Perturbations of higher dimensional black holes

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A *singly-spinning* Myers-Perry black hole (i.e. one with $J_1 \neq 0$, other $J_i$ vanishing) has superficial resemblance to Kerr. *But* Kerr has upper bound on $J$ for given $M$. This is not true for MP with $d \geq 6$. Such black holes can have arbitrarily large $J$ for given $M$: these are called *ultraspinning* black holes.

In limit of large $J$, ultraspinning MP black hole resembles black *brane*, suggests instability (Emparan & Myers 03)

These arguments are heuristic. Demonstrating existence of an instability requires a study of linearized gravitational perturbations. Hard! But progress has been made in past 18 months.
Linearized gravitational perturbations: known results

- Schwarzschild solution is stable for all \( d > 4 \) (Ishibashi & Kodama 03)

- Singly spinning MP: time-independent mode appears at critical value of \( J \) (for given \( M \)). Checked for \( 6 \leq d \leq 11 \). Black hole expected to be unstable for larger \( J \). (Dias et al 09,10)

- Most symmetrical MP black hole has equal angular momenta \( (J_i = J) \) with odd \( d \). Solution is cohomogeneity-1. There is an upper bound on \( J \). Saturating this bound gives extreme black hole, as for Kerr. For \( d = 9 \), solutions close to extremality exhibit linearized perturbations growing exponentially with time. (Dias et al 10).
This talk.

So far, work on linearized perturbations has been limited to particular Myers-Perry black holes which exhibit *symmetry enhancement*. A general MP solution has $\mathbb{R} \times U(1)^N$ isometry group, $N = [(d - 1)/2]$. Singly spinning MP solutions, and equal angular momenta, odd $d$, solutions have much larger isometry groups. This makes the study of linearized perturbations tractable. How can we make progress with the general case?

In 4d, the tractability of the study of linearized gravitational perturbations of Kerr derives from the remarkable decoupling property discovered by Teukolsky (1972). We shall explore the extent to which this extends to $d > 4$ dimensions.
As discovered by Teukolsky, and emphasized by Stewart and Walker (1974), decoupling is closely related to the existence of gauge-invariant local variables describing metric perturbations.

Consider linearized perturbations of a 4d spacetime. We can separate any scalar $X$ into a background value $X^{(0)}$ and a perturbation $X^{(1)}$. There is gauge freedom corresponding to infinitesimal diffeomorphisms, under which $X^{(1)} \rightarrow X^{(1)} + \xi \cdot \partial X^{(0)}$. Hence $X^{(1)}$ is gauge-invariant iff $X^{(0)} = \text{const}$. 
Teukolsky’s approach employs the Newman-Penrose formalism, based on a null tetrad \( \{\ell, n, m, \bar{m}\} \). Let \( \Psi_A, A = 0, 1, 2, 3, 4 \) denote the NP scalars encoding the Weyl tensor. Now we have gauge freedom corresponding to infinitesimal tetrad rotations. However:

\( \Psi_0^{(1)} \) is invariant under infinitesimal coordinate transformations and infinitesimal tetrad rotations iff \( \Psi_0^{(0)} = \Psi_1^{(0)} = 0 \), i.e., iff the background spacetime is algebraically special.

Other quantities, e.g. \( \Psi_1^{(1)} \), are not gauge invariant.

Summary: for linearized metric perturbations of an algebraically special spacetime, there exists a local quantity, linear in the metric perturbation, that is invariant under infinitesimal diffeomorphisms and infinitesimal changes of basis.
Review of 4d results: decoupling

Gauge-invariant quantity $\Psi_0^{(1)}$ is a complex scalar: 2 physical degrees of freedom, just as for gravitational field. Plausible that it encodes all information about metric perturbation. Furthermore, one can decouple $\Psi_0^{(1)}$ from the other $\Psi_A^{(1)}$:

$\Psi_0^{(1)}$ satisfies a second order, linear, homogeneous, hyperbolic, partial differential equation: the Teukolsky equation.

This fact renders tractable the study of Kerr perturbations.

Does any of this extend to $d > 4$ dimensions?
Higher dimensions: null bases

Start by figuring out the $d > 4$ analogue of $\Psi_0$.

Introduce a null basis $\{\ell, n, m_i\}$, $i = 2, \ldots, d - 1$, where $\ell$, $n$ are null, $m_i$ are spacelike, and $\ell \cdot m_i = n \cdot m_i = 0$, $m_i \cdot m_j = \delta_{ij}$, $\ell \cdot n = 1$.

