The Cosmological Constant Problem
(and its sequester)

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The Cosmological Constant Problem

General Covariance & Equivalence Principle $\Rightarrow$ Vacuum Energy Gravitates

$$-V_{\text{vac}} \int \sqrt{-g} d^4 x \implies T_{\mu\nu} = -V_{\text{vac}} g_{\mu\nu}$$
The Cosmological Constant Problem

General Covariance & Equivalence Principle $\Rightarrow$ Vacuum Energy Gravitates

$$-V_{vac} \int \sqrt{-g} d^4 x \quad \Longrightarrow \quad T_{\mu\nu} = -V_{vac} g_{\mu\nu}$$

...add a bare cosmological constant...

$$-(V_{vac} + \Lambda_{bare}) \int \sqrt{-g} d^4 x \quad \Longrightarrow \quad T_{\mu\nu} = -\Lambda_{tot} g_{\mu\nu}$$

where $\Lambda_{tot} = V_{vac} + \Lambda_{bare} \lesssim (m eV)^4$
Estimating the vacuum energy

\[ V_{\text{vac}} \supset \sum_m \int d^3 k \frac{1}{2} \hbar \sqrt{k^2 + m^2} \]

\[ \sim c_\nu m_\nu^4 + c_e m_e^4 + c_\mu m_\mu^4 + \ldots + M_{\text{cut-off}}^4 \]

\[ = \quad \text{vacuum loops involving virtual gravitons} \]
\[ + \text{counterterms} \]
<table>
<thead>
<tr>
<th>Event Type</th>
<th>Scale (Units)</th>
<th>Fine Tuning</th>
</tr>
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<tr>
<td>Quantum Gravity cut-off</td>
<td>$(10^{18} \text{ GeV})^4$</td>
<td>fine tuning to 120 decimal places</td>
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<tr>
<td>SUSY cut-off</td>
<td>$(T \text{eV})^4$</td>
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<td>Standard Model phase</td>
<td>$(200 \text{ GeV})^4$</td>
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<td>QCD phase transition</td>
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<td>Muon</td>
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<td>Electron</td>
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<tr>
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<td>observed value</td>
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</table>
Contrast to electron mass

electron mass, $\Delta m \sim m \log(M_{\text{cutoff}}/m)$

protected by chiral symmetry in massless limit
Electron mass & vacuum are both UV sensitive — cannot be predicted in EFT, must be measured!

Electron mass is only mildly sensitive to unknown UV

Vacuum energy is extremely sensitive to unknown UV

naturalness ensures that low energy EFTs agree on low energy couplings.
How can we make the cosmological constant radiatively stable?

Within particle physics, SUSY would do the job, but not in a way that is compatible with pheno.

Look to gravity: perhaps the radiative corrections are there, but they simply don’t gravitate.
Global Vacuum Energy Sequester
Introduce **global** dynamical variables: $\Lambda, \lambda$

$$S = \int d^4 x \sqrt{g} \left[ \frac{M_{Pl}^2}{2} R - \Lambda - \mathcal{L}_{m}^{(g^2 \lambda, \Phi)} \Phi \right] + \sigma \left( \frac{\Lambda}{\lambda^4 \mu^4} \right)$$

$\Lambda$ is the CC counterterm  
$\lambda$ sets hierarchy between matter scales & Planck mass
Equations of motion

Λ equation : \( \frac{\sigma'}{\lambda^4 \mu^4} = \int d^4 x \sqrt{g} \)

λ equation : \( 4\Lambda \frac{\sigma'}{\lambda^4 \mu^4} = \int d^4 x \sqrt{g} \lambda^4 \tilde{T}^\mu_\mu \)

g_{\mu\nu} equation : \( M^2_{pl} G^\mu_\nu = -\Lambda \delta^\mu_\nu + \lambda^4 \tilde{T}^\mu_\nu \)

\( \tilde{T}_{\mu\nu} = \lambda^4 \tilde{T}_{\mu\nu} \)
Equations of motion

$\Lambda$ equation: $\Lambda = \frac{1}{4} \langle T^\alpha_{\alpha} \rangle$

$\lambda$ equation: $\langle Q \rangle = \frac{\int d^4 x Q \sqrt{g}}{\int d^4 x \sqrt{g}}$

