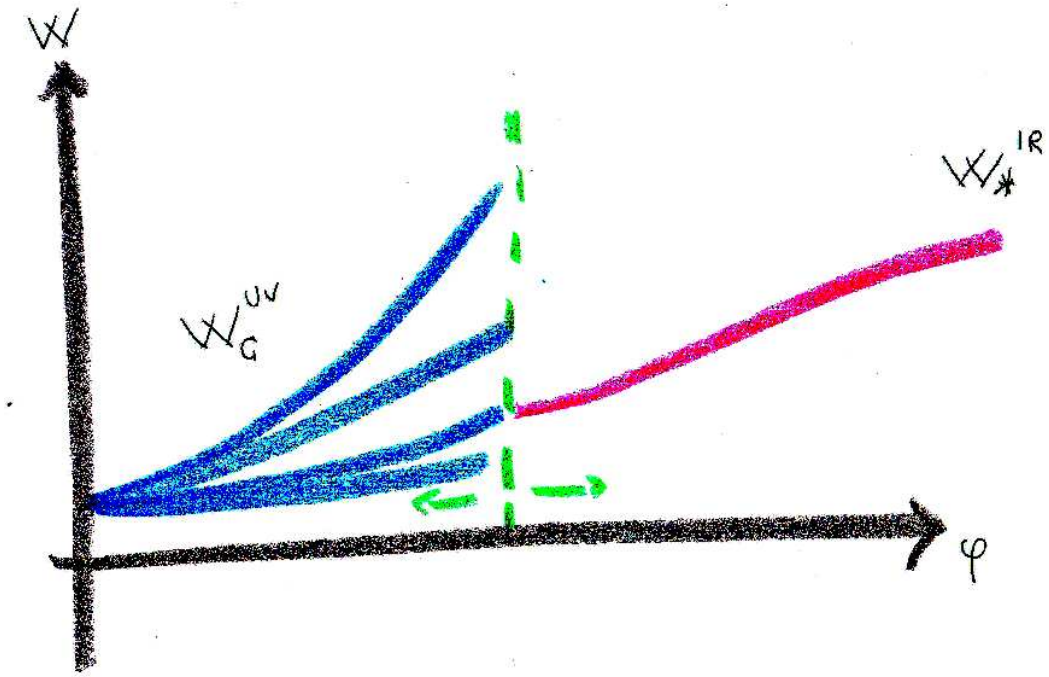
Equilibrium solution



$$W^{UV}(\varphi_0) = W_*^{IR}(\varphi_0) - W_B(\varphi_0), \qquad \frac{dW^{UV}}{d\varphi}(\varphi_0) = \frac{dW_*^{IR}}{d\varphi}(\varphi_0) - \frac{dW_B}{d\varphi}(\varphi_0)$$

Two equations for two unknowns C_{UV}, φ_0. Generically there exist a unique (or a discrete set of) solutions with C_{UV}, φ_0 determined.

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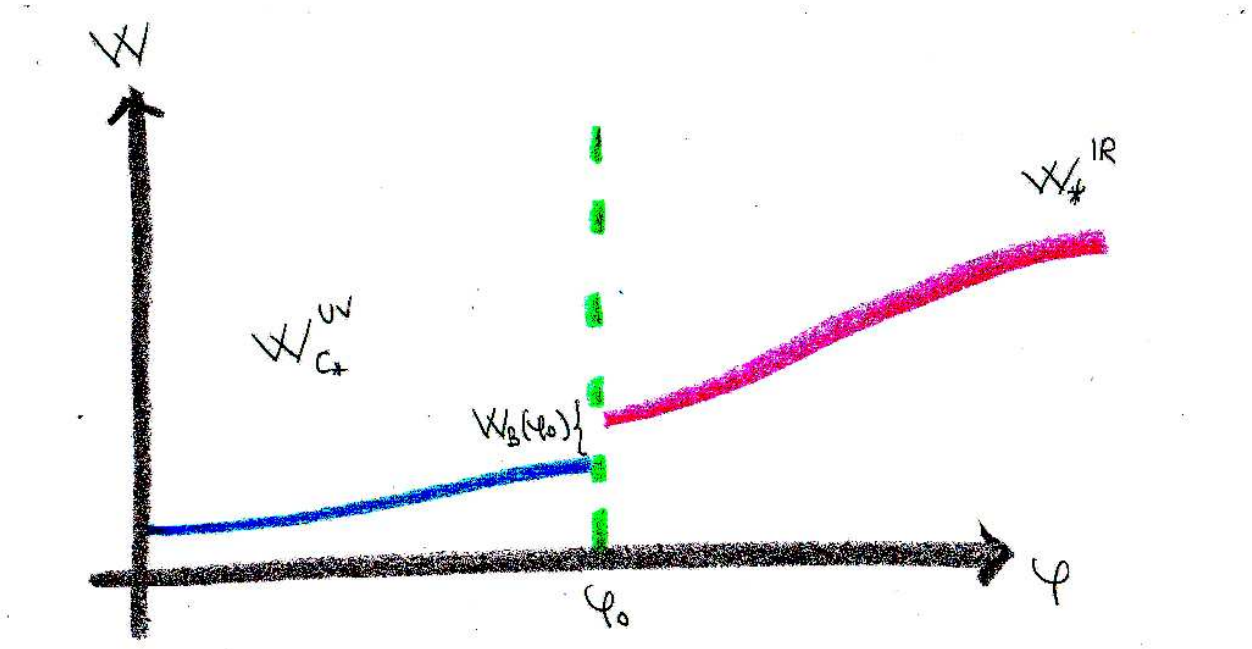

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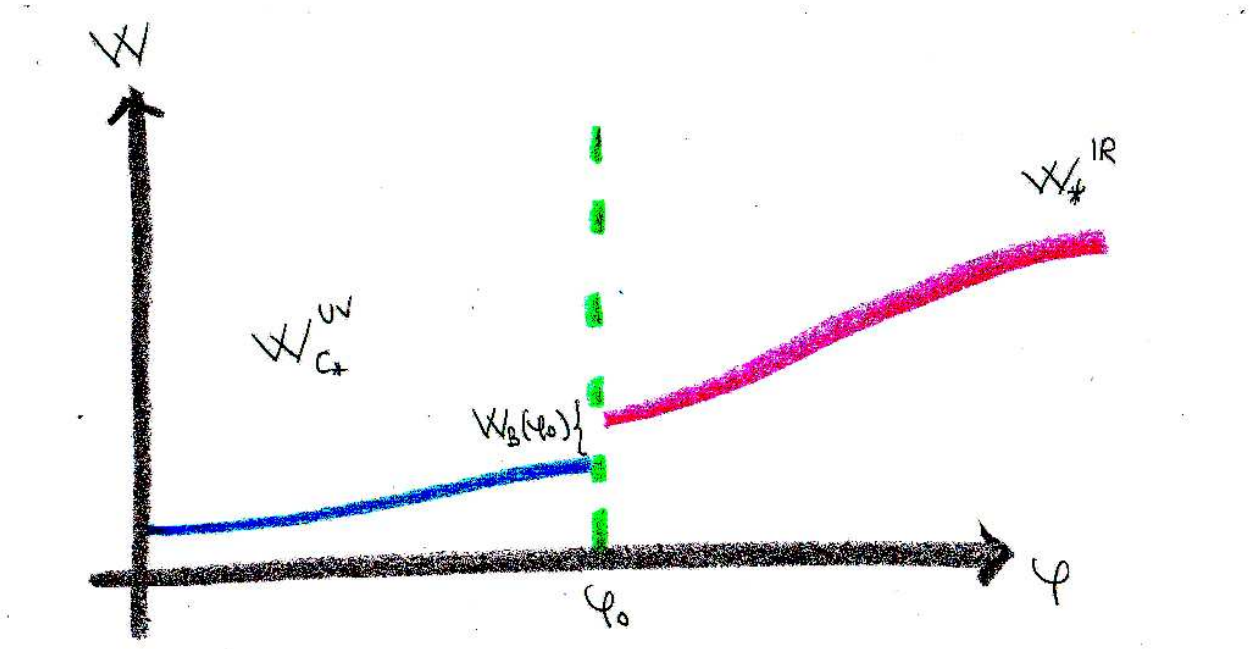