In 4d, $\Psi_0 \equiv C_{abcd} \ell^a m^b \ell^c m^d$. The $d > 4$ analogue is therefore

$$\Omega_{ij} = C_{abcd} \ell^a m_i^b \ell^c m_j^d$$

This is a $(d - 2) \times (d - 2)$ traceless, symmetric, matrix: same number of degrees of freedom as gravitational field.
Gauge invariance

Consider linearized perturbations, decompose quantities into background (e.g. \( \Omega_{ij}^{(0)} \)) and perturbation (\( \Omega_{ij}^{(1)} \)). Follow same argument as in 4d:

\( \Omega_{ij}^{(1)} \text{ is invariant under infinitesimal coordinate transformations and infinitesimal basis transformations iff the spacetime is algebraically special.} \)

The Petrov classification was extended to \( d \) dimensions by Coley et al (2004).

The Myers-Perry solutions are algebraically special (type D). Black rings are not algebraically special.
Summary so far

- $\Omega_{ij} = C_{abcd} \ell^a m^b_i \ell^c m^d_j$ is the $d$-dimensional analogue of $\Psi_0$
- $\Omega^{(1)}_{ij}$ has the same number of degrees of freedom as the gravitational field.
- $\Omega^{(1)}_{ij}$ is invariant under infinitesimal coordinate transformations and infinitesimal basis transformations iff the background spacetime is algebraically special.
- The Myers-Perry solutions are algebraically special.

Does $\Omega^{(1)}_{ij}$ satisfy a decoupled equation?
Decoupling in higher dimensions

$d = 4$: $\Omega_{ij}^{(1)} \sim \Psi_{0}^{(1)}$ satisfies a decoupled equation (in vacuum) iff $\ell$ is geodesic and shearfree. Guaranteed by Goldberg-Sachs theorem!

$d > 4$: $\Omega_{ij}^{(1)}$ decouples iff $\ell$ is geodesic with vanishing expansion, rotation and shear.

A spacetime admitting a null geodesic congruence with vanishing expansion, rotation and shear is called a *Kundt spacetime*. Any such spacetime is algebraically special.

Unfortunately, black hole spacetimes are not Kundt spacetimes.
Near-horizon geometries: motivation

The near-horizon geometry of an extreme black hole is a Kundt spacetime: can study gravitational perturbations using our decoupled equation.

Why is this interesting?

1. We will suggest that, under certain circumstances, an instability of a near-horizon geometry implies instability of the full extreme black hole.

Instability of an extreme black hole suggests that near-extreme black holes also will be unstable.

2. Can determine operator dimensions if there exists a dual CFT.
Near-horizon geometry

The near horizon geometry of any *known* extreme black hole solution has a universal form. With $n$ commuting rotational Killing vector fields, it is (Kunduri et al 07,08)

$$
\begin{align*}
\text{ds}^2 &= L(y)^2 \left( -R^2 \, dT^2 + \frac{dR^2}{R^2} \right) + g_{MN}(y)dy^M dy^N \\
&+ g_{IJ}(y) \left( d\phi^I - k^I R dt \right) \left( d\phi^J - k^J RdT \right)
\end{align*}
$$

$-dT \pm dR/R^2$ both are tangent to null geodesics with vanishing expansion, rotation and shear: this spacetime is ”doubly Kundt”. 

”Kaluza-Klein” reduction to $AdS_2$ on compact space $\mathcal{H}$ (horizon cross-section). KK gauge fields $-k^I RdT$: homogeneous electric fields.
KK reduction

Separate variables:

$$\Omega_{ij}(T, R, \phi, y) = \text{Re} (\chi(T, R)Y_{ij}(\phi, y))$$

$Y_{ij}$: tensor eigenfunction of a complicated operator $\mathcal{O}$ on $\mathcal{H}$

$$\mathcal{O}Y = \lambda Y, \quad Y_{ij} \propto e^{im_l \phi^l}$$

$\chi$: scalar field in $AdS_2$ with homogeneous electric field. Charge $q$, mass $\mu$:

$$q = m_l k^l + 2i, \quad \mu^2 - q^2 = \lambda$$

Can define inner product on $\mathcal{H}$ so that $\mathcal{O}$ self-adjoint hence $\lambda$ real.

"Breitenlohner-Freedman" stability bound $\mu^2 - q^2 \geq -1/4$. If violated then we say the near-horizon geometry is unstable.
Perturbations of near-horizon geometries

Proceed phenomenologically: consider perturbations of near-horizon geometries and see how results correlate with stability properties of full black hole.

Start with near-horizon extreme Kerr (NHEK) geometry (Bardeen & Horowitz 99). $\mathcal{H} = S^2$ (inhomogeneous). Gravitational perturbations studied by Amsel et al, Dias et al, 09.

Eigenvectors of $\mathcal{O}$ labelled by integers $(l, m)$, $|m| \leq l$, $l \geq 2$.

$m = 0$ modes (axisymmetric) respect effective BF bound.

Modes with $|m| \sim l$ violate effective BF bound: unstable.