$g_{\mu\nu}$ equation: $M_{pl}^2 G_{\mu\nu} = -\Lambda \delta_{\mu\nu} + T_{\mu\nu}$
\[ M_{pl}^2 G_{\mu \nu} = T_{\mu \nu} - \frac{1}{4} \delta_{\mu \nu} \langle T^{\alpha \alpha} \rangle \]

\[ T_{\nu}^{\mu} = -V_{\nu \text{vac}} \delta_{\nu}^{\mu} + \tau_{\nu}^{\mu} \]
\[ M_{pl}^2 G^{\mu \nu} = \tau^{\mu \nu} - \frac{1}{4} \delta^{\mu \nu} \langle \tau^{\alpha \alpha} \rangle \]

\[ T^\mu_{\nu} = -V_{vac} \delta^\mu_{\nu} + \tau^\mu_{\nu} \]
\[ M_{pl}^2 G_{\mu \nu} = \tau_{\mu \nu} - \frac{1}{4} \delta_{\mu \nu} \langle \tau^{\alpha \alpha} \rangle \]

Vacuum energy drops out at each and every loop order

No hidden equations — this is everything!

Residual CC is radiatively stable, value should be measured
Symmetries?

Approximate scaling
\[ \delta_\epsilon \lambda = \epsilon \lambda, \quad \delta_\epsilon \Lambda = 4 \epsilon \Lambda, \quad \delta_\epsilon \left( \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_{\text{pl}}} \right) = -2 \epsilon \eta_{\mu\nu} - \epsilon \frac{h_{\mu\nu}}{M_{\text{pl}}} \]

\[ \delta_\epsilon S = O \left( \frac{1}{M_{\text{pl}}} \right) \]

Approximate shift
\[ \delta_\alpha \Lambda = \alpha m^4 \lambda^4, \quad \delta_\alpha \mathcal{L}_m = \alpha m^4 \]

\[ \delta_\alpha S = \alpha \frac{m^4}{\mu^4} \sigma' \]

\[ \frac{\Lambda_{\text{eff}}}{M_{\text{pl}}^2} \sim \frac{1}{4} \frac{\langle \tau \rangle}{M_{\text{pl}}^2} \]

0 as \( M_{\text{pl}} \to \infty \)

0 as \( \mu \to \infty \) (for fixed volume) - conformal limit, vanishing \( \Lambda \)
An Etude on Global Vacuum Energy Sequester
Guido D'Amico, Nemanja Kaloper, Antonio Padilla, David Stefanyszyn, Alexander Westphal, George Zahariade

e-Print: arXiv:1705.08950
Tseytlin’s original idea ....

$$S_T = \frac{\int d^4 x \sqrt{g} \left[ \frac{M_{Pl}^2}{2} R - \mathcal{L}(g^{\mu\nu}, \Phi) \right]}{\left[ \mu^4 \int d^4 x \sqrt{g} \right]}.$$}

$$\mathcal{L}(g^{\mu\nu}, \Phi) = V_{vac} + \ldots$$

*higher dim operators*
In QFT, powers of $\hbar$ (generically) count loops

$$S_{\text{eff}} = \frac{S_0}{\hbar} + S_1 + \hbar S_2 + \hbar^2 S_3 + \ldots$$

Tseytlin introduces an effective $\hbar$ proportional to spacetime volume, $\Omega = \mu^4 \int d^4x \sqrt{g}$, so:

$$S^T_{\text{eff}} = \frac{S^T_0}{\Omega} + S^T_1 + \Omega S^T_2 + \Omega^2 S^T_3 + \ldots$$

vacuum energy loops not cancelled
VES fixes this by exploiting universality of matter coupling, eg

\[ \Omega^{-1} \int d^4 x \sqrt{g} \left[ \frac{M_{Pl}^2}{2} R - \mathcal{L}(g^{\mu \nu}, \Phi) \right] \]

\[ \downarrow \]

\[ \int d^4 x \sqrt{g} \left[ \frac{M_{Pl}^2}{2} R - \Omega^{-1} \mathcal{L}(\Omega^{\frac{1}{2}} g^{\mu \nu}, \Phi) \right] \]

now all matter loop corrections scale as \( \Omega^{-1} \)
Carroll & Remmen’s recent idea ....