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For generic brane vacuum energy $\sim \Lambda^4$, geometry and brane position adjust so that the brane is flat and the UV glues to the regular IR (*self-tuning*).

Emergent gravity on the brane

In the model considered, solutions with flat 4d brane are generic. Do gravitational interactions between brane sources look 4d?

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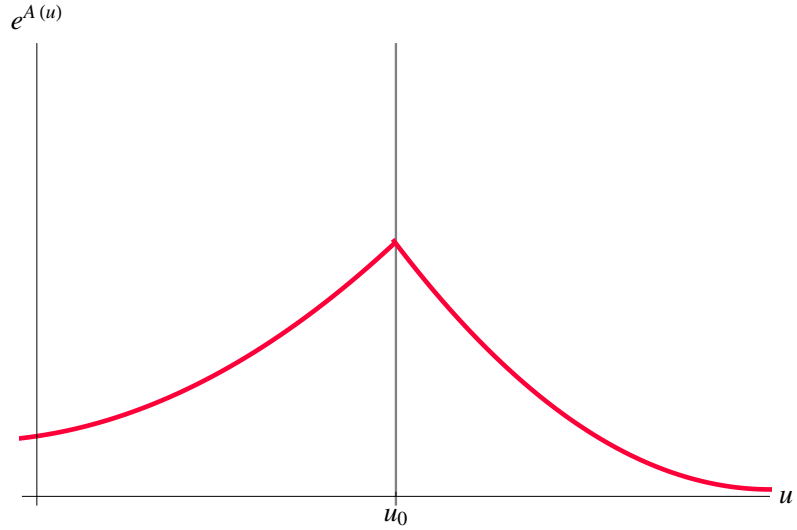

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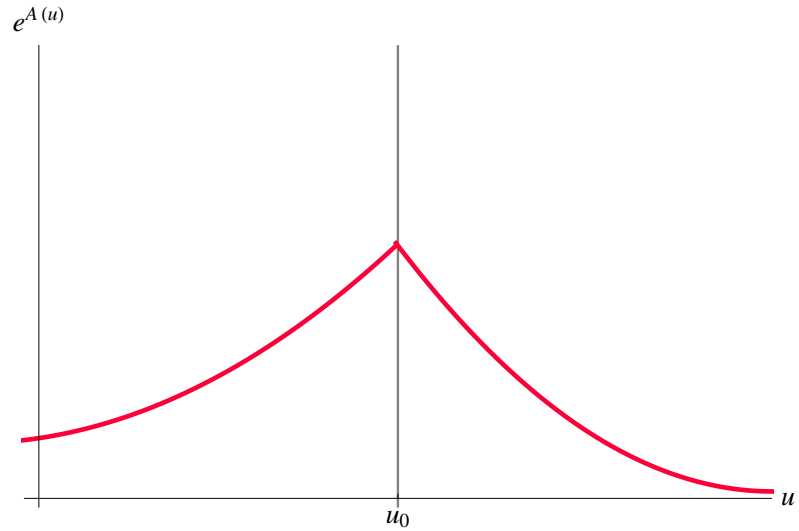
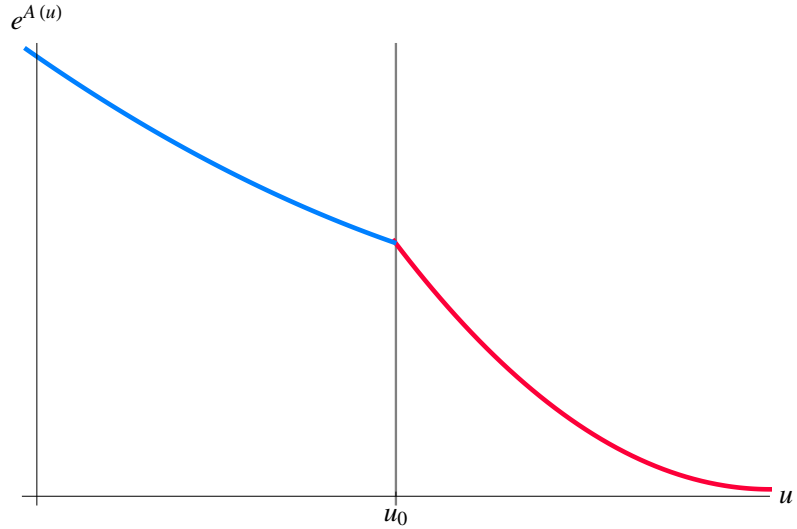


- Volume is finite on both sides $\Rightarrow$ Normalizable 4d graviton zero mode mediates 4d gravity at large distances
- Brane connects two "IR" special solutions $\Rightarrow$ Need fine-tuning of the brane tension for the brane to stay flat.
  $\Rightarrow$ self-tuning impossible

Emergent gravity on the brane

"Holographic" asymmetric braneworld:



