Black Holes, Fluids, and their Instabilities

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“Black Holes are like Fluids”
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What does this mean?
What is it good for?
“Black Holes are like Fluids”
What does this mean?

• Some of the dynamics of a black hole spacetime takes the form of the equations for some kind of fluid system
“Black Holes are like Fluids”
What does this mean?

• Some of the dynamics of a black hole spacetime takes the form of the equations for some kind of fluid system
  – What part of the dynamics?
    • All of the spacetime? Only the region near/inside the horizon? Only long wavelength fluctuations?
  – What kind of fluid?
    • Relativistic/Non-relativistic
    • Incompressible/Barotropic/Conformal...
    • With/without a surface boundary
“Black Holes are like Fluids”
What is it good for?

- Einstein eqs: **Coupled non-linear PDEs**
- Navier-Stokes: **Coupled non-linear PDEs**
“Black Holes are like Fluids”
What is it good for?

- Einstein eqs: Coupled non-linear PDEs
- Navier-Stokes: Coupled non-linear PDEs
- But typically, in the mapping the fluid lives in fewer dimensions than the bh – “holographic reduction”
- Lots of theoretical/numerical/exptl work on fluids
- Universal results for black holes
“Black Holes are like Fluids”
What is it good for?

This talk:

• BH perturbations and stability analyses are very complicated
• Fluid description may simplify the system
• Case study: Gregory-Laflamme-type instabilities
Gregory-Laflamme instability

\[ \lambda = \frac{2\pi}{k} \gtrsim r_0 \]

\[ \delta r_0 \sim e^{\Omega t + ikz} \]

Linearized perturbation analysis

\[ D = n + 4 \]

(plot data: Figueras)
Fluid dynamics

• *Effective theory* for fluctuations away from thermodynamic equilibrium, whose frequency $\omega \to 0$ as wavelength $\lambda \to \infty$

• Long-wavelength expansion, relative to
  – molecular mean free path: theory comes with a scale
  – thermal wavelength: scale set by state

• Fluid eqs: $\nabla_\mu T^{\mu\nu}=0$
  
  ($T^{\mu\nu}$ from constitutive relations)
General Relativity as Fluid dyn?

- Classical vacuum theory \((R_{\mu\nu}=0)\) is scale-invariant
- Black holes set a length scale \(r_H \sim GM\)
- It’d seem (naively) that a fluid description should capture only fluctuations with \(\lambda \gg r_H\) : can’t fit!
- Instead, theory may have a scale, eg, a cosmo constant. Then fluid may capture \(\lambda \gg L_{\text{cosmo}}\)
Membrane paradigm

• An outside observer interacting with a black hole (perturbing it) can replace it with a membrane endowed with sources of stress-energy (viscous fluid), charge (resistive) current etc

• Fluid lives on (stretched) horizon

Damour Thorne et al
Bredberg et al
Membrane paradigm

• Fluid stress-tensor: quasilocal Brown-York

$$8\pi G T_{\mu \nu} = K (h_{\mu \nu} - K_{\mu \nu})$$

• Captures dynamics of horizon (+inside), and its coupling with outside -- not full spacetime
Membrane paradigm

• Fluid dyn obtains (exactly!) because of huge redshift near horizon:
  – acceleration-length $a^{-1} \to 0$
  – local temperature for fidos $\to \infty$

• Near-horizon is Rindler: universal, erases differences among distinct black holes (eg black holes vs black branes): good/bad

• Not good for studying bh instabilities
BHs as fluid droplets/lumps?

• Analogy: $\sim$

• Fluid drop with surface boundary tends to extremize (minimize) surface area
• Surface is elastic
• Fluid drops evaporate when microphysics included, evaporation rate greater for smaller drops
BHs as fluid droplets/lumps?

• Rayleigh-Plateau / Gregory-Laflamme

Cardoso+Dias
BHs as fluid droplets/lumps?

