Glueballs:
At the interface between lattice QCD and constituent models

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Prologue

Lattice QCD

Constituent models

AdS/CFT

Large N expansion

Deconfinement

QGP

Phase diagram

Glueballs

Mesons, baryons, ...

Mass gap

Confinement

Nonperturbative

Nonabelian

Nonperturbative

\[ \mathcal{L}_{QCD} = -\frac{1}{4} F^{\mu\nu}_a F^{\alpha}_{\mu\nu} + \sum_f \bar{\psi}_f (i \gamma^\mu D_\mu + m_f) \psi_f \]
Outline

A summary about glueballs

- Experimental data
- Lattice QCD
  - Mass spectrum
  - Wave function
- Effective approaches

How to build a constituent approach?

- Why?
- Informations from the lattice
  - Constituent gluons
  - Potential

Glueball mass spectrum

- Two- and three-body states
- Large N limit

Thermodynamics
Conclusions
A summary about glueballs
Experimental data

Nothing unambiguous yet

- Too many $0(0^{++})$ states for the quark model
  - PDG: $f_0(600)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$

- Mixed states involving: $|u\bar{u}\rangle + |d\bar{d}\rangle$, $|s\bar{s}\rangle$, $|G\rangle$

- Unclear status of the $0(0^{-+})$ state $\eta(1405)$

- Future: PANDA, GlueX, ...

Lack of unquenched QCD results

- Mixings in Fock space
- Decay widths
Lattice results (I)

Mass spectrum: Pure gauge, SU(3)


Lattice results (II)

Pure gauge, SU(3) and SU(8)

Lattice results (III)

Large N limit

Lattice results (IV)

Structure of the spectrum
- Lightest states with $C = +$: $0^{++}$, $2^{++}$, $0^{-+}$
  - Scalar always the lowest-lying, 1400-1800 MeV
  - Range of some $f_0$ states
- No light $1^{p+}$ state

Large N limit
- Behavior in $M_G(N) = M_G(\infty) + \frac{\theta}{N^2}$
- $\theta$ compatible with 0
Lattice results (V)

0^{++} Wave functions

- Bethe-Salpeter for a two-gluon state

\[ \chi(r) = \langle 0 | s_{\mu \nu} \int d\hat{r} Y_{lm}(\hat{r}) A_\mu^\dagger(x) A_\nu(x + r) | G \rangle \]

P. de Forcrand and K. F. Liu,

M. Loan and Y. Ying,
Effective approaches

Large amount of theoretical works

- Potential models / Constituent approaches
- Effective lagrangians
- AdS/QCD
- String theory
- ...

Lattice QCD

Experiment
How to build a constituent approach?
Why « constituent » ? (I)

Correlators

\[ \langle \Theta^+_G(\tau) \Theta_G(0) \rangle \propto e^{-m_G \tau} \]

- Glueball operators
  - \( 0^{++} = E^2_a \pm B^2_a \)
  - \( 1^{+-} = d_{abc}(E_a \cdot E_b) B_c \)

- Gluelumps
  - \( 1^{+-} = B_a \)

Large N

- Constituent approach as good (or bad) for \( N = 3 \) than for large \( N \)

Mass gap

Why « constituent » ? (II)

Assumption: Glueball = bound state of gluons

- Hamiltonian approach?
Gluon's features (I)

Color octet

- Singlet if more than 2 gluons
- Charge conjugation
  \[ \hat{C} A_\mu \hat{C}^{-1} = -A^T_\mu \]
- Glueball's C

Gluon mass

- 0 bare mass
- Generated
  - About 600 MeV at \( q^2 = 0 \)
  - Quite small above \( q^2 = 1 \) GeV

Gluon's features (II)

Spin degree of freedom

- Early works: spin 1, $S_z = -1, 0, +1$
  - Usual LS basis like quark models
  - Too many states when compared to lattice


- Our approach: transverse gluons
  - Zero mass
  - Helicity 1, $\lambda = \pm 1$
  - Jacob and Wick's helicity formalism
  - Only the lattice states

Helicity formalism (I)

Two-gluon states

\[ |\lambda_1, \lambda_2; J^P, M, \epsilon\rangle = \frac{1}{\sqrt{2}} \left\{ \Omega^J_{M, \lambda_1-\lambda_2} \left[ |\psi(p, \lambda_1)\rangle \otimes |\psi(-p, \lambda_2)\rangle \right] \right. \\
+ \epsilon \left. \Omega^J_{M, \lambda_2-\lambda_1} \left[ |\psi(p, -\lambda_1)\rangle \otimes |\psi(-p, -\lambda_2)\rangle \right] \right\} \]

\[ \Omega^J_{M, \lambda}[X] = \left[ \frac{2J + 1}{4\pi} \right]^{1/2} \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin \theta \\
\times D^J_{M, \lambda}(\phi, \theta, -\phi) R(\phi, \theta, -\phi) X(\phi, \theta) \]

Quantum numbers

\[ J \geq |\lambda_1 - \lambda_2| \]

\[ P = \epsilon(-)^J, \quad C = + \]
Helicity formalism (II)

Helicity states + Pauli principle

- Color symmetric, spin-space symmetric
- No $1^{++}$ and $1^+$ states
  - Yang's theorem, no $\rho \rightarrow \gamma\gamma$
  - Lattice, no light J = 1 glueball
- No $3^+$, $5^+$, $7^+$, ...
- Matrix elements $\langle \vec{L}^2 \rangle = J(J + 1) + 2\lambda_1\lambda_2$
- Examples
  
  $$|0^{++}\rangle = \sqrt{\frac{2}{3}} |L = 0, S = 0\rangle + \sqrt{\frac{1}{3}} |L = 2, S = 2\rangle$$
  $$|0^{-+}\rangle = - |L = 1, S = 1\rangle$$
**Interaction potential (I)**