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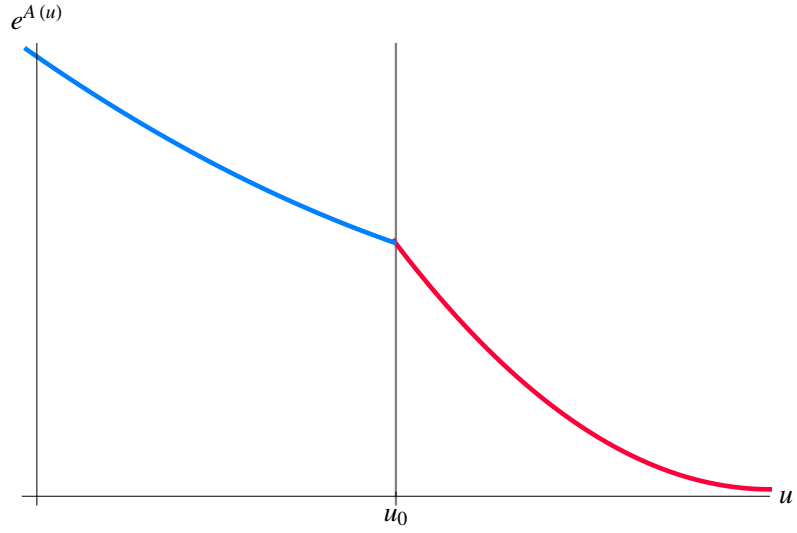
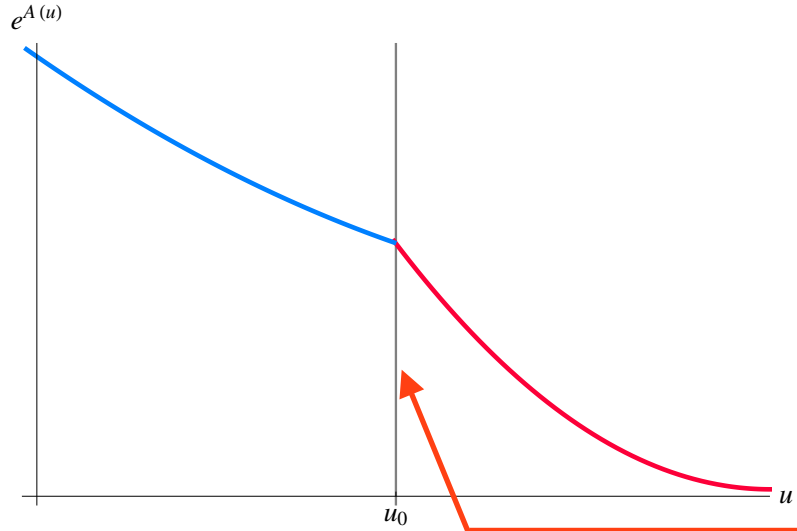


- Can choose generic "UV" solutions $\Rightarrow$ self-tuning possible
- Volume is infinite on the UV side $\Rightarrow$ No Normalizable 4d graviton zero mode.

Emergent gravity on the brane

"Holographic" asymmetric braneworld:



$$S = M^3 \int du\, d^4x \sqrt{g} R_5 + \ldots + \boxed{M^3 \int_{u=u_0} d^4x \sqrt{\gamma} U(\varphi_0) R_4}$$

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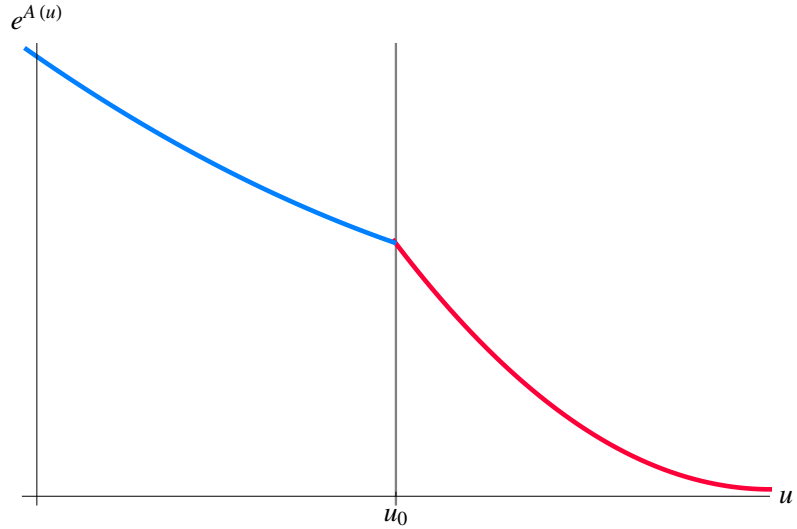

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- Localized Einstein-Hilbert term on the brane $\Rightarrow$ 4d-like graviton resonance (Dvali,Gabadadze,Porrati, '00): gravity is effectively 4d at short distances.
- Bulk curvature $\Rightarrow$ 4d massive graviton at *very* large distances.

Scales of braneworld gravity

Two competing scales:

1. "DGP" transition length: $r_c \approx U(\varphi_0)$
2. Bulk curvature length $r_t = (e^{A_0}\mathcal{R}_0)^{-1}$, $\qquad \mathcal{R}_0 \approx W_{UV}(\varphi_0)$

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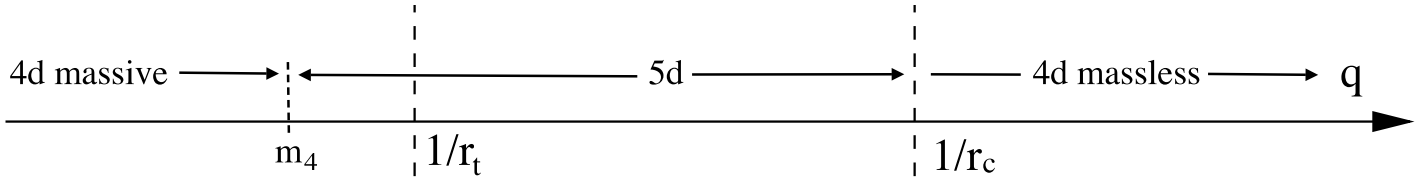

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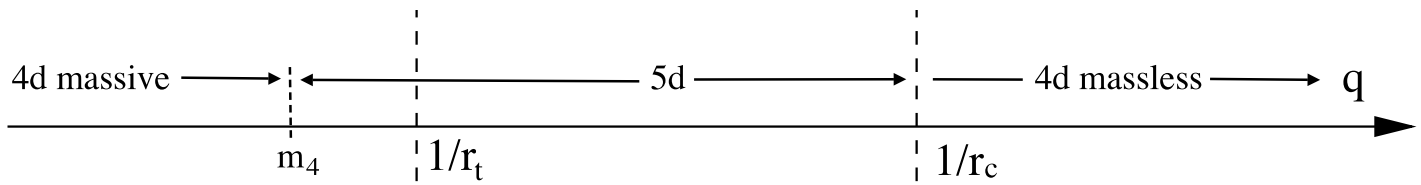

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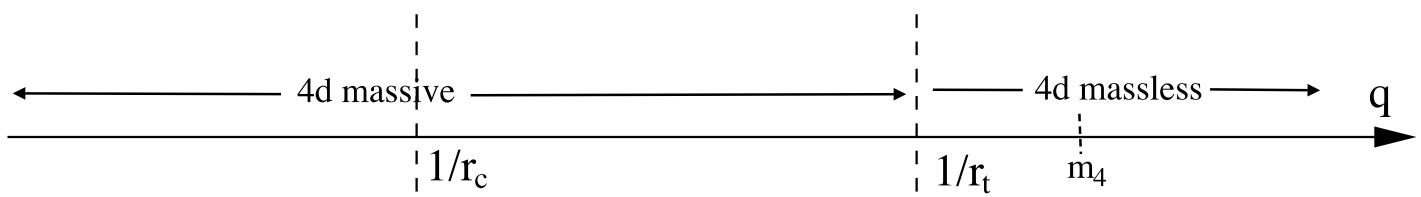