\[ S_{CR} = \eta \int d^4 x \sqrt{\tilde{g}} \left[ \frac{M_{Pl}^2}{2} \tilde{R} - \mathcal{L}(\tilde{g}^{\mu\nu}, \Phi) - \frac{1}{48} \tilde{F}_{\mu\nu\lambda\sigma}^2 + \frac{1}{6} \tilde{\nabla}_\mu (\tilde{F}_{\mu\nu\lambda\sigma} \tilde{A}_{\nu\lambda\sigma}) \right] \]

**Global** variable \( \eta \) is originates as magnetic dual of a 4 form field, \( H \)

**Dual of the 4 form** \( F \) acts like the CC counterterm

**Claim:** Global constraint from \( \eta \) ... forces action to vanish ... forcing CC counterterm to cancel radiative corrections
\[ S_{CR} = \eta \int d^4x \sqrt{\tilde{g}} \left[ \frac{M_{Pl}^2}{2} \tilde{R} - \mathcal{L}(\tilde{g}^{\mu\nu}, \Phi) - \frac{1}{48} \tilde{F}_{\mu\nu\lambda\sigma}^2 + \frac{1}{6} \tilde{\nabla}_\mu (\tilde{F}_{\mu\nu\lambda\sigma} \tilde{A}_{\nu\lambda\sigma}) \right] \]

**CC counterterm cannot adjust if**

- it is an integration constant, i.e. fixed by boundary condition
- it stems from a quantised 4 form (in presence of membrane sources)

**OK, so assume CC counterterm can adjust i.e. not fixed by BCs, there are no membrane sources**

**Are we good then?**

No!!

- Effective hbar is proportional to \( \eta \), so loops will spoil the constraint
Can fix this theory by moving $\eta$

\[
\eta \int d^4 x \sqrt{\tilde{g}} \left[ \frac{M_{Pl}^2}{2} \tilde{R} - \mathcal{L}(\tilde{g}^{\mu\nu}, \Phi) - \frac{1}{48} \tilde{F}_{\mu\nu\lambda\sigma}^2 + \frac{1}{6} \tilde{\nabla}_{\mu}(\tilde{F}_{\mu\nu\lambda\sigma} \tilde{A}_{\nu\lambda\sigma}) \right]
\]

Improved version....

- places global constraint on geometry (as opposed to geometry + radiatively unstable matter)
- is just a **hybrid** of global VES and local VES (see later)
Global VES in “Jordan frame”

\[ S = \int d^4 x \sqrt{g} \left[ \frac{M_{Pl}^2}{2} R - \Lambda - \lambda^4 \mathcal{L}_m (\lambda^{-2} g^{\mu\nu}, \Phi) \right] + \sigma \left( \frac{\Lambda}{\lambda^4 \mu^4} \right) \]

Vary over metric and constants \( \kappa \) and \( \Lambda \)

\( \kappa \) variation yields a global constraint on \( R \), \( \Lambda \) adjusts accordingly
Local Vacuum Energy Sequester
Why bother?

Consistency with QM requires action to be additive

\[ S_{AC} = S_{AB} + S_{BC} \]

\[ \mathcal{A}_{A \rightarrow B} = \langle B, t_B | A, t_A \rangle = \int dx_1 \ldots dx_{N-1} \langle B, t_B | x_{N-1}, t_{N-1} \rangle \langle x_{N-1}, t_{N-1} | x_{N-2}, t_{N-2} \rangle \ldots \langle x_1, t_1 | A, t_A \rangle \\
= \int dx_1 \ldots dx_{N-1} e^{\frac{i}{\hbar} \sum t_i \delta t} = \int Dx e^{\frac{i}{\hbar} S_{AB}[x]} \]
Hint: UMG a la Henneaux & Teitelboim

\[ S_{UMG} = \int d^4x \sqrt{g} \left[ \frac{M_{pl}^2}{2} R - \mathcal{L}_m(g^{\mu\nu}, \Phi) \right] - \int d^4x \Lambda(x)(\sqrt{g} - 1) \]

non-gravitating but breaks diffs

\[ S_{HT} = \int d^4x \sqrt{g} \left[ \frac{M_{pl}^2}{2} R - \mathcal{L}_m(g^{\mu\nu}, \Phi) \right] - \int \Lambda(x)(\sqrt{g} d^4x - F_4) \]

