The Casimir effect at finite temperature: inert versus living conductors

**The historical calculation: zero temperature**
(Casimir, 1948)

Electromagnetic field is enclosed in an empty metallic box with perfectly conducting walls

Macroscopic boundary conditions:
vanishing of the tangential component of the electric field

Field and charge fluctuations inside the conductors are ignored:

**inert (or dead) conductor**

Eigenmodes are determined by the Helmotz equation

\[
\nabla^2 E(x, \omega) = -\frac{\omega^2}{c^2} E(x, \omega), \quad \nabla \cdot E(x, \omega) = 0, \quad E_{tg}(x, \omega) = 0, \quad x \in \partial \Lambda
\]

The vacuum energy

\[
\frac{1}{2} \sum_{k, \lambda}' \hbar \omega_k
\]

depends on the plate separation \(d\)

Casimir force (per unit surface)

\[
f_{\text{vac}}(d) = -\frac{\pi^2 \hbar c}{240d^4}.
\]
Temperature dependence

A new length, the thermal wave length of the photon

\[ \alpha = \frac{\beta \pi \hbar c}{d} \]

\( \alpha \gg 1 \) ↔ Low temperature or small distance

\( \alpha \ll 1 \) ↔ High temperature or large distance

Treat the photon modes as thermalized oscillators with free energy

\[ -\beta^{-1} \ln \left( \sum_{n=0}^{\infty} e^{-\beta \hbar \omega_k n} \right) \]

Long distance Casimir force

\[ f_T(d) \approx -\frac{\zeta(3)}{4\pi \beta d^3} \]

Independent of Planck constant and speed of light
Two questions

Why the Casimir force is universal i.e independent of the nature of the conductors?

What is the role of charge and field fluctuations inside the conductors?

A semi microscopic theory

Lifshitz theory of the force between dielectric bodies (1956):

characterizes the physical properties of the dielectrics by their frequency dependent dielectric functions

\[
\begin{array}{c|c|c}
A & y & B \\
\epsilon_A(\omega) & 0 & \epsilon_B(\omega) \\
\hline
& d & \\
\end{array}
\]

\(\circ\) The actual fields are realisations of a stochastic process generated by a random polarization

\[
P(x, \omega) = \overline{P}(x, \omega) + \frac{K(x, \omega)}{\omega 4\pi}
\]

and obey stochastic Maxwell equations.

\(\circ\) The random polarization obeys the fluctuation-dissipation theorem.
The Lifshitz result is $\frac{1}{2}$ smaller than that of the standard calculation with inert conductors.

\[
  f_T(d) \approx -\frac{\zeta(3)}{8\pi\beta d^3}, \quad d \to \infty
\]

**Controversies regarding the temperature dependence of the Casimir force**


He treats the Casimir effect for a scalar field and adds:

"An inserted polarization multiplicity factor of 2 will suffice to produce the electromagnetic result."


"Lifshitz found a temperature dependence which disagrees with that found in other calculations. We show that the error arises only in the limit taken to recover the conductor case."

What should be the order of the limits?

Large separation $\quad d \to \infty$ \quad Perfect conductor $\quad \epsilon \to \infty$
The predictions of the Lifshitz theory are ambiguous in the metallic limit depending on the choice of $\varepsilon(\omega)$.

13 dec. 2005 J. S. Hoye et al:

**Drude model, including dissipation**

\[ \varepsilon(\omega) \sim \frac{4\pi i\sigma}{\omega} \quad f_T(d) \approx -\frac{\zeta(3)}{8\pi\beta d^3}, \quad d \to \infty \]

16 dec 2005: V.M. Mostepanenko et al

**Plasma model, perfect metal, free plasma oscillation**

\[ \varepsilon(\omega) \sim 1 - \frac{\omega_p^2}{\omega^2} \quad f_T(d) \approx \frac{-\zeta(3)}{4\pi\beta d^3}, \quad d \to \infty \]

A full statistical mechanical treatment of the metallic plates is needed

Two companion letters


P.R. Buentzli, Ph.A. Martin, Europhys. Lett. 72, 42 (2005).
First studies:
Conductors are treated as classical Coulomb fluid.
The interaction is purely Coulombic.

At high temperature, matter becomes classical and decouples from the radiation Field (Bohr-van Leeuwen theorem). Only Coulomb interaction matters.

Full statistical quantum mechanical treatment with quantized electromagnetic field
Buenzli and Martin, Phys.Rev. E, 76, 2007:

Result: in all cases \( f_T(d) \approx \frac{\zeta(3)}{8\pi\beta d^3}, \ d \rightarrow \infty \)

Central problem: calculate the charge charge correlation \( C(r_A, r_B) \)
for large separation of the slabs A and B

- Use the technique of screened Mayer bonds.
- In the quantum case use the Feynman-Kac-Ito representation of the thermal state
- Universality follows from the electroneutrality sum rule:

\[
\int dr \sum_\gamma e_\gamma \rho(\gamma, r) \tilde{h}(\gamma r, \gamma' r') = -e_\gamma
\]
Conclusions

The charge and field fluctuations inside the conductors cannot be ignored when the temperature is different from zero.

In the large separation high temperature limit, they cause a reduction by a factor $\frac{1}{2}$ of the Casimir force calculated with macroscopic boundary conditions.

In this limit the Casimir force is of pure electrostatic origin. Transverse degrees of freedom of the electromagnetic field play no role.

Universality is a consequence of **electroneutrality sum rules** in conductors.

Problems

How to extend the analysis in the intermediate temperature range and interpolate with the zero temperature Casimir formula?

What is the role of quantum fluctuations on the Casimir force in the metal at zero temperature?