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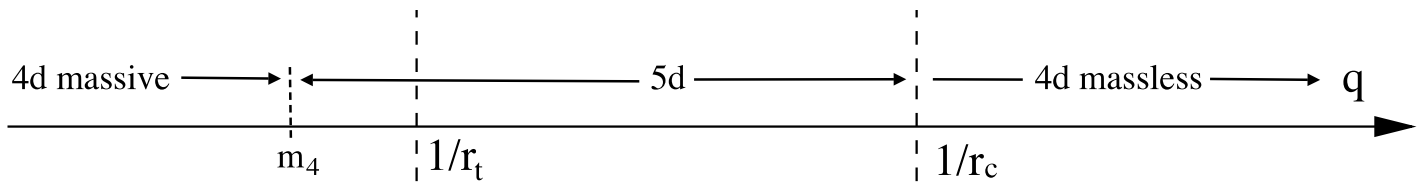

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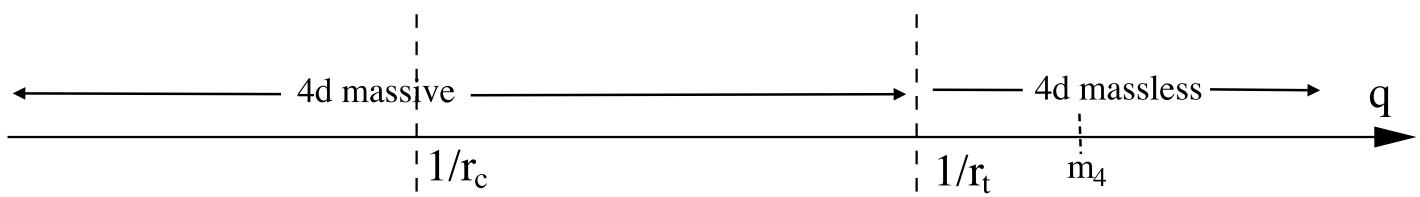

$$M_p^2 \approx M^3 U_0, \qquad m_g^2 \approx \frac{\mathcal{R}_0}{U_0}$$

Effective 4d Green's function

Introuce tensor perturbations:

$$\delta g_{\mu\nu} = e^{2A(r)} h_{\mu\nu}(r, x^\alpha), \quad h^\mu_\mu = \partial^\mu h_{\mu\nu} = 0$$

Solve classical linearized equation for tensor fluctuations with localized source:

$$h_{\mu\nu}(x, r) = \int d^4x G_{\mu\nu}{}^{\rho\sigma}(x - x'; r, r_0) T_{\rho\sigma}(x', r_0),$$

Tree-level interaction described in purely 4d terms by an effective Green's function:

$$S_{int}(T) = \int \frac{d^4p}{(2\pi)^4} \tilde{G}_4(p) \left[T_{\mu\nu}(p) T^{\mu\nu}(-p) - \frac{1}{3} T(p) T(-p) \right]$$

$$G_4(x) \equiv G(x, r_0, r_0).$$

Brane-to-brane propagator

Solution in terms of Green's function:

$$h(x,r) = \int d^4x G(x-x';r,r_0)\hat{T}(x',r_0),$$

$$\tilde{G}(r,p;r_0) = -\frac{1}{M^3}\frac{D(p,r)}{1+[U_0\,D(p,r_0)]p^2}$$

$$\left[\partial_r e^{3A(r)}\partial_r - e^{3A(r)}p^2\right] D(p,r) = -\delta(r-r_0)$$

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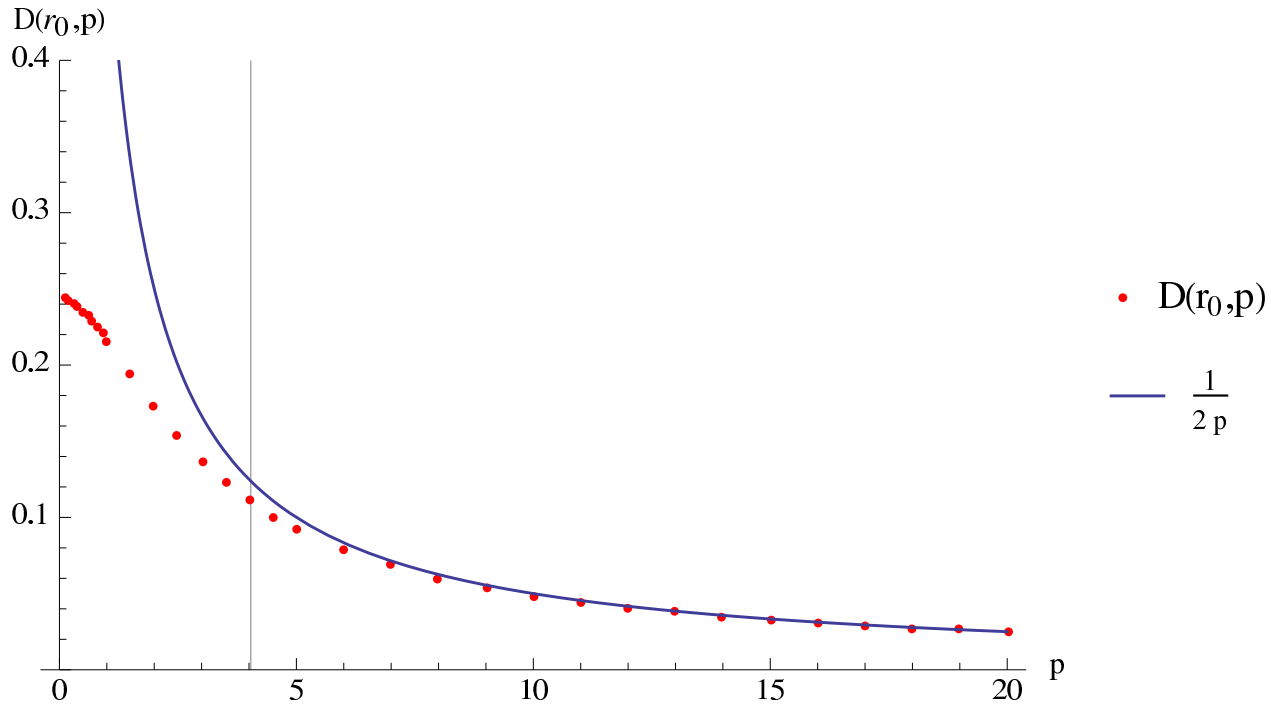
$D(p,r)$ is the bulk-to-bulk propagator, and depends only on properties of bulk modes.