alternative measure: the 4 form

retains diffs

does not gravitate

exact 4 form \( F_4 = dA_3 \) forces constant \( \Lambda \)
Local VES

\[ S = \int d^4 x \sqrt{g} \left[ \frac{\kappa^2(x)}{2} R - \Lambda(x) - \mathcal{L}_m(g^{\mu\nu}, \Phi) \right] + \int \sigma \left( \frac{\Lambda(x)}{\mu^4} \right) F_4 + \hat{\sigma} \left( \frac{\kappa^2(x)}{M_{Pl}^2} \right) \hat{F}_4. \]
\[
\kappa^2 G^\mu_\nu = (\nabla^\mu \nabla_\nu - \delta^\mu_\nu \nabla^2) \kappa^2 + T^\mu_\nu - \Lambda(x) \delta^\mu_\nu
\]
\[
\frac{\sigma'}{\mu^4} F_4 = \sqrt{g} d^4 x, \quad \frac{\hat{\sigma}'}{M_{Pl}^2} \hat{F}_4 = -\frac{1}{2} R \sqrt{g} d^4 x, \\
\frac{\sigma'}{\mu^4} \partial_\mu \Lambda = 0, \quad \frac{\hat{\sigma}'}{M_{Pl}^2} \partial_\mu \kappa^2 = 0.
\]

\[\implies \quad \kappa^2 G^\mu_\nu = \left(T^\mu_\nu - \frac{1}{4} \delta^\mu_\nu \langle T^\alpha_\alpha \rangle \right) - \Delta \Lambda \delta^\mu_\nu
\]

constant

radiatively stable

\[
\Delta \Lambda = -\frac{\mu^4}{2} \frac{\kappa^2 \hat{\sigma}'}{M_{Pl}^2 \sigma'} \int \frac{\hat{F}_4}{F_4}
\]
Is \( \Delta \Lambda = -\frac{\mu^4}{2} \frac{\kappa^2 \hat{\sigma}'}{M_{Pl}^2 \sigma'} \int \hat{F}_4 \) radiatively stable?

Effect of vacuum loops:

\[
\Lambda \rightarrow \Lambda + M_{UV}^4 \quad \Rightarrow \quad \sigma' \rightarrow \mathcal{O}(1) \sigma' \\
\kappa^2 \rightarrow \kappa^2 + M_{UV}^2 \quad \Rightarrow \quad \hat{\sigma}' \rightarrow \mathcal{O}(1) \hat{\sigma}'
\]

\[
\int F_4, \quad \int \hat{F}_4 \quad \text{geometric, IR quantities, not UV sensitive}
\]

\[
\Rightarrow \quad \Delta \Lambda \rightarrow \mathcal{O}(1) \Delta \Lambda
\]

smooth functions

\( \mu > M_{UV} \)

\( M_{pl} > M_{UV} \)
Global trace equations

\[ \sigma' \left\langle \ast F_4 \right\rangle = 1 \]

\[ \frac{\kappa^2 \sigma'}{\mu^4} \left\langle \ast \hat{F}_4 \right\rangle = -\frac{1}{4} \kappa^2 \left\langle R \right\rangle \]

\[ 4\Lambda + 4V_{vac} = \left\langle \tau^\alpha_{\alpha} \right\rangle + \kappa^2 \left\langle R \right\rangle \]

GR fixed by assumption

Sequester fixed by ratio of fluxes
Key points

Λ is sink for vacuum energy, channelled there by non-gravitating 4 forms

Works to any order in matter loops — gravity loops on the other hand…..

Equivalence Principle violated GLOBALLY — local theory is GR, Weinberg no go evaded

Residual CC is radiatively stable— like any relevant coupling, should be measured.
And finally, an aside on self-tuning …
How can we make the cosmological constant radiatively stable?

Within particle physics, SUSY would do the job, but not in a way that is compatible with pheno.