- Rayleigh-Plateau vs Gregory-Laflamme

Cardoso+Dias
BHs as fluid droplets/lumps?

• Highly suggestive! But...
BHs as fluid droplets/lumps?

- Highly suggestive! But...
- Problematic for vacuum black holes/branes
- Fluid drops have an intrinsic scale: can tell large and small drops
- Vacuum bhs (eg Schw) don’t have any intrinsic scale: all equal
- We’re trying to relate systems with different numbers of parameters!
BHBs as fluid droplets/lumps?

Possible fixes:

- Consider not vacuum bhs, but bhs in AdS
  - Can tell large and small bhs
  - Large black holes can fit long-wavelength fluctuations
- Small parameter \(1/N\) (number of dimensions): large \(N\) expansion
  - large-N bhs are large

Caldarelli+Dias+RE+Klemm
Miyamoto+Maeda
BHs as fluid drops/lumps?

- Large BHs in AdS: fluid lumps
- BHs in gravitational duals to confining theories: AdS with “infrared bottom”
- BH is dual to plasma ball: with surface boundary
- “Analogy” upgraded to precise “duality”
  – although, actually, no independent construction of gravitational side
  – fluid “prediction” for GL instab of black branes

Aharony+Minwalla+Wiseman
Fluid/gravity correspondence(s)

- A spin-off of AdS/CFT, but conceptually independent of it
- Long-wavelength fluctuations of AdS black brane captured by effective fluid at asymptotic timelike boundary
- Relativistic conformal fluid
Fluid/gravity correspondence(s)

- Stress-tensor is again quasilocal Brown-York
  
  \[ 8\pi \, G \, T_{\mu \nu} = K \, h_{\mu \nu} - K_{\mu \nu} \]

- Full spacetime reconstructed from fluid dyn solution + horizon regularity

- Fluid not exact:
  derivative expansion

- Conformal fluid is stable
Fluid/gravity correspondence(s)

Blackfold approach:

- long-wavelength dynamics of black holes, incl. vacuum, charged etc
- black hole is locally like a black brane

- Extrinsic dynamics: elastic
- Intrinsic dynamics: fluid
General Classical Brane Dynamics

• Dynamics of a fluid on a dynamical worldvolume

• Fluid-dynamics: long-wavelength deviations from thermodynamic equilibrium

• Generalized geodesic eqn (Carter): elastic deformations at long wavelengths
Blackfold fluid

• Effective stress tensor $T_{\mu \nu}$
  – Quasilocal Brown-York, measured in AF region of planar black brane

• Leading-order ‘black brane effective fluid’:

\[
T_{\mu \nu} = (\varepsilon + P) u_\mu u_\nu + Ph_{\mu \nu}
\]

\[
\varepsilon = (n+1) r_0^n
\]

\[
P = -r_0^n
\]

• (viscosity at next order also computed)
Blackfold effective theory

\[ \nabla_\mu T^{\mu\nu} = 0 \rightarrow \text{Blackfold equations} \]

for: \( u_\mu \) (wv boost), \( r_0 \) (thickness), \( K^\rho(X^\mu) \) (embedding)

Coupled elastic and fluid dynamics:

\[ \dot{u}^\mu + \frac{1}{n+1} u^\mu \nabla_\nu u^\nu = \frac{1}{n} K^\mu + \nabla^\mu \ln r_0 \]
Blackfold effective theory

• Can be used for finding new stationary black holes
• Also for studying their dynamics at long-wavelengths
  – much simpler stability analysis
  – can go beyond linear perturbations
• Simplest instability: Gregory-Laflamme-type
Gregory-Laflamme as sound-mode instability

\[ \delta r_0 \sim e^{\Omega t + ikz} \]

Hydrodynamic regime
\[ \Omega \sim 0, \quad k \sim 0 \]

\[ \lambda \gg r_0 \]

threshold zero-mode \( k = k_{GL} \)

\[ \lambda_{GL} \approx r_0 \]
Gregory-Laflamme as sound-mode instability

- Sound waves of “black string fluid”:
  \[ \delta P \rightarrow \delta r_0 \]