Brane-to-brane propagator

$$D(r_0, p) \simeq \begin{cases} \dfrac{1}{2p} & p \gg \mathcal{R}_0, \\ \\ d_0 + d_2 p^2 + \ldots & p \ll \mathcal{R}_0, \end{cases}$$

$\mathcal{R}_0 =$ bulk curvature around the brane position $\approx W_0$



DGP-like behavior

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$$\approx \frac{1}{2M^3 U_0} \frac{1}{p^2} \qquad p \gg U_0^{-1}$$

- Four dimensional interaction at distances $\ll r_c \equiv U_0/2$
- Effective four-dimensional Planck scale

$$M_p^2 = U_0 M^3$$

Massive gravity

$$D(r_0, p) \simeq \begin{cases} \dfrac{1}{2p} & p \gg \mathcal{R}_0, \\ \\ d_0 + d_2 p^2 + \ldots & p \ll \mathcal{R}_0, \end{cases}$$

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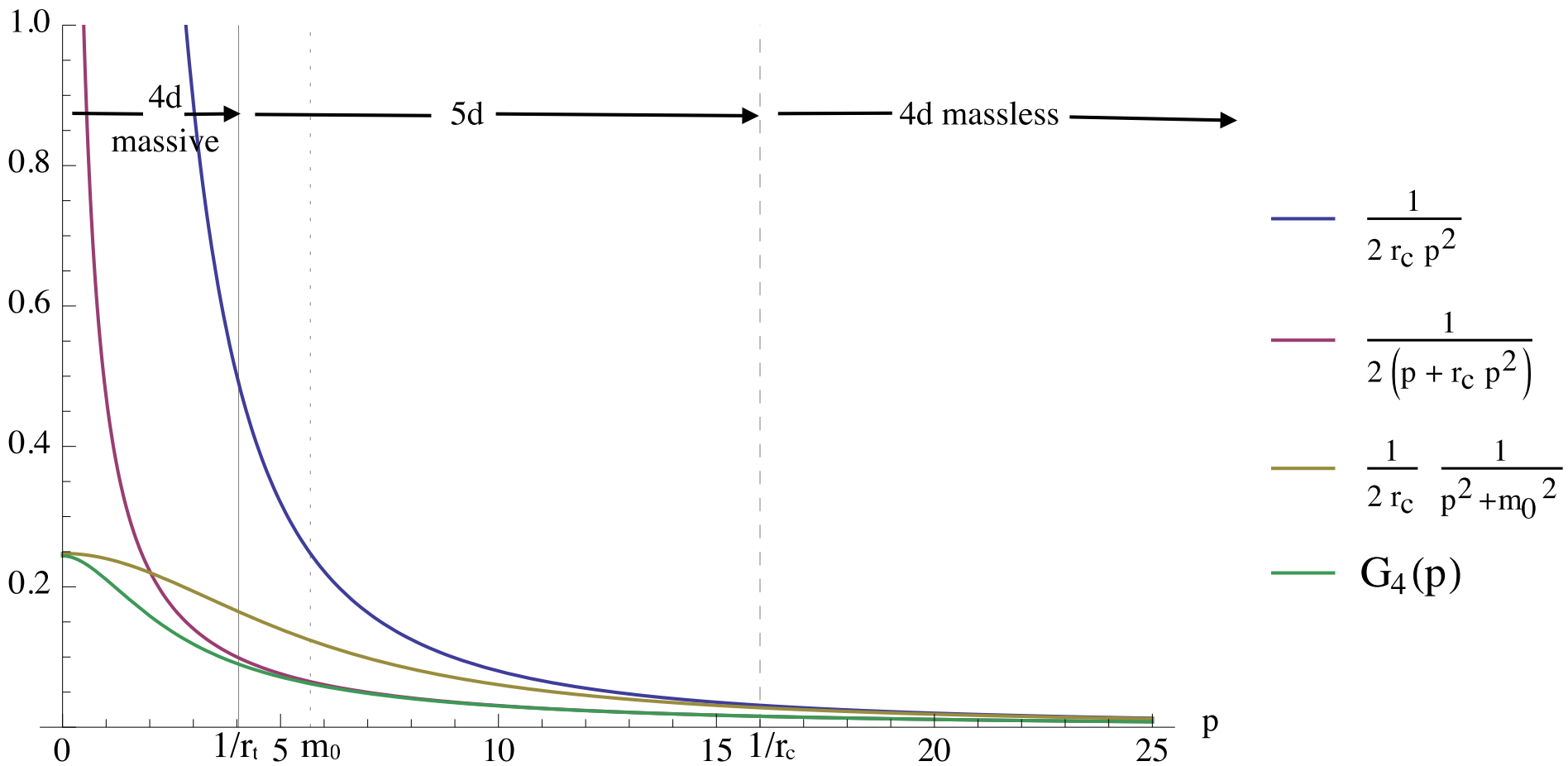
- Massive gravity at large distances,

$$m_g^2 \simeq \frac{1}{d_0 \tilde{U}_0}, \qquad \tilde{M}_p^2 = M^3 \tilde{U}_0$$

$$\tilde{U}_0 \equiv U_0 \left(1 - \frac{d_2}{U_0 d_0^2} \right).$$

4d-5d transition

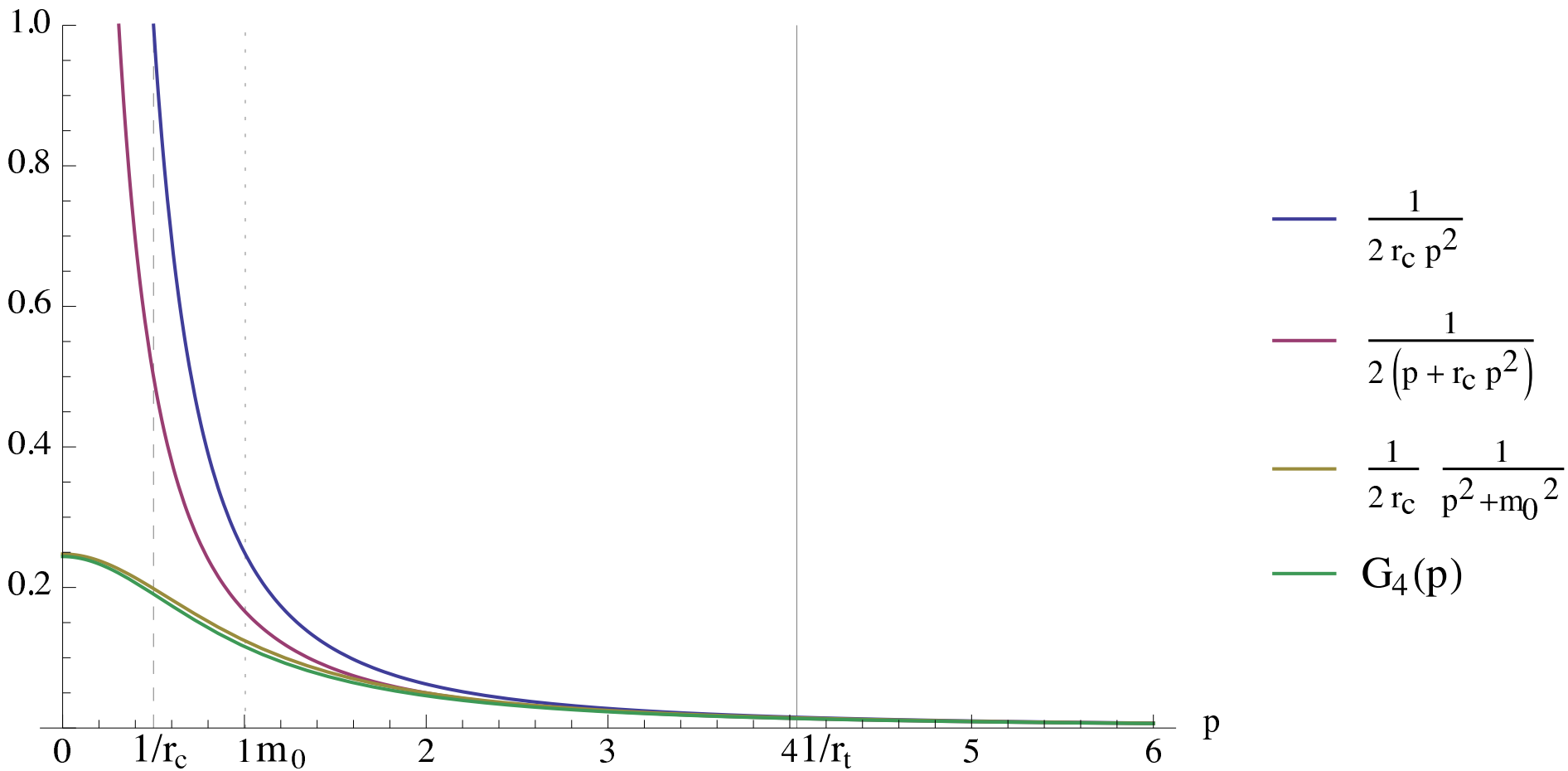
$r_c < r_t$: DGP-like transition, at intermediate distances.