Look to gravity: perhaps the radiative corrections are there, but they simply don’t gravitate.
Self-tuning:

“admits Minkowski solution for any value of vacuum energy”

Weinberg makes assumptions e.g.
- local 4D effective theory
- All fields are Poincare invariant

Relax these assumptions and self-tuning can be possible e.g.
- branes in 6D
- Fab Four
- VES

But
- 6D models cannot recover 4D phenomenology without spoiling the self-tuning
- Fab Four has a light scalar, bad for pheno.
- many other models run into problems with ghosts, singularities etc
General approach to seek out consistent field theoretic completions of self-tuning

- use standard Kallen Lehmann spectral representation to describe generic exchange amplitudes
- impose unitarity and Lorentz invariance
- require self-tuning of long wavelength sources
- require closeness to GR for short wavelength sources.
The AdS loop hole

Result generalises to dS but not AdS … can find explicit examples of AdS self-tuning that tick every box.

The VES loop hole

VES does not admit a standard KL spectral representation in terms of canonical free-field propagators…it decapitates!
Decapitation

\[
\frac{-i}{k^2 + i\epsilon} \rightarrow \frac{-i(1 + F(k))}{k^2 + i\epsilon}
\]

\[
F(k) = \begin{cases} 
0 & k^{\mu} \neq 0 \\
-1 & k^{\mu} = 0
\end{cases}
\]

\[
\text{divergence} + \text{counterterm} = 0
\]

\[
\text{divergence} + \text{guillotine} = 0
\]
GR

single massless graviton

VES

massless graviton

massless scalar

Fab Four

massless graviton

massless scalar
Vacuum energy sequestering

is a new mechanism through which loop corrections to vacuum energy can be rendered gravitationally harmless

is an effective field theory and, in keeping with standard ideas behind renormalisation, makes no prediction for precise value of the CC, rather

— it should be measured

— it’s value is only mildly sensitive to the details of the unknown UV

...... just like the mass of electron.

a new way to tackle problems of naturalness

LOTS MORE EXCITING DEVELOPMENTS ON THE WAY. STAY TUNED!
Back up slides
What about graviton loops?

These introduce new $\kappa$ dependence in renormalised 1PI effective potential

\[ \sim \frac{M^6_{\text{cutoff}}}{\kappa^2} \]

\[ \sim \frac{M^8_{\text{cutoff}}}{\kappa^4} \]

Screws up the $\kappa$ EoM - no longer able to constrain $R$ at large wavelength with four form fluxes
\[ S = \int d^4x \sqrt{g} \left[ \frac{\kappa^2(x)}{22} R - R A A(x_m) (g^{\mu} D^\nu m \Phi) \right] + \int d^4x \left[ \left( \frac{\Lambda(x)}{\mu^4} \right)^4 c^4 \right] F_4 \left( + \frac{\kappa^2(x)}{\mu^2} \frac{\kappa^2(x)}{M^2_{Pl}} \right) \right] \hat{F}_4. \]

\[ - \left[ a_0 M^4 + a_1 \frac{M^6}{\kappa^2} + a_2 \frac{M^8}{\kappa^4} + \ldots \right] \int \sqrt{g} d^4x \]

Variation wrt to rigid \( \kappa \) protects ROTHING at long wavelength.
Two key insights

to avoid undesirable corrections to the effective potential, need an unbroken shift symmetry

any curvature invariant that is NOT scale invariant can be used to constrain R at large wavelength

\[ \int d^4 x \sqrt{-g} \theta(x) \left( R_{\mu\nu}^2 - 4R_{\mu\nu}^2 + R^2 \right) \]
\[ S = \int d^4x \left\{ \sqrt{g} \left[ \frac{M_{Pl}^2}{2} R + \theta(x) R_{GB} - \Lambda(x) - \mathcal{L}_m \right] + \frac{\epsilon_{\mu\nu\lambda\sigma}}{4!} \left[ \sigma \left( \frac{\Lambda}{\mu^4} \right) F_{\mu\nu\lambda\sigma} + \hat{\sigma} \left( \theta \right) \hat{F}_{\mu\nu\lambda\sigma} \right] \right\}. \]

\[ M_{Pl}^2 G^{\mu\nu} = T^{\mu\nu} - \frac{1}{4} \delta^{\mu\nu} \langle T^\alpha_\alpha \rangle - \Delta \Lambda \delta^{\mu\nu}, \]

\[ \Delta \Lambda^2 = \frac{3 M_{Pl}^4}{8} \left[ \langle R_{GB} \rangle - \langle W_{\mu\nu\alpha\beta}^2 \rangle + \frac{2}{M_{Pl}^4} \langle (T_{\mu\nu} - \frac{1}{4} T g_{\mu\nu})^2 \rangle - \frac{1}{6M_{Pl}^4} \left( \langle T^2 \rangle - \langle T \rangle^2 \right) \right]. \]