Hamiltonian: ansatz \[ H_{gg} = 2\sqrt{p^2} + V(r) \]

- 0^{++} Mass and wave function from the lattice
- Inverse problem

**Funnel-like**

- Standard,
  - Casimir scaling

\[
V(r) = \frac{9}{4}\sigma r - 3\frac{\alpha_s}{r} - D
\]

\[
\sigma = 0.185 \text{ GeV}^2, \quad \alpha_s = 0.4
\]

\[
D = 0.45 \text{ GeV}
\]

Interaction potential (II)

Instanton-induced forces

- Suggestion
  - Attractive in the scalar channel
  - Repulsive (same magnitude) in the pseudoscalar one


- Negative D-constant

Spin-effects

- Neglected in first approximation
Interaction potential (III)

Adjoint string
- Funnel potential
- Creation of two gluelumps at too large distance

Glueball mass spectrum
Two gluons (I)

Mass spectrum

$M_{g\bar{g}}$ (GeV)

$0^{++}$  $0^{-+}$  $2^{++}$  $2^{-+}$  $3^{++}$  $4^{++}$  $6^{++}$

$f_0(1710)$  $f_2(2340)$
Two gluons (II)

Transverse gluons

- No light $J = 1$ state
- Expected number of states
- Good agreement
- Needed: relativistic kinematics

Longitudinal gluons

- Too many states
- Poor agreement

Three gluons (I)

**Color**

- $[[8, 8]^{8_S}]^{1_S}$, $C = -$, symmetric spin-space
  - Lightest states
  - Like three photons
  - Transverse: No light (pseudo)scalar state
    F.G. Fumi, L. Wolfenstein, Phys. Rev. 90, 498 (1953)

- $[[8, 8]^{8_A}]^{1_A}$, $C = +$, antisymmetric spin-space

**Problem: Wick's formalism**

- Not available yet for three-gluon glueballs
Three gluons (II)

Mass spectrum with spin-1 gluons

**Four gluons?**

A heavy $0^+$ state seen on the lattice

- Highly excited three-gluon state
- Low-lying four-gluon state
  - Proposal, color function $[[8, 8]^10, [8, 8]^{10}]^1$
  - Symmetry

Mass estimate, ok with lattice QCD

Many-boy helicity formalism needed
Large N limit

Strong coupling \[ \sigma = \frac{C_R}{N} \sigma_0 \]


- Invariant with N if R = Adjoint

One gluon exchange \[ \propto C_R \alpha_s \propto \frac{C_R}{N} \alpha_0 \]

- Invariant with N if R = Adjoint

Spectrum roughly invariant with N

- OK with recent lattice studies, up to SU(8)

Thermodynamics
Warming up

Increasing the temperature

$T = 0$
- « Usual QCD »
- Confinement
- Hadron gas

$T > T_c$
- QGP
- Deconfinement
- Quark – gluon gas

$T = T_c$
- Phase transition

Pure Yang-Mills similar to QCD
Equation of state

Results from the lattice

G. Boyd et al., PRL 75, 4169 (1995)
M. Panero, PRL 103, 232001 (2009)

Trace anomaly $\sim e^{-3p}$

Phase transition, « weakly first order »
Quasiparticle models

Well above $T_c$
- Ideal gas of deconfined gluons
  - Thermal masses from perturbation theory
  - Scaling in $(N^2 - 1)$ as expected

Around $T_c$
- Strongly interacting gas of deconfined gluons
  - Maybe presence of glueballs
  - Not fully understood

Below $T_c$
- Glueball gas
  - Not studied a lot
Simple glueball gas

Basic model: Ideal Bose gas

- Input, lattice spectrum + $T_c = 300$ MeV
Hagedorn spectrum (I)

Pressure underestimated
- Glueball pressure suppressed $\propto (2J+1) e^{-m_G/T}$
- Negligible contribution of high-lying states

String picture of glueballs
- String theory predicts a Hagedorn spectrum: $e^{+m_G/T}$
- Degeneracy growing like $e^{+m_G/T}$
- Relevant contribution of high-lying states
- Might be suggested by experimental data (mesons and baryons)

Hagedorn spectrum (II)

Agreement with lattice data

H. B. Meyer, PRD 80, 051502(R) (2009)

Entropy of the confined phase ($N_c=3$, $N_f=0$)

\[ \frac{T_h}{T_c} = 1.024 \]
Conclusions
Summary (I)

Glueballs: « QCD only »
- Pure gauge bound states

Lattice: various data available
- Mass spectrum in different cases
- Wave function
- Thermodynamics

Constituent models
- Successfull for mesons and baryons
- Mass spectrum partly agrees with lattice data
  - Standard Hamiltonian
  - Transverse gluons with relativistic kinematics
- Glueball gas for gluonic matter below Tc
Outlook

Three-gluon bound states
- Need to deal with helicity states for three identical transverse bodies
- C = - sector in lattice still not understood

Experimental candidates
- Possibly seen in $f_0$ and $f_2$ resonances
- Probably not pure glue state
- Issue : « unquenching » the existing models
  - Much remains to be done
Very last slide

Lattice
- More fundamental
- Gives « all »
- Numerical

Constituent models
- Less fundamental
- Capture essential features
- Intuitive / analytical

Complete each other