$$r_c = U_0,\ r_t = \frac{e^{-A_0}}{\mathcal{R}_0}, \quad M_p^2 \approx M^3 U_0, \quad m_0^2 \approx \frac{\mathcal{R}_0}{U_0},$$

Massless/Massive gravity transition

$r_c > r_t$ massive graviton propagator all the way.



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Scalar perturbations

- Determine whether vacuum solution (flat brane at $r = r_0$) is stable.
- Possible light scalar mediated interactions (fifth force, violations of equivalence principle) $\Rightarrow$ pheno constraints.
- Analysis of linear flucutations show that there exist conditions on the background solution which guarantee stability.

1.

$$\tau_0 > 0, \quad Z_0 > 0, \quad Z_0 \tau_0 > 36 \left(\frac{dU_B}{d\varphi}\Big|_{\varphi_0} \right)^2$$

$$\tau_0 \equiv 6 \left(6 \frac{W_B}{W_{IR} W_{UV}} - U \right)_{\varphi_0}, \quad Z_0 \equiv Z(\varphi_0)$$

$\Rightarrow$ No ghost instabilities

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2.

$$\tilde{\mathcal{M}}^2 \equiv \left(\frac{d^2 W_B}{d\varphi^2}(\varphi_0) - \left[\frac{d^2 W}{d\varphi^2} \right]_{UV}^{IR} \right) \geq 0$$

$\Rightarrow$ No tachyonic instabilities.

Conclusion and outlook

We constructed a framework where Self-tuning of the CC is generically realized. Challenge now is model-building: construct phenomenologically viable model.

- Acceptable values of M_p, r_c, m_g given large UV cutoff;
- Compliance with stability requirements;
- Deal with vDVZ discontinuity (Role of non-linearities, Veinshtein mechanism);
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- Avoidance of fifth force constraints;

If this all goes through, one can do more phenomenology:

- Add SM and Higgs field (ongoing work with Lukas Witkowski)
- Study the space of solutions: non-flat brane, time-dependent solutions (cosmology) (ongoing work with Lukas Witkowski and Jewel Ghosh)
- Understand self-tuning from a *dinamical* perspective

What about Weinberg?

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$$\frac{\delta S}{\delta \gamma^{\mu\nu}} = \frac{\delta S}{\delta \phi} = 0 \quad \Rightarrow \quad V(\phi) = V'(\phi) = 0$$

System is over-constraint and will generically have no solution, unless $V(\phi)$ is fine-tuned.

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$$V_{eff}(\varphi_0) = W_{IR}(\varphi_0) - W_{UV}(\varphi_0; C_{UV}) - W_B(\varphi_0)$$

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$$V_{eff} = 0, \quad V'_{eff} = 0$$

Two equations for two unknowns (C_{UV}, φ_0).

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$$\delta g_{rr} = e^{2A}\phi, \quad \delta g_{r\mu} = e^{2A}\partial_\mu B,$$

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- Brane:

$$r = r_0 + \rho(x)$$

Scalar perturbations

Gauge fixing, constraints

$$\Downarrow$$

Reduce to the single bulk variable $\psi(x, r)$

Linearized equation in the bulk:

$$\psi'' + \left(3A' + 2\frac{z'}{z}\right)\psi' + \Box\psi = 0$$

$$z \equiv \frac{\bar{\varphi}'}{A'}$$

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$$\bar{\varphi}(r_0)\rho(x) = -\frac{[\psi]}{[z^{-1}]},$$

$[X] \equiv X_{IR}(r_0) - X_{UV}(r_0)$

Scalar perturbations

Linearized matching conditions:

$$[z\psi'] = -\left(\frac{6}{a_0}\frac{dU_B}{d\varphi}\Big|_{\varphi_0}\right)\Box\frac{[z\,\psi]}{[z]} - \frac{1}{a_0}\left(Z_0\Box - a_0^2\tilde{\mathcal{M}}^2\right)\frac{[\psi]}{[z^{-1}]}$$

$$[z^2\psi'] = 6\left(2\frac{U_0}{a_0} - \left[\frac{a}{a'}\right]\right)\Box\frac{[z\,\psi]}{[z]} - \left(\frac{6}{a_0}\frac{dU}{d\varphi}\Big|_{\varphi_0}\right)\Box\frac{[\psi]}{[z^{-1}]}$$

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both ψ and ψ' must be discontinous at the brane.