- Gauss-Bonnet integral anchored in place by four forms
- Weyl tensor
- Vacuum energy drops out

\[ T^\mu_\nu = -V_{vac} \delta^\mu_\nu + \tau^\mu_\nu \]

\[ \langle R_{GB} \rangle = -\mu^A \hat{\sigma}' \frac{\hat{F}_A}{\sigma'} \int \hat{F}_A. \]
How big is $\Lambda_{\text{eff}}$?

For standard matter, space-time integrals dominated by time when universe is largest

$$\int d^4x \sqrt{-g} \sim \frac{1}{H_{\text{age}}^4} \quad \text{where lifetime } t_{\text{age}} \sim \frac{1}{H_{\text{age}}} \gtrsim 13.7 \text{ Gyrs}$$

$$\langle \tau^\alpha_\alpha \rangle \sim \rho_{\text{age}} \sim \text{energy density at largest size} < \rho_c$$

$\Rightarrow \Lambda_{\text{eff}} \text{ is not dark energy ... too small!}$
Observational consequences?
Universe has finite spacetime volume

\[ \frac{\sigma'}{\lambda^4 \mu^4} = \int d^4 \sqrt{g} \]

space-time volume must be finite or else \( \lambda \to 0 \)

\[ \frac{m_{phys}}{M_{pl}} = \frac{\lambda m}{M_{pl}} \]

if \( \lambda \to 0 \) particle masses go to zero

**Ends in a crunch**

w=-1 is transient

\( \Omega_k > 0 \)

**circles in the sky?**

possible correlation between 1+w and \( \Omega_k \)
Assume translation invariant solution for ANY vacuum energy:

\[
S[\pi, g_{\mu\nu}] = \int d^4x \sqrt{-g} R + \Delta L(\pi, g_{\mu\nu}, \text{derivatives})
\]

On shell field eqns: \[
\left. \frac{\partial \Delta L}{\partial g_{\mu\nu}} \right|_{g, \pi = \text{const}} = 0, \quad \left. \frac{\partial \Delta L}{\partial \pi} \right|_{g, \pi = \text{const}} = 0
\]
Scalar eqn $\implies$ trace of gravity eqn

$$2g_{\mu \nu} \frac{\partial \Delta L}{\partial g_{\mu \nu}} - f(\pi) \frac{\partial \Delta L}{\partial \pi} \equiv 0$$

If $g_{\mu \nu}$ and $\pi$ are constant then $\Delta L$ is invariant under

$$\delta g_{\mu \nu} = 2\epsilon g_{\mu \nu}, \ \delta \phi = -\epsilon$$

where we define $\phi = \int \frac{d\pi}{f(\pi)}$

Then $\Delta L = \sqrt{-\hat{g}}\mathcal{L}(\hat{g}_{\mu \nu}, \text{derivatives})$ where $\hat{g}_{\mu \nu} = e^{2\phi} g_{\mu \nu}$
\[
\implies \frac{\partial \Delta L}{\partial g_{\mu\nu}} \bigg|_{g,\pi=\text{const}} = \frac{1}{2} g^{\mu\nu} \Delta L \bigg|_{g,\pi=\text{const}}
\]

Recall \( \Delta L_{g,\pi=\text{const}} = -V_0 \sqrt{-\hat{g}} \)

So translation invariant EOMs give \( \Delta L = 0 \implies V_0 e^{4\phi} = 0 \)

\[
\implies V_0 = 0 \text{ or } e^{2\phi} \rightarrow 0
\]

Fine tuning Scale invariance
Phase transitions?
Sequestering works best in domains that dominate spacetime volume

\[ V_{\text{before}} \quad \downarrow \quad V_{\text{after}} \]

\[
\tau_{\mu \nu} - \frac{1}{4} \delta_{\mu \nu} \langle \tau^\alpha \alpha \rangle = \begin{cases} 
- \langle V_{\text{before}} - V \rangle \delta_{\mu \nu} & t < t_*, \\
- \langle V_{\text{after}} - V \rangle \delta_{\mu \nu} & t > t_*.
\end{cases}
\]