- Sound velocity \( c_s^2 = \frac{dP}{d\varepsilon} < 0 \): Unstable

\[ \Omega = \sqrt{-c_s^2} k + O(k^2) = \frac{k}{\sqrt{n+1}} + O(k^2) \]

Almost effortless!
Viscous damping of sound

- Viscosity allows to compute next order

\[ \Omega = \frac{k}{\sqrt{n + 1}} \left(1 - \frac{n + 2}{n\sqrt{n + 1}}kr_0\right) \]

Viscous 1+1 fluid

(\textit{Camps+RE+Haddad})

Numerical Gregory-Laflamme

(\textit{P.Figueras})
Viscous damping of sound

• Agreement is impressively good for large $n$

$$\Omega = \frac{k}{\sqrt{n + 1}} \left(1 - \frac{n + 2}{n\sqrt{n + 1}} kr_0\right)$$

$n=100$

(discrepancy $\leq 1/n$)

$$T = \frac{n}{4\pi r_0}$$

“Large-$n$ limit of black holes = fluid dynamics”
Gregory-Laflamme as sound-mode instability

• This is not:
  – Rayleigh-Plateau instability of fluid tubes
    (no surface boundary)
  – Jeans instability of gravitating fluid
    (no gravity acting on fluid)

• It is:
  – Sound-mode instability
  – Thermodynamic instability: $c_s^2 = s / C_v < 0$
  – Proof of Gubser-Mitra conjecture at long wavelength
Gregory-Laflamme from fluid dynamics

• Huge simplification from full Einstein equations:
  – radial dependence holographically solved
  – linearized analysis becomes algebraic
  – non-linear analysis is 1+1 instead of 2+1

• Easily extended to other branes:
  – leading-order instab only requires eq of state: no perturbations
  – can do charged, rotating... branes
Gregory-Laflamme from fluid dynamics

- Simple, explicit proof of Gubser-Mitra, applicable to generic black branes
- G-M applies only to perturbations that have a hydrodynamic limit: infinitely extended branes
Gregory-Laflamme from fluid dynamics

• Applies to blackfold (brane-like) regime of hi-d black holes
• Includes black holes with compact horizons: MP bhs, black rings etc
Gregory-Laflamme from fluid dynamics

• Can we follow non-linear evolution?
  – Hard-core numerics of Lehner+Pretorius cry out for fluid interpretation/simplification

black string

Lehner+Pretorius

fluid column

Brenner et al
Gregory-Laflamme from fluid dynamics

• Can we follow non-linear evolution?
  – Hard-core numerics of Lehner+Pretorius cry out for fluid interpretation/simplification
  – Not only simplify/reinterpret L+P for vacuum black strings: extend to other black branes with qualitatively different properties

• Capitalize on theory of drop formation
Gregory-Laflamme from fluid dynamics

Captured exactly by effective 1+1 theory (like blackfold description of black string)

self-similar evolution of pinch-down
End-point of instability: A proposal

- Fluid tube breaks when thin-neck dynamics dominated by evaporation
- Black string will break by quantum evaporation effects: breaking controlled by same physics as in endpoint of bh evaporation
- In (weakly coupled) string theory, this happens at "correspondence point" (before Planck scale)
Conclusions

• Several meanings to “black holes are like fluids”

• Fluid/gravity correspondences: fluid from quasilocal stress tensor at timelike boundary, long wavelengths, seems to encompass (almost) all versions

• BHs as fluid lumps: dual “plasma balls”, bh in AdS w/ IR bottom
Conclusions

• Fluid/AdS gravity and blackfolds put fluid at asymptotic boundary
• Membrane paradigm puts it at horizon
• Holographic RG flow interpolates between them
• Fluid/gravity correspondences: very efficient way to study horizon instabilities when bh has brane-like regime