Scalar perturbations

Recast as a 2-component Sturm-Liouville problem:

$$\Psi(r,x) = \begin{pmatrix} \psi_{UV}(r,x) \\ \psi_{IR}(r,x) \end{pmatrix}$$

$$\Big(\mathcal{B}(r)\Psi'\Big)' + \mathcal{B}(r)\partial_\mu\partial^\mu\Psi = 0, \qquad r \neq r_0$$

$$\Psi'(r_0) = \Big(\Gamma_1 + \Gamma_2\,\partial^\mu\partial_\mu\Big)\Psi(r_0),$$

$\mathcal{B}(r)$, Γ_1, Γ_2 are 2×2 matrices:

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$$\Gamma_1 = \frac{a_0\tilde{\mathcal{M}}^2}{[z]^2}\begin{pmatrix} -z_{IR}^2 & z_{IR}^2 \\ -z_{UV}^2 & z_{UV}^2 \end{pmatrix}$$

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$$\Gamma_2 = \frac{1}{[z]^2 a_0}\begin{pmatrix} -12 z_{IR}\frac{dU_B}{d\varphi} + \tau_0 + Z_0 z_{IR}^2 & 6\left(\frac{z_{IR}^2}{z_{UV}} + z_{IR}\right)\frac{dU_B}{d\varphi} - \tau_0\frac{z_{IR}}{z_{UV}} - Z_0 z_{IR}^2 \\ -6\left(\frac{z_{UV}^2}{z_{IR}} + z_{UV}\right)\frac{dU_B}{d\varphi} + \tau_0\frac{z_{UV}}{z_{IR}} + Z_0 z_{UV}^2 & 12 z_{UV}\frac{dU_B}{d\varphi} - \tau_0 - Z_0 z_{UV}^2 \end{pmatrix}$$

$$\tau_0 \equiv 6\left(6\frac{W_B}{W_{IR}W_{UV}}\bigg|_{\varphi_0} - U_0\right)$$

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Solve with normalizable boundary conditions in the UV for ψ_{UV} and in the IR for ψ_{IR}.

Action for scalar fluctuations

$$S_5^{(2)} = -\frac{M^3}{2}\int d^4x \Bigg[\int dr\, \left[\partial_r\Psi^\dagger\, \mathcal{B}(r)\Theta(r)\, \partial_r\Psi \,+\, \partial_\mu\Psi^\dagger\, \mathcal{B}(r)\Theta(r)\, \partial^\mu\Psi\right]$$

$$+\Psi^\dagger(r_0)\, \Sigma\Gamma_1\, \Psi(r_0) \,-\, \partial_\mu\Psi^\dagger(r_0)\, \Sigma\Gamma_2\, \partial^\mu\Psi(r_0)\Bigg]$$

$$\Sigma \equiv \begin{pmatrix} -e^{3A_{UV}}\, z_{UV}^2 & 0 \\ 0 & e^{3A_{IR}}\, z_{IR}^2 \end{pmatrix}_{r_0}, \quad \Theta \equiv \begin{pmatrix} \theta(r_0 - r) & 0 \\ 0 & \theta(r - r_0) \end{pmatrix}$$

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Localized mass term Localized kinetic term

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Decompose into radial eigenmodes with fixed "energy" m^2:

$\Psi(r, x^\mu) = \Psi(r)\phi(x)$,

$$-\mathcal{B}^{-1}\frac{d}{dr}\left(\mathcal{B}(r)\frac{d\Psi(r)}{dr}\right) = m^2\Psi(r), \quad \Psi'(r_0) = \Gamma_1 + m^2\Gamma_2\Psi(r_0)$$

Positivity (no ghosts)

$\Rightarrow$ Effective 4-d action for massive scalar modes:

$$S_4^{(2)} = -\frac{1}{2}\mathcal{N}\int d^4x\,\left(\partial^\mu\phi\partial_\mu\phi + m^2\phi^2\right)$$

$$\mathcal{N} = \int dr\,\Psi^\dagger\,\mathcal{B}\Theta\,\Psi \;+\; \begin{pmatrix} \frac{[z\psi]}{[z]} & -\frac{[\psi]}{[1/z]} \end{pmatrix}\mathcal{K}\begin{pmatrix} \frac{[z\psi]}{[z]} \\ -\frac{[\psi]}{[1/z]} \end{pmatrix}$$

$$\mathcal{K} \equiv a_0^2\begin{pmatrix} \tau_0 & -6\frac{dU_B}{d\varphi}\Big|_{\varphi_0} \\ -6\frac{dU_B}{d\varphi}\Big|_{\varphi_0} & Z_0 \end{pmatrix}, \quad \tau_0 \equiv 6\left(6\frac{W_B}{W_{IR}W_{UV}}\Big|_{\varphi_0} - U_0\right)$$

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- Absence of ghost instabilities is guaranteed if:

$$\tau_0 > 0, \quad Z_0 > 0, \quad Z_0\tau_0 > 36\left(\frac{dU_B}{d\varphi}\Big|_{\varphi_0}\right)^2$$

Stability (no tachyons)

The mass eigenvalues satisfy the relation:

$$m^2\mathcal{N} - \Psi^\dagger(r_0)\Sigma\Gamma_1\Psi(r_0) \geq 0, \quad \Sigma\Gamma_1 = a_0^4 \frac{\tilde{\mathcal{M}}^2}{[1/z]^2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

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Eigenvalues: $0, 2$

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- Sufficient condition for absence of tachyonic instabilities ($m^2 > 0$):

$$\tilde{\mathcal{M}}^2 \geq 0$$

$$\tilde{\mathcal{M}}^2 = \left(\frac{d^2 W_B}{d\varphi^2}(\varphi_0) - \left[\frac{d^2 W}{d\varphi^2} \right]_{UV}^{IR} \right)$$

Scalar-mediated interaction

Define metric and dilaton sources:

$$T_{\mu\nu} = -\frac{2}{\sqrt{\gamma}}\frac{\delta S_m}{\gamma^{\mu\nu}}, \qquad O = \frac{\delta S_m}{\delta\varphi}.$$

Interaction between brane-localized sources:

$$S_{int} = -\frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\mathcal{T}^\dagger(q)G_s(q)\mathcal{T}(-q), \qquad \mathcal{T} \equiv \left(T^\mu_\mu,\, O\right)$$

$$G_s(q) \equiv \frac{1}{2M^3}P\left[\Sigma\left(\Gamma_1 + q^2\Gamma_2\right) + \mathcal{D}^{-1}(r_0;q)\right]^{-1}P^\dagger$$

$$P \equiv -\frac{z_{IR}z_{UV}}{[z]}\begin{pmatrix} \frac{1}{z_{IR}} & -\frac{1}{z_{UV}} \\ 1 & 1 \end{pmatrix}.$$

- Modes coupling to O can be parametrically heavy, $m \simeq \mathcal{M}$.
- Modes coupling to T remain light.

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localized mass

localized kinetic term

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Example

$$V(\varphi) = -12 - \left(\frac{\Delta(4-\Delta)}{2} - \frac{b^2}{4}\right)\varphi^2 - V_1 \sinh^2 \frac{b\varphi}{2},$$

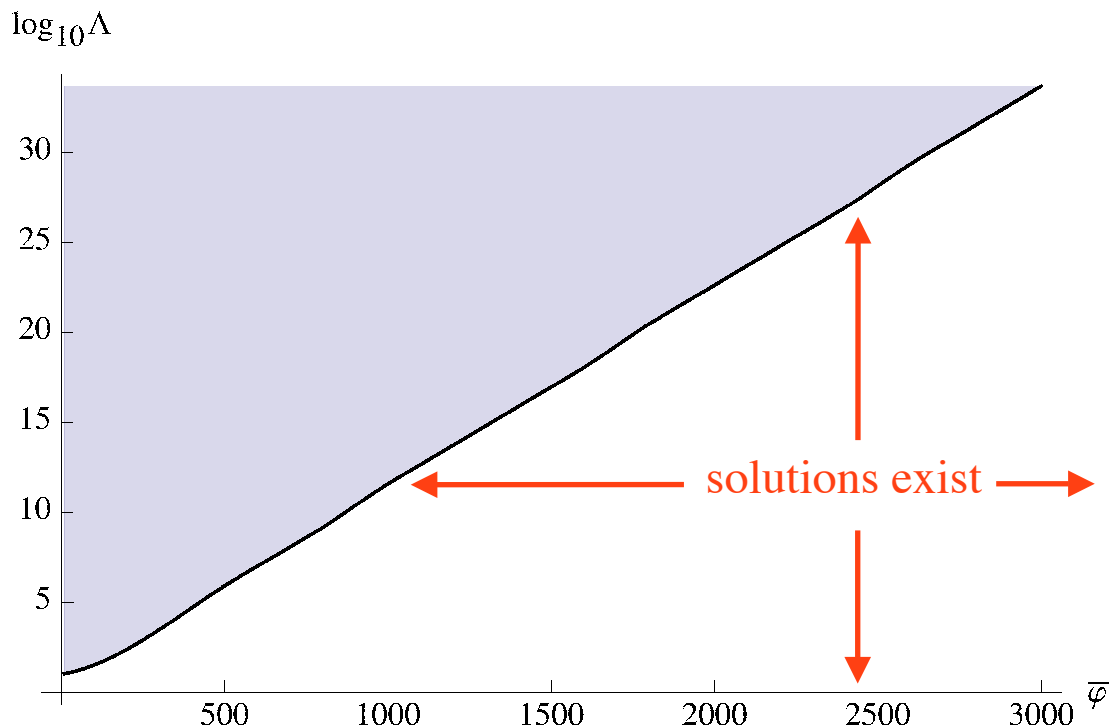
- supports an AdS fixed point at $\varphi = 0$ ($\ell_{UV} = 1$)
- good IR solution:

$$W_{IR}(\varphi) \sim \sqrt{\frac{2}{(32/3) - b^2}} \exp \frac{b\varphi}{2}, \qquad \varphi \to +\infty.$$

How large can Λ be?

$$W_B(\varphi) = \Lambda^4 \left[-1 - \frac{\varphi}{s} + \left(\frac{\varphi}{s}\right)^2 \right]$$

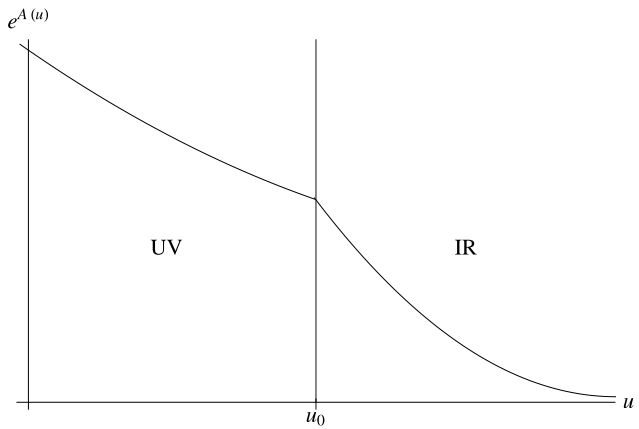
$$b = \frac{1}{\sqrt{6}}, \ \Delta = 3, \ V_1 = 1$$



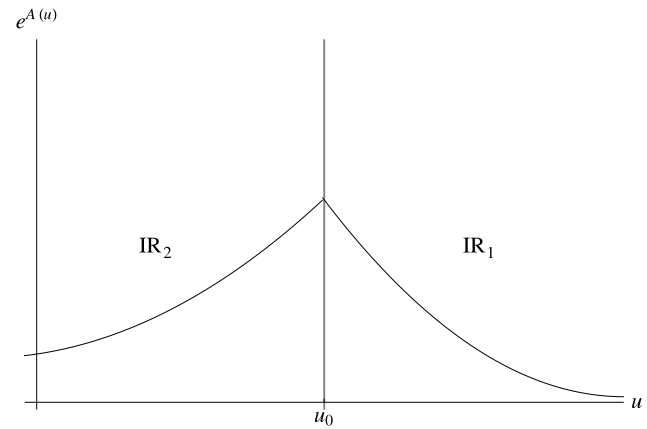
$$\varphi_0 \simeq \bar{\varphi} \approx 1.6\, s$$

Consistent self-tuning

Two possibilities:



$W_{UV} > 0$



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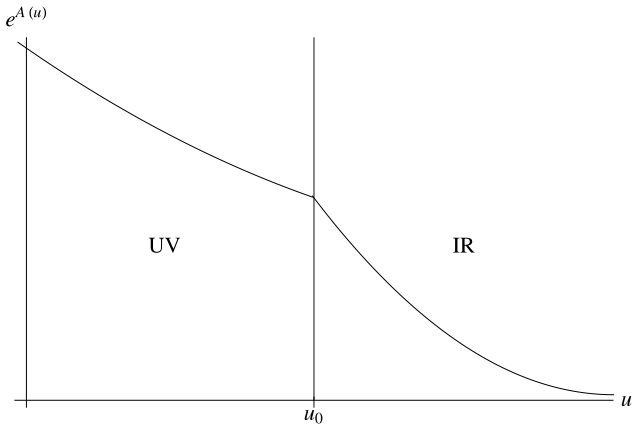

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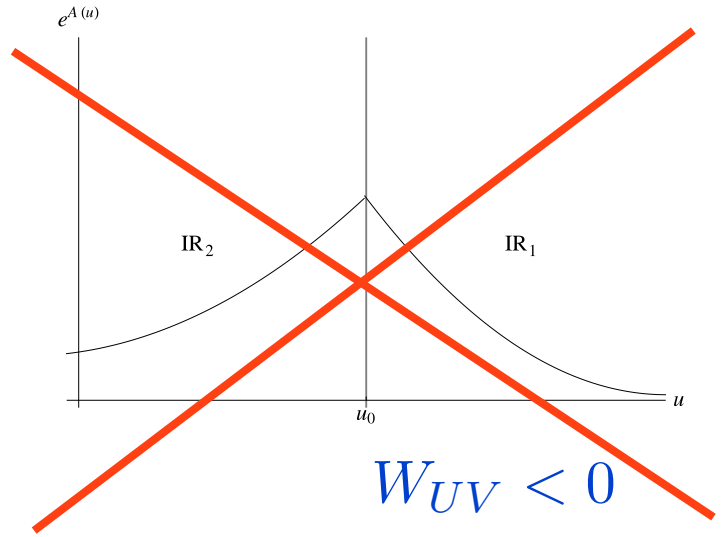

$W_{UV} < 0$

Needs fine tuning of the brane potential to join two "special" solutions

Cfr. Randall-Sundrum setup