For an early transition….

\[
\langle V_{\text{after}} - V \rangle = -\Delta V \frac{\int_{t_{\text{fin}}}^{t_*} dt a^3}{\int_{t_{\text{in}}}^{t_{\text{fin}}} dt a^3} \ll \Delta V
\]

\[
\langle V_{\text{before}} - V \rangle \sim \mathcal{O}(1) \Delta V \quad \text{short burst of inflation in build up to a transition}
\]
CDL bubbles

Sequestering works best in near Minkowski vacua

tunnelling rates

from ds to Minkowski - generically enhanced compared wrt GR
from Minkowski to AdS — generically suppressed wrt GR
\[ S = \int d^4x \sqrt{g} \left[ \frac{\kappa_0^2}{2} R - \Lambda_0 - \mathcal{L}(g^{\mu\nu}, \Phi) \right] - \Lambda \int d^4x \sqrt{g} + \sigma_1 \left( \frac{\Lambda}{\mu^4} \right) S_1 + \theta S_g[g_{\mu\nu}] + \sigma_2(\theta) S_2 \]

1. A globally adjustable CC counterterm

2. A second global variable for constraining the global geometry

3. \( S_1 \) and \( S_2 \) should have no variation wrt SM fields or metric, \( g \)

4. \( S_1 \) should not vanish on shell (to avoid unphysical constraint on spacetime volume)

5. \( S_g \) chosen
   - so as not to screw up gravitational phenomenology
   - so as to yield a constraint on the scale dependent part of the geometry

\[ \text{e.g.:} \quad \int d^4y \sqrt{f} [\alpha R(f) + \beta] \]

\[ \text{e.g.: Einstein-Hilbert, Gauss-Bonnet} \]
Graviton loops?
Start with local VES

\[ S = \int d^4 x \sqrt{g} \left[ \frac{\kappa^2(x)}{2} R - \Lambda(x) - \mathcal{L}_m(g^{\mu\nu}, \Phi) \right] + \int \sigma \left( \frac{\Lambda(x)}{\mu^4} \right) F_4 + \hat{\sigma} \left( \frac{\kappa^2(x)}{M_{Pl}^2} \right) \hat{F}_4. \]

Integrate out \( \Lambda \) and \( \hat{A} \)

\[ S = \int d^4 x \sqrt{g} \left[ -\mu^4 \epsilon \left( \frac{\star F_4}{\mu^4} \right) + \frac{1}{2} \kappa^2 R - \mathcal{L}_m(g^{\mu\nu}, \Phi) \right] \]

where \( \kappa \) is now global and function \( \epsilon \) is related to Legendre transform of \( \sigma \)

\[ \int d^4 x \sqrt{g} \left[ \frac{M_{Pl}^2}{2} \eta \tilde{R} - \mathcal{L}(\tilde{g}^{\mu\nu}, \Phi) - \frac{1}{48} \tilde{F}^2_{\mu\nu\lambda\sigma} + \frac{1}{6} \tilde{\nabla}_{\mu}(\tilde{F}^{\mu\nu\lambda\sigma} \tilde{A}_{\nu\lambda\sigma}) \right] \]

compare with
The End
COLLAPSE TRIGGER = DARK ENERGY

Linear potential \( V = m^3 \phi \)

form protected by shift symmetry, size of \( m^3 \) technically natural

If \( \phi_{in} > M_{pl} \), then when scalar dominates, does so in SLOW ROLL until collapse time

\[
 t_{\text{collapse}} \sim \sqrt{\frac{M_{pl}}{m^3}}
\]
Radiatively stable choice of collapse time?

Yes, thanks to $m^3$

Radiatively stable choice of $\varphi_{in}$?

Yes, thanks to shift symmetry

But its not even a “choice”.... $\langle R \rangle = 0$ picks out precisely those solutions with $\varphi_{in} > M_{pl} !!!!!!
WHY NOW?

Because the end is nigh!!!

Why is it nigh?

Because the radiatively stable parameter
\( m^3 \sim M_{\text{pl}} H_0^2 \)

Prediction: \( 1+w \sim \Omega_k^2 \)