But Kerr is (believed to be) stable!

Our proposal: instability of near-horizon geometry implies instability of full black hole only for modes preserving certain symmetry. In the case of Kerr, the symmetry is axisymmetry.
Near-horizon Myers-Perry stability

Myers-Perry with equal angular momenta in odd $d$ is cohomogeneity-1. Near-horizon geometry is homogeneous, with $\mathcal{H} = S^{d-2}$ (squashed).

Write Killing field tangent to horizon generators as $k + \Omega_H m$.

Certain modes that violate symmetry generated by $m$ violate effective BF bound for any $d$: analogous to non-axisymmetric unstable modes in NHEK. We claim that this tells us nothing about stability of full black hole.

Modes that respect symmetry generated by $m$ respect BF bound for $d = 5$.

A few modes that respect symmetry generated by $m$ violate BF bound for $d \geq 7$. We predict that these correspond to instability of full extreme black hole.
Comparison with known results

Previous studies of perturbations of cohomogeneity-1 Myers-Perry:

\( d = 5 \): Murata & Soda (2008) found no evidence of any instability.

\( d > 5 \): Dias \textit{et al} (2010) found instability for \( d = 9 \), conjectured to exist also for \( d > 9 \). Instability respects symmetry generated by \( m \).

Appears to be a discrepancy between these results and prediction from near-horizon geometry.

J. Santos has repeated the analysis of Dias \textit{et al} for black holes very close to extremality. He finds instability for \( d = 7, 9, 11, 13, 15 \). In all cases, the modes that we predict to be unstable do indeed turn out to be unstable.
Why does this work?

Consider a scalar field in a stationary rotating black hole background. Impose symmetry conditions on the scalar to reduce its eq of motion to the form

$$-\frac{\partial^2 \Phi}{\partial t^2} = \mathcal{A}\Phi$$

where $\mathcal{A}$ involves only spatial derivatives. Introduce scalar product $(,)$ so that $\mathcal{A}$ self-adjoint. Let $\lambda_0$ denote lowest eigenvalue of $\mathcal{A}$. Then, if $\lambda_0 < 0$, $\Phi$ is unstable, growing as $\exp(\sqrt{-\lambda_0} t)$. 
\( \lambda_0 \) can be estimated using the Rayleigh-Ritz formula:

\[
\lambda_0 \leq \frac{(\psi, A\psi)}{\langle \psi, \psi \rangle}
\]

where \( \psi \) is any trial function.

For an extreme black hole, if the scalar field violates the BF bound in the near-horizon geometry, then can find a trial function \( \psi \) such that RHS above is negative hence full black hole is unstable.

Key step of argument was to impose symmetry condition on scalar field to eliminate first time derivatives. In 4d, this is just axisymmetry, for cohomogeneity-1 it is invariance under symmetry generated by \( m \).

Variational formula may give easy way of demonstrating instability of non-extreme black holes.
Gravitational perturbations

How do we extend this argument to gravitational perturbations? Need to impose symmetry conditions on perturbation to bring linearized Einstein eq to form

$$-\frac{\partial^2 \Phi_\alpha}{\partial t^2} = A_{\alpha\beta} \Phi_\beta$$

where $\Phi_\alpha$ encodes perturbation. Then need to find scalar product to make $A$ self-adjoint. Can this be done?

Kerr: yes, for axisymmetric perturbations (Friedman & Schutz, Chandrasekhar).

Cohomogeneity-1 or singly-spinning MP: probably, for $m$-invariant modes. Why else would unstable modes have purely imaginary $\omega$?
Guica et al 08 proposed that NHEK is dual to a CFT.

We’ve reduced everything to scalar field in $\text{AdS}_2$. What happens if we apply usual AdS/CFT rules to determine dimensions of operators dual to gravitational perturbations?

NHEK: non-axisymmetric modes violating effective BF bound $\rightarrow$ complex $\Delta$. Problem?

$m = 0$ (axisymmetric) modes $\rightarrow \Delta = l + 1, \ l = 2, 3, \ldots$ integers!

5d extreme Cohomogeneity-1 MP: operators dual to $m$-invariant perturbations also have integer $\Delta$. (Not true for $d > 5$ but then BH unstable).

What is protecting dimensions of these operators?
Summary

- Many higher-dimensional black holes are unstable if they rotate sufficiently rapidly.
- There exist a gauge-invariant quantity for describing perturbations of algebraically special spacetimes, e.g., Myers-Perry black holes.
- This quantity satisfies a decoupled equation only in a Kundt background.
- This decoupled equation can be used to study gravitational perturbations of near-horizon geometries of extreme black holes: much easier than studying full black hole!
- It appears that, if certain symmetries are respected, then instability of a near-horizon geometry implies instability of full black hole.
- Dual CFT has infinite families of operators with protected